

B · Algebra and functions · supported focus

AQA · A-Level · Mathematics

Scope and prerequisites

AQA A-level Mathematics 7357, section B (Algebra and functions), version 1.3 (first assessment 2018). Original teaching notes; objective-level mapping lives in content/teaching/aqa_a_level_mathematics/objective-review.yaml.

Prerequisites: GCSE algebra: indices, surds, factorising and solving quadratics. Each lesson develops its own method before independent transfer.

These are original teaching examples, not official AQA material.

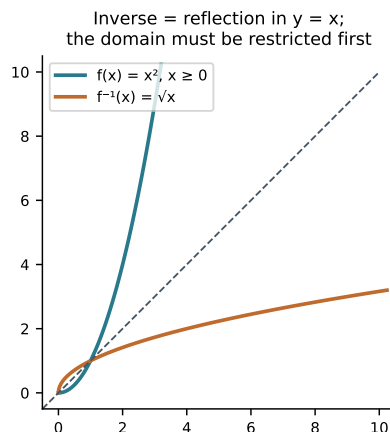
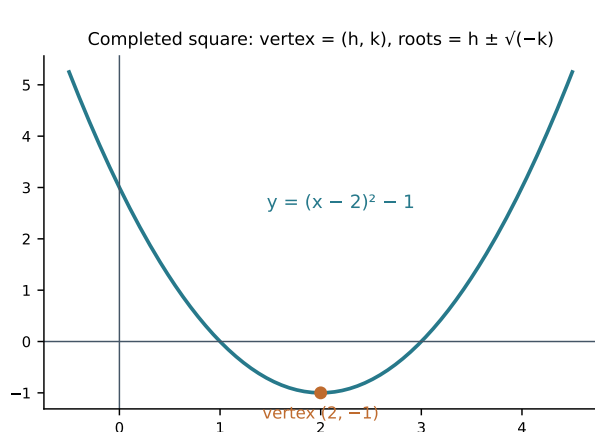
Algebra and functions

■ What would explain this observation?

- Two students solve $x^2 - 4x + 1 = 0$. One completes the square and gets exact answers; the other substitutes into the formula and rounds early, drifting off by a tenth. What separates an exact method from a drifting one?
- Start with a prediction. State what information each solution method needs, and how you would check a final answer.

■ Build the model

- A quadratic can be solved by factorising, **completing the square** $(a(x - h)^2 + k$ giving vertex (h, k)) or the formula. The **discriminant** $b^2 - 4ac$ counts real roots before any solving.
- A **function** assigns each input of its **domain** exactly one output. An inverse exists only if the function is one-to-one on its domain – restrict x^2 to $x \geq 0$ first. Composition $fg(x) = f(g(x))$ applies g first, and fg and gf generally differ. Graph transformations read from the factored form $a \cdot f(b(x - h)) + k$: scale by $1/b$ and a , then shift by h and k .
- completing the square: Rewriting a quadratic as $a(x-h)^2+k$ to read the vertex and solve exactly; discriminant: The quantity b^2-4ac deciding how many real roots exist; function: A rule giving exactly one output for each allowed input; domain: The set of allowed inputs of a function.



■ Choose evidence that can test it

- Check every candidate solution in the ORIGINAL equation, especially after squaring (which can add false roots) or dividing by an expression (which can lose the zero case). Keep surds exact until the requested rounding; rounding mid-calculation is how a tenth is lost.
- For a claimed inverse, verify $f(f^{-1}(x)) = x$ on the inverse's domain, and state that domain explicitly – the inverse of x^2 exists only on a restricted branch.

■ Work from known quantities

- State the known values and their units. Choose the relation because its assumptions fit this case, then rearrange before substitution.
- Known: $x^2 - 4x + 1 = 0$. Complete the square: $(x - 2)^2 - 4 + 1 = 0$, so $(x - 2)^2 = 3$. Rearranged: $x - 2 = \pm\sqrt{3}$. Substitution gives $x = 2 + \sqrt{3} \approx 3.732$ or $x = 2 - \sqrt{3} \approx 0.268$. Check: $(2 - \sqrt{3})^2 - 4(2 - \sqrt{3}) + 1 = 7 - 4\sqrt{3} - 8 + 4\sqrt{3} + 1 = 0 \checkmark$.

Example:

For $f(x) = x^2 - 4$ with domain $x \geq 0$, find the inverse and its domain using the same sequence: known relation $\rightarrow$ rearrange $\rightarrow$ select branch $\rightarrow$ state domain and range. (Answer: from $y = x^2 - 4$, $x = \sqrt{y + 4}$ using the $x \geq 0$ branch, so $f^{-1}(x) = \sqrt{x + 4}$ with domain $x \geq -4$ and range $y \geq 0$.)

■ Check the conclusion and its limits

- The formula always works but exact forms are preferred when the roots are surds; a decimal is an approximation. Composition is order-sensitive, and a transformed graph must be checked by tracking one known point, not just the formula.
- Return to the original observation. Explain what the result supports, which conditions it assumes, and one way to test a competing explanation.

Warn:

The function $f(x) = x^2$ has an inverse over all real numbers. This claim is false: two inputs share each output (2 and -2 both give 4), so the reverse mapping is not a function – the domain must first be restricted, for example to $x \geq 0$, giving $f^{-1}(x) = \sqrt{x}$.

Key:

Algebra and functions: complete the square or use the formula with the discriminant as a root counter; check candidates in the original equation. Functions need stated domains; inverses need one-to-one branches, and graph transformations read from the factored form.