

Topic 13

AQA · A-Level · Mathematics

Scope and prerequisites

AQA A-level Mathematics 7357, section M (Probability), version 1.3 (first assessment 2018). Original teaching notes; objective-level mapping lives in content/teaching/aqa_a_level_mathematics/objective-review.yaml.

Prerequisites: GCSE probability trees and Venn diagrams. Each lesson develops its own method before independent transfer.

These are original teaching examples, not official AQA material.

Probability

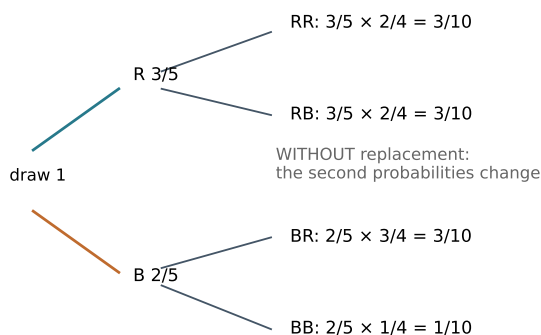
■ What would explain this observation?

- Two counters leave a bag of 3 red and 2 blue, one after another. The second draw's probabilities are different from the first – is that an error or the point?
 - Start with a prediction. State what “without replacement” changes, and how a tree shows it.
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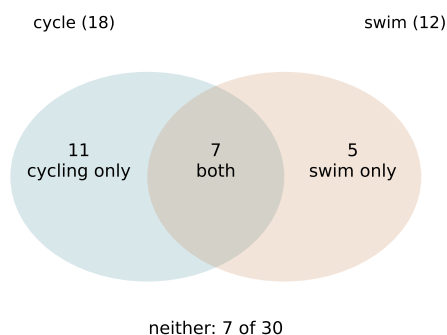
■ Build the model

- Multiply along a tree branch and add disjoint branches for one outcome. **Conditional probability** restricts the sample space: $P(A \text{ given } B) = P(A \cap B)/P(B)$ for $P(B) > 0$. Without replacement, the composition changes for the next draw.
- **Mutually exclusive** means no overlap; **independent** means knowing one event does not change the other's probability. Two disjoint events with positive probability are never independent: if A happens, $P(B)$ drops to 0.
- conditional probability: The probability of an event within the group selected by given information; mutually exclusive: Events that cannot happen together; independent: Events where knowing one leaves the other's probability unchanged.

Multiply along a branch, add the branches of one outcome: $P(\text{one of each}) = 6/10$



Fill the overlap first:
at least one = $11 + 7 + 5 = 23$



■ Choose evidence that can test it

- Fill a Venn diagram from the OVERLAP outward: with 18 cycling, 12 swimming and 7 doing both, the only-regions are 11 and 5, so at least one is 23 and neither is 7 of 30. Adding $18 + 12$ double-counts the 7.
- A conditional probability uses the conditioned group's total as its denominator: $P(\text{swim given cycle}) = 7/18$, not $7/30$. Reversing the condition usually changes the value.

■ Work from known quantities

- State the known values and their units. Choose the relation because its assumptions fit this case, then rearrange before substitution.
- Known: 3 red and 2 blue, two draws without replacement. $P(\text{two red}) = 3/5 \times 2/4 = 3/10$. $P(\text{one of each}) = 3/5 \times 2/4 + 2/5 \times 3/4 = 3/10 + 3/10 = 6/10$. The branches multiply and the disjoint routes add.

Example:

Given $P(A \cap B) = 0.12$ and $P(B) = 0.3$, find $P(A \text{ given } B)$ using the same sequence: identify the conditioned group $\rightarrow$ divide $\rightarrow$ interpret. (Answer: $P(A \text{ given } B) = 0.12/0.3 = 0.4$. The denominator is B's probability, not the whole space.)

■ Check the conclusion and its limits

- Independence and exclusivity answer different questions: disjoint events with positive probability are always dependent. Expected counts (np) are long-run averages, not guarantees for a single run.
- Return to the original observation. Explain what the result supports, which conditions it assumes, and one way to test a competing explanation.

Warn:

Mutually exclusive events with positive probabilities must be independent. This claim is false: if A and B cannot happen together, then $P(B \text{ given } A) = 0 \neq P(B)$ – knowing A happened changes B’s probability completely, the opposite of independence.

Key:

Probability: multiply along branches, add disjoint routes; conditional probability divides by the conditioned group; without replacement the probabilities evolve; disjoint with positive probability means dependent, never independent.