F · Exponentials and logarithms · supported focus

AQA · A-Level · Mathematics

Scope and prerequisites

AQA A-level Mathematics 7357, section F (Exponentials and logarithms), version 1.3 (first assessment 2018). Original teaching notes; objective-level mapping lives in content/teaching/aqa_a_level_mathematics/objective-review.yaml.

Prerequisites: index laws and scientific calculators from GCSE. Each lesson develops its own method before independent transfer.

These are original teaching examples, not official AQA material.

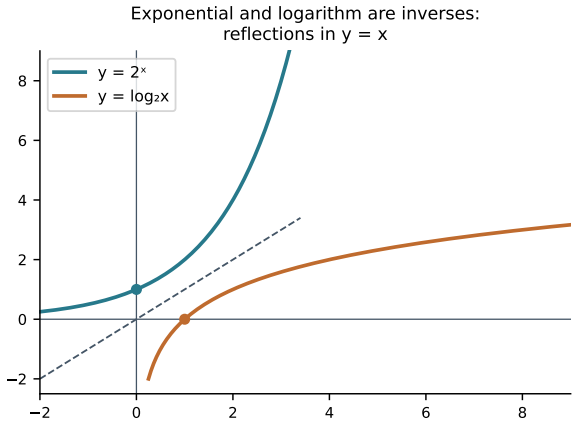
Exponentials and logarithms

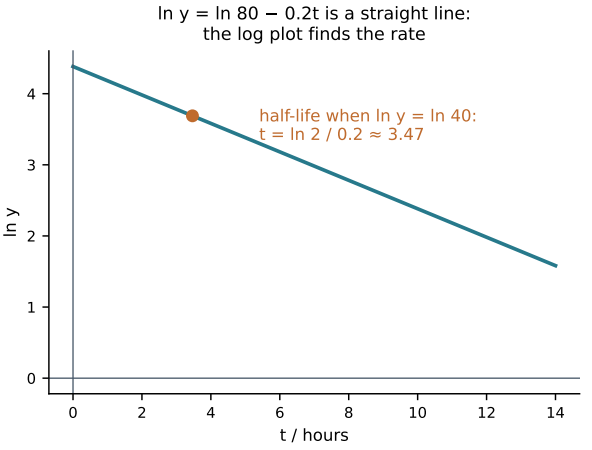
■ What would explain this observation?

- A sample halves every 3.5 hours, but the laboratory needs the level at 5 hours – no table gives it. How does one formula replace the missing table?
- Start with a prediction. State what the model y = Ae^(kt) needs as input, and how a log plot could recover k from data.

■ Build the model

- In y = Ae^(kt), A is the initial amount and k is the proportional rate: positive for growth, negative for decay. The **half-life** is ln 2/|k|; doubling time is ln 2/k.
- The **logarithm** is the inverse of the exponential: y = a^x means x = log_a y, defined only for positive y. The log laws follow from the index laws: log(uv) = log u + log v, log(u/v) = log u − log v, log(u^k) = k log u. Solving a^x = b gives x = ln b / ln a. Plotting ln y against t turns y = Ae^(kt) into the straight line ln y = ln A + kt.
- half-life: The time for a decaying quantity to fall to half its initial value; logarithm: The exponent to which a stated base must be raised to produce a given positive number.





■ Choose evidence that can test it

- A fitted exponential is a model: check its stated time unit and range before predicting, and compare a prediction with a fresh measurement (a residual) before trusting it. A negative k means decay; a negative starting amount usually contradicts the context.
- In log equations, record the original domain first: every argument of every log must be positive, so candidates produced after combining logs must be checked against it.

■ Work from known quantities

- State the known values and their units. Choose the relation because its assumptions fit this case, then rearrange before substitution.
- Known: y = 80e^(−0.2t), y in grams, t in hours. Half the amount means 40 = 80e^(−0.2t), so e^(−0.2t) = 1/2. Taking ln: −0.2t = −ln 2, so t = ln 2/0.2 ≈ 3.466 hours. The same k gives the half-life for any starting amount.

Example:

Solve 3^x = 20 using the same sequence: take logs → apply the power law → divide → interpret. (Answer: x ln 3 = ln 20, so x = ln 20/ln 3 ≈ 2.727. Check: 3^2.727 ≈ 20.)

■ Check the conclusion and its limits

- Log laws apply only to positive arguments; ln(x + y) has NO sum law (ln 2 + ln 3 = ln 6 ≠ ln 5). The exponential model needs its unit of time matched to k; an effective yearly rate of 10% means k = ln 1.1, not k = 0.1.
- Return to the original observation. Explain what the result supports, which conditions it assumes, and one way to test a competing explanation.

Warn:

For positive x and y, $\ln(x + y) = \ln x + \ln y$. This claim is false: the product law is $\ln(xy) = \ln x + \ln y$; there is no law for a sum inside a logarithm, as $x = y = 1$ shows ($\ln 2 \approx 0.69$ but $\ln 1 + \ln 1 = 0$).

Key:

Exponentials and logarithms: $y = Ae^{(kt)}$ with half-life $\ln 2/|k|$; logs invert exponentials, follow the product/quotient/power laws, and turn an exponential fit into a straight line. Check domains and units before solving.