1. A · Proof

Starter Quiz ·

AQA · A-Level · Mathematics Name: _____ Score: _____

1. Evaluate n^2+n+41 at $n=41$.

   _____

2. How many remainder cases are needed for division by 3?

   _____

3. If $p=2r$ and $p^2=2q^2$, what coefficient multiplies r^2 in q^2?

   _____

4. For $2a+1$ and $2b+1$ with $a=3, b=4$, find their sum.

   _____

5. What remainder does 8^2 have on division by 3?

   _____

6. Proof by contradiction means a proof that assumes the negation of a claim and derives an impossibility.

   ☐ True ☐ False

7. Checking the first ten integers proves a claim for every positive integer.

   ☐ True ☐ False

8. If a claim works for the first hundred integers, it has been proved for all integers.

   ☐ True ☐ False

9. In the $\sqrt{2}$ proof, why does $p^2=2q^2$ force p to be even?

   ☐ p^2 is even, and an odd square $(2r+1)^2=4r(r+1)+1$ is odd, so p cannot be odd

   ☐ Even numbers have no whole square roots, so p^2 cannot be even

   ☐ Because q must be odd, and an odd q always makes $2q^2$ an odd number

10. Why does $n=41$ disprove "n^2+n+41 is prime for every positive integer n"?

    ☐ Because the first forty cases also failed, so the claim was never true

    ☐ Because 41 is itself prime, so any formula using 41 must fail somewhere

    ☐ It is a single valid case where the value $1763=41\times43$ is not prime

1. A · Proof

Answers ·

AQA · A-Level · Mathematics

1. **1763.0**

   At n=41 the value is $41^2+41+41=41\times43=1763$.

2. **3.0**

   The possible remainders are 0,1,2, giving three cases.

3. **2.0**

   $4r^2=2q^2$, so $q^2=2r^2$.

4. **16.0**

   The two odd integers are 7 and 9, whose sum is 16.

5. **1.0**

   $64=3\times21+1$.

6. **True**

   True: A proof that assumes the negation of a claim and derives an impossibility.

7. **False**

   A pattern can hold for the first ten cases and then fail; a proof must cover every case.

8. **False**

   Checking cases proves a claim only when every case is checked; integers go on forever.

9. **p^2 is even, and an odd square $(2r+1)^2=4r(r+1)+1$ is odd, so p cannot be odd**

   $p^2=2q^2$ makes p^2 even; since odd integers square to odd results, p itself must be even. Writing p=2r then forces $q^2=2r^2$, so q is even too - contradicting coprimality.

10. **It is a single valid case where the value $1763=41\times43$ is not prime**

    A universal claim falls to one counterexample. At n=41 the value is 41×43, divisible by 41, so not prime - no further cases are needed.