

J · Vectors · supported focus

AQA · A-Level · Mathematics

Scope and prerequisites

AQA A-level Mathematics 7357, section J (Vectors), version 1.3 (first assessment 2018). Original teaching notes; objective-level mapping lives in `content/teaching/aqa_a_level_mathematics/objective-review.yaml`.

Prerequisites: coordinate geometry from section C and Pythagoras. Each lesson develops its own method before independent transfer.

These are original teaching examples, not official AQA material.

Vectors

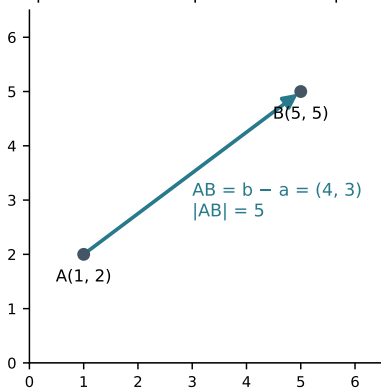
■ What would explain this observation?

- A drone flies from A(1, 2) to B(5, 5), but the mission software asks for the return leg too. Which single formula gives both the outward displacement and its exact reverse?
 - Start with a prediction. State what a displacement vector needs beyond a distance, and how the return leg differs.
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■ Build the model

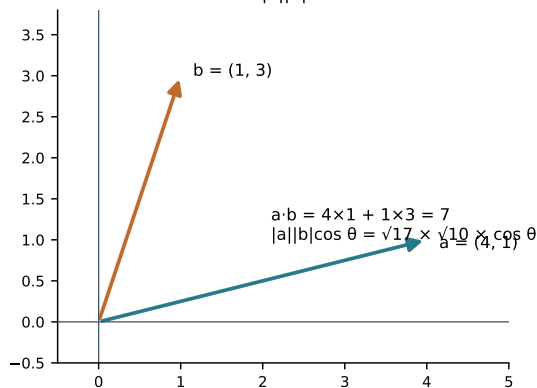
- A **vector** carries magnitude and direction. Position vectors locate points from one origin; the **displacement** from A to B is $\mathbf{AB} = \mathbf{b} - \mathbf{a}$, and $\mathbf{BA} = -\mathbf{AB}$ reverses it. Add and subtract components; a scalar multiple keeps a vector parallel.
- The **scalar product** $\mathbf{a} \cdot \mathbf{b} = a_1b_1 + a_2b_2 = |\mathbf{a}||\mathbf{b}| \cos \theta$ measures the angle: perpendicular vectors have $\mathbf{a} \cdot \mathbf{b} = 0$, and $\mathbf{a} \cdot \mathbf{a} = |\mathbf{a}|^2$. Magnitude comes from Pythagoras: $|(4, 3)| = 5$.
- vector: A quantity with magnitude and direction; displacement: The vector from one point to another, $\mathbf{b} - \mathbf{a}$; scalar product: The sum of component products, equal to $|\mathbf{a}||\mathbf{b}| \cos \theta$.

Displacement = end position – start position



The scalar product measures the angle:

$$a \cdot b = |a||b| \cos \theta$$



■ Choose evidence that can test it

- Keep the order of subtraction: $BA = -AB$, so swapping endpoints reverses every component. A claimed parallel needs a scalar-multiple relation between the vectors; a sketch alone is not a proof.
- For collinearity of three points, show two displacements are parallel AND share a common point; parallel vectors at different locations prove nothing about one line.

■ Work from known quantities

- State the known values and their units. Choose the relation because its assumptions fit this case, then rearrange before substitution.
- Known: $a = (4, 1)$ and $b = (1, 3)$. The scalar product is $a \cdot b = 4 \times 1 + 1 \times 3 = 7$. The magnitudes are $|a| = \sqrt{17}$ and $|b| = \sqrt{10}$, so $\cos \theta = 7 / (\sqrt{17} \times \sqrt{10}) \approx 0.537$, giving $\theta \approx 57.5^\circ$. Since $a \cdot b \neq 0$, the vectors are not perpendicular.

Example:

With $A(1, 2)$ and $B(5, 5)$, find the displacement AB , its magnitude and the return displacement, using the same sequence. (Answer: $AB = (5, 5) - (1, 2) = (4, 3)$; $|AB| = \sqrt{4^2 + 3^2} = 5$; $BA = (-4, -3)$, the same magnitude in the opposite direction.)

■ Check the conclusion and its limits

- A vector answers “how far AND which way”; a distance is only a scalar. The scalar product sees direction: it is largest when the vectors align, zero when perpendicular, and negative when they oppose.
- Return to the original observation. Explain what the result supports, which conditions it assumes, and one way to test a competing explanation.

Warn:

AB and BA have the same components. This claim is false: $BA = -AB$ reverses every

component – from B back to A is $(-4, -3)$, not $(4, 3)$. Only the magnitudes agree.

Key:

Vectors: displacement = end – start, reversed by the return leg; components add; scalar multiples stay parallel; the scalar product gives the angle, with perpendicularity exactly at $\mathbf{a} \cdot \mathbf{b} = 0$.