G · Differentiation · supported focus

AQA · A-Level · Mathematics

Scope and prerequisites

AQA A-level Mathematics 7357, section G (Differentiation), version 1.3 (first assessment 2018). Original teaching notes; objective-level mapping lives in content/teaching/aqa_a_level_mathematics/objective-review.yaml.

Prerequisites: algebraic manipulation and function notation from section B. Each lesson develops its own method before independent transfer.

These are original teaching examples, not official AQA material.

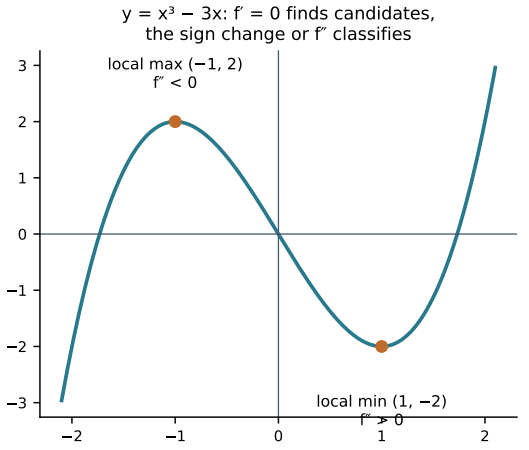
Differentiation

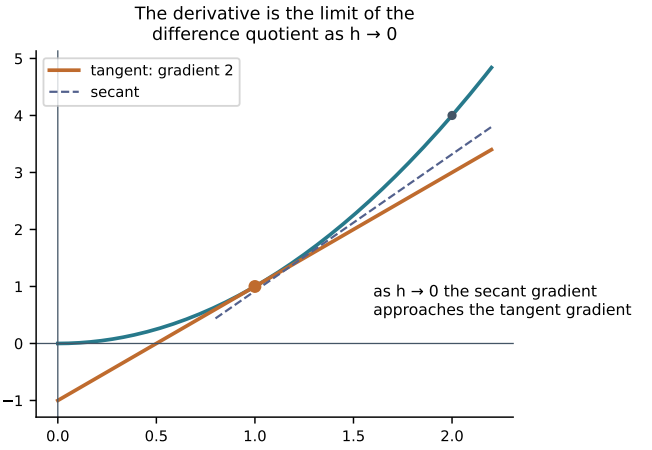
■ What would explain this observation?

- A car's distance-time curve keeps bending, so no single "speed" fits the whole journey – yet the speedometer shows one number now. What does that number mean geometrically?
- Start with a prediction. State how a tangent gradient differs from an average speed between two times.

■ Build the model

- The **derivative** is the limit of the difference quotient $[f(x+h) - f(x)]/h$ as $h \to 0$: the tangent gradient and instantaneous rate. For $y = ax^n$, $dy/dx = anx^{n-1}$; trigonometric and exponential forms follow: $(\sin x)' = \cos x$, $(\cos x)' = -\sin x$, $(e^x)' = e^x$, $(\ln x)' = 1/x$, in radians.
- A **stationary point** has $dy/dx = 0$; classify it by the sign change of the derivative or the sign of the **second derivative** (positive: local minimum; negative: local maximum). The chain, product and quotient rules handle composite forms, with every inner-rate factor retained.
- derivative: The instantaneous rate of change, the tangent gradient; stationary point: A point where the derivative is zero; second derivative: The derivative of the derivative, measuring how the gradient changes.





Choose evidence that can test it

- A zero derivative is only a candidate: $y = x^3$ is stationary at 0 yet keeps rising, so classification needs the sign change or f''. On a restricted interval, compare boundary values too – the global extreme can sit at an endpoint.
- After differentiating a product such as xe^x, check at an easy input: the product rule gives $e^x + xe^x$, which at $x = 0$ is 1, not 0. Multiplying the separate derivatives would give the wrong value there.

Work from known quantities

- State the known values and their units. Choose the relation because its assumptions fit this case, then rearrange before substitution.
- Known: $y = x^3 - 3x$, so $dy/dx = 3x^2 - 3$. Stationary where $3x^2 - 3 = 0$: $x = \pm 1$. The second derivative is $6x$: at $x = 1$ it is $6 > 0$, so $(1, -2)$ is a local minimum; at $x = -1$ it is $-6 < 0$, so $(-1, 2)$ is a local maximum.

Example:

For $y = x^3 - 3x$ at $x = 2$, find the tangent and normal equations using the same sequence: point from $f \to$ gradient from $f' \to$ line equations. (Answer: $y = 2$ at $x = 2$ and the gradient is 9, so the tangent is $y - 2 = 9(x - 2)$ and the normal is $y - 2 = -(1/9)(x - 2)$.)

Check the conclusion and its limits

- The derivative supplies the gradient, not the y-coordinate – the point always comes from the original curve. A horizontal tangent (gradient 0) has a vertical normal, stated as $x =$ constant, since the negative reciprocal of zero does not exist.
- Return to the original observation. Explain what the result supports, which conditions it assumes, and one way to test a competing explanation.

Warn:

Every point with zero derivative is a local maximum or minimum. This claim is false: $y = x^3$ has $dy/dx = 0$ at $x = 0$ but the derivative $3x^2$ stays positive on both sides – a stationary point of inflection, with no extremum at all.

Key:

Differentiation: the derivative is the limit of the secant gradient, giving the tangent gradient and rate. Stationary candidates come from $f' = 0$ and are classified by the sign change or f''; tangents and normals use the point from f and the gradient from f'.