

N · Statistical distributions · supported focus

AQA · A-Level · Mathematics

Scope and prerequisites

AQA A-level Mathematics 7357, section N (Statistical distributions), version 1.3 (first assessment 2018). Original teaching notes; objective-level mapping lives in `content/teaching/aqa_a_level_mathematics/objective-review.yaml`.

Prerequisites: probability from section M. Each lesson develops its own method before independent transfer.

These are original teaching examples, not official AQA material.

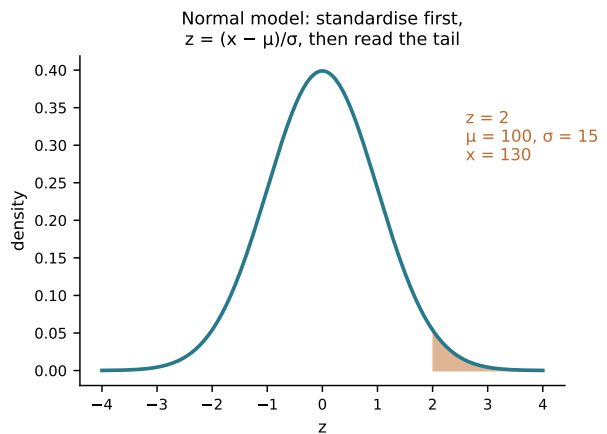
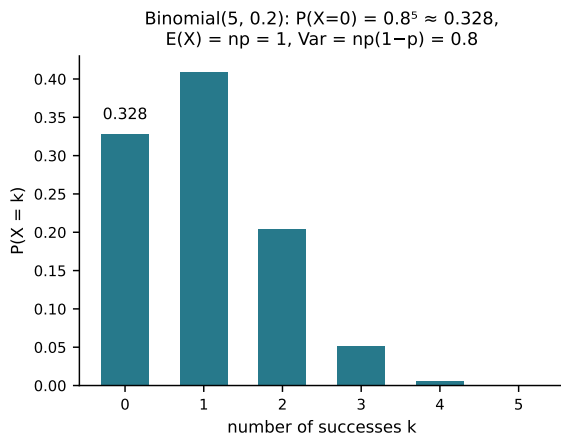
Statistical distributions

■ What would explain this observation?

- A machine fills bottles with a little scatter around 500 ml, while a machine-count of defects is always a whole number. One model uses areas under a smooth curve; the other counts discrete probabilities. Which fits which?
 - Start with a prediction. State what makes a count binomial, and what a continuous model needs.
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■ Build the model

- A **binomial distribution** needs fixed n , independent trials, two outcomes and a constant p : $P(X = k) = C(n,k) p^k(1 - p)^{n-k}$, with $E(X) = np$ and $\text{Var}(X) = np(1 - p)$. The **expected value** is the probability-weighted mean.
- A continuous quantity follows a density curve: probability is AREA, and $P(X = x) = 0$ for any exact value. The **normal distribution** standardises with $z = (x - \mu)/\sigma$ before reading a tail probability. A Poisson model counts events at a constant rate, with mean equal to variance.
- binomial distribution: The count of successes in n independent two-outcome trials with constant probability p ; expected value: The probability-weighted mean of a random variable; normal distribution: The symmetric bell-shaped model for continuous quantities, standardised by $z = (x - \mu)/\sigma$.



■ Choose evidence that can test it

- Check the binomial conditions before using the formula: sampling without replacement makes trials dependent and changes p , so the model fails. Not every count is binomial.
- For a continuous variable, never ask $P(X = \text{exactly } 130)$: compute a tail or an interval. State the event before calculating, and draw the shaded region first.

■ Work from known quantities

- State the known values and their units. Choose the relation because its assumptions fit this case, then rearrange before substitution.
- Known: $X \sim \text{binomial}(5, 0.2)$. $P(X = 0) = 0.8^5 = 0.32768$. $E(X) = np = 1$ and $\text{Var}(X) = np(1 - p) = 0.8$. For a normal quantity with $\mu = 100$, $\sigma = 15$, the value 130 has $z = (130 - 100)/15 = 2$, so it lies two standard deviations above the mean.

Example:

For $X \sim \text{binomial}(5, 0.2)$, find $P(X = 1)$ using the same sequence: choose the model $\rightarrow$ formula $\rightarrow$ substitution. (Answer: $P(X = 1) = C(5,1) \times 0.2 \times 0.8^4 = 5 \times 0.2 \times 0.4096 = 0.4096$.)

■ Check the conclusion and its limits

- Binomial moments follow only when all four conditions hold. A normal model needs roughly symmetric data; a continuity correction matters when approximating a discrete distribution by a normal one.
- Return to the original observation. Explain what the result supports, which conditions it assumes, and one way to test a competing explanation.

Warn:

Every count of successes has a binomial distribution regardless of dependence. This

claim is false: the model requires independent trials with constant p – sampling without replacement or a changing success rate breaks it, so the formula would report a false precision.

Key:

Statistical distributions: binomial for fixed- n independent counts ($E = np$, $\text{Var} = np(1 - p)$); normal for continuous quantities via $z = (x - \mu)/\sigma$; probabilities are areas for continuous models and exact terms for discrete ones.