1. Which expression shows that the product of two odd numbers is odd?

   ☐ $(2a+1)(2b+1) = 2(2ab+a+b)+1$

   ☐ $(2a+1)+(2b+1) = 2(a+b+1)$

   ☐ $(2a)(2b) = 4ab$

   ☐ $3 \times 5 = 15$

2. Which set of cases covers every integer?

   ☐ $4k$, $4k+1$, $4k+2$, $4k+3$

   ☐ $3k+1$, $3k+2$

   ☐ $4k$, $4k+2$

   ☐ $2k+1$, $4k+1$

3. Suppose the only primes were 2, 3 and 5. What is Euclid's N?

   _____

4. To prove "there is no largest even number" by contradiction, what do you assume?

   ☐ $N+2$ is odd

   ☐ Every number is even

   ☐ There is a largest even number N

   ☐ There is no largest even number

5. Checking $n=1$, $n=2$ and $n=3$ proves that $n^3 - n$ is always even.

   ☐ True ☐ False

6. What remainder does $N = 2 \times 3 \times 5 \times 7 + 1$ leave on division by 7?

   _____

7. In the proof that $\sqrt{2}$ is irrational, "a^2 is even, so a is even" is a valid step.

   ☐ True ☐ False

8. For $n = 3k+2$, what is n^2 in the form $3m+r$?

   ☐ $3(3k^2+4k+1)+1$

   ☐ $3(3k^2+2k)+2$

   ☐ $9k^2+4$

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9. Euclid's proof shows that $p_1 p_2 \cdots p_k + 1$ is always prime.

   ☐ True ☐ False

10. Claim: n^2+n+1 is prime for every positive integer n. Give the smallest n that is a counterexample.

    _____

1. $(2a+1)(2b+1) = 2(2ab+a+b)+1$

   It has the form $2k+1$ for every a, b.

2. $4k$, $4k+1$, $4k+2$, $4k+3$

   Every remainder on division by 4.

3. **31**

   $2 \times 3 \times 5 + 1$.

4. **There is a largest even number N**

   Assume the opposite of the statement.

5. **False**

   Use $2k$ and $2k+1$ algebraically.

6. **1**

   7 divides the product exactly.

7. **True**

   If a were odd, a^2 would be odd.

8. $3(3k^2+4k+1)+1$

   $9k^2+12k+4$.

9. **False**

   $30031 = 59 \times 509$.

10. **4**

    $n = 4$ gives $21 = 3 \times 7$.

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