

E · Trigonometry · supported focus

AQA · A-Level · Mathematics

Scope and prerequisites

AQA A-level Mathematics 7357, section E (Trigonometry), version 1.3 (first assessment 2018). Original teaching notes; objective-level mapping lives in `content/teaching/aqa_a_level_mathematics/objective-review.yaml`.

Prerequisites: GCSE right-triangle trigonometry and exact values. Each lesson develops its own method before independent transfer.

These are original teaching examples, not official AQA material.

Trigonometry

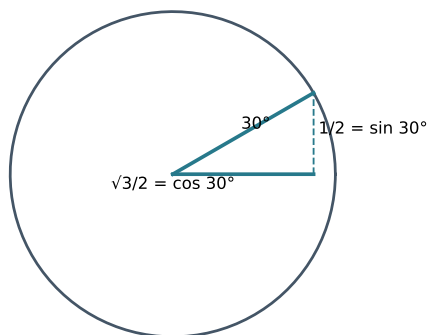
■ What would explain this observation?

- Two students are given $A = 30^\circ$, $a = 7$ and $b = 10$. One reports one triangle; the other insists there are two, and both are right. What information separates the cases?
 - Start with a prediction. State which rule pairs sides with opposite angles, and what extra check an inverse-sine answer needs.
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■ Build the model

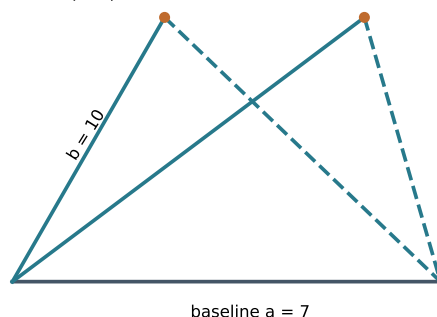
- On the **unit circle**, the point at angle θ is $(\cos \theta, \sin \theta)$, so signs follow the quadrant and exact values come from the 30-60-90 and 45-45-90 triangles. A **radian** is the angle whose arc equals the radius: $s = r\theta$ and sector area $= r^2\theta/2$ require radians.
- The **sine rule** pairs opposite sides and angles: $a/\sin A = b/\sin B$. The **cosine rule** $a^2 = b^2 + c^2 - 2bc \cos A$ uses the angle opposite a . In the **ambiguous case**, $\sin B = b \sin A / a$ can give B or its supplement $180^\circ - B$ – both must be tested against the remaining angle.
- unit circle: The circle of radius 1 on which $(\cos \theta, \sin \theta)$ is the point at angle θ ; radian: The angle subtended by an arc of length equal to the radius; sine rule: The rule $a/\sin A = b/\sin B$ pairing opposite sides and angles; cosine rule: The rule $a^2 = b^2 + c^2 - 2bc \cos A$; ambiguous case: The sine-rule situation where two triangles may fit the same data.

Unit circle: $(\cos \theta, \sin \theta)$ with reference triangles for exact values



Sine-rule ambiguous case:
B and $180^\circ - B$ can both work

two valid apex positions from the same $A = 30^\circ$, $a = 7$, $b = 10$



■ Choose evidence that can test it

- Choose the rule from the known information: a matching side-angle pair calls for the sine rule; two sides with the INCLUDED angle, or three sides, call for the cosine rule. For area, $ab \sin C / 2$ needs the angle BETWEEN a and b .
- Test each candidate angle: the remaining angle $C = 180^\circ - A - B$ must be positive, and every side positive. A calculator's inverse sine returns only the acute principal value – the supplement is a separate candidate.

■ Work from known quantities

- State the known values and their units. Choose the relation because its assumptions fit this case, then rearrange before substitution.
- Known: sides 6 and 8 enclose 60° . Use the cosine rule: $c^2 = 6^2 + 8^2 - 2 \times 6 \times 8 \times \cos 60^\circ = 36 + 64 - 96 \times 0.5 = 52$. So $c = \sqrt{52} = 2\sqrt{13} \approx 7.21$. The area is $(1/2) \times 6 \times 8 \times \sin 60^\circ = 24 \times (\sqrt{3}/2) = 12\sqrt{3}$.

Example:

For $A = 30^\circ$, $a = 7$, $b = 10$, find the possible values of B using the same sequence: known $\rightarrow$ sine rule $\rightarrow$ both branches $\rightarrow$ validity check. (Answer: $\sin B = 10 \times \sin 30^\circ / 7 = 5/7$, so $B \approx 45.6^\circ$ or $B \approx 180^\circ - 45.6^\circ = 134.4^\circ$. Both leave a positive third angle, so TWO triangles fit.)

■ Check the conclusion and its limits

- The ambiguous case needs the count: with $h = b \sin A$ and acute A , $a < h$ gives none, $a = h$ one right triangle, $h < a < b$ gives two, and $a \geq b$ gives one. Trig equations on an interval must list every solution: $\sin x = 1/2$ on $[0, 2\pi)$ has both $\pi/6$ and $5\pi/6$.

- Return to the original observation. Explain what the result supports, which conditions it assumes, and one way to test a competing explanation.

Warn:

The calculator's inverse-sine value is the only possible angle in a sine-rule problem. This claim is false: sine is positive in two quadrants, so the supplement $180^\circ - B$ is a second candidate and must be checked, which is exactly the ambiguous case.

Key:

Trigonometry: unit circle for values and signs, radians for arcs and sectors, sine rule with its ambiguous case, cosine rule for included angles, and $ab \sin C / 2$ for area – always with a validity check on the remaining angle.