

Course Companion

AQA · A-Level · Mathematics

Every topic handout in one volume

exercises.lol

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A · Proof · supported focus

AQA · A-Level · Mathematics

Scope and prerequisites

AQA A-level Mathematics 7357, section A (Proof), version 1.3 (first assessment 2018). Original teaching notes; objective-level mapping lives in `content/teaching/aqa_a_level_mathematics/objective-review.yaml`.

Prerequisites: algebraic manipulation from GCSE. Each lesson develops its own method before independent transfer.

These are original teaching examples, not official AQA material.

Proof

■ What would explain this observation?

- The formula $n^2 + n + 41$ gives prime numbers for $n = 1, 2, 3$, all the way to 39. A student concludes it is always prime. How can one number destroy forty confirmations?
 - Start with a prediction. State what kind of argument could settle a claim about EVERY case, and what could settle one about A SINGLE case.
-

■ Build the model

- A **deductive proof** starts from stated definitions and follows logically: odd integers $2a+1$ and $2b+1$ sum to $2(a+b+1)$, which is even – proved for every pair.
- A **counterexample** refutes an all-values claim with one valid failing case: at $n = 41$, the value is $41 \times 43 = 1763$, not prime. A **proof by contradiction** assumes the negation and derives an impossibility: if $\sqrt{2} = p/q$ in lowest terms, then $p^2 = 2q^2$ forces p even, and writing $p = 2r$ forces q even too – contradicting lowest terms, so $\sqrt{2}$ is irrational.
- deduction: Proving a claim by logical steps from accepted facts; counterexample: A single valid case that disproves a universal claim; contradiction: Assuming the negation and deriving an impossibility.

Three ways to be sure: each has a required structure

Deduction	Counterexample	Contradiction
$2a+1$ and $2b+1$ are odd. Their sum is $2(a+b+1)$, which is even — for EVERY pair.	Claim: n^2+n+41 is always prime. $n=41$ gives $41 \times 43 = 1763$, not prime — claim dead in one case.	Suppose $\sqrt{2} = p/q$ in lowest terms. Then $p^2=2q^2$ forces p and q both even — impossible. So $\sqrt{2}$ is irrational.

Checking many cases is evidence, never a proof — one counterexample kills a universal claim

■ Choose evidence that can test it

- Checking $n = 0, 1, 2, \dots, 39$ is strong evidence but not a proof: the claim concerns EVERY positive integer. A proof must cover all cases at once – by general algebra, by exhaustion of a complete finite list, or by contradiction.
- Before accepting a proof, check its structure: does it start from assumptions rather than from the conclusion? Does the contradiction attack the stated assumption? Checking examples verifies arithmetic but cannot replace the general argument.

■ Work from known quantities

- State the known values and their units. Choose the relation because its assumptions fit this case, then rearrange before substitution.
- Known: the candidate counterexample $n = 41$. Substitution: $n^2 + n + 41 = 41^2 + 41 + 41 = 41(41 + 1 + 1) = 41 \times 43 = 1763$. The value has a factor of 41, so it is not prime – the universal claim fails at exactly one case.

Example:

Test whether the sum of two even numbers is always even using deduction: write the two numbers as $2a$ and $2b$ and show the structure. (Answer: $2a + 2b = 2(a + b)$, which is a multiple of 2, hence even for every pair – no examples needed.)

■ Check the conclusion and its limits

- Each method has a required shape: deduction needs general variables, exhaustion needs a genuinely complete case list, and contradiction needs the impossibility to follow from the negation itself. A proof about integers does not automatically cover real numbers, and an identity proved for all values differs from an equation true only at particular solutions.
- Return to the original observation. Explain what the result supports, which conditions it assumes, and one way to test a competing explanation.

Warn:

Checking a claim for the first hundred integers proves it for every positive integer. This claim is false: a finite sample is evidence, never a proof – $n^2 + n + 41$ survives 40 checks and fails at the 41st, so only a general argument or a counterexample settles a universal claim.

Key:

Proof: deduction works from definitions with general variables; a counterexample kills an all-values claim with one case; contradiction assumes the negation and derives an impossibility. Examples verify and disprove – they never prove a universal.

B · Algebra and functions · supported focus

AQA · A-Level · Mathematics

Scope and prerequisites

AQA A-level Mathematics 7357, section B (Algebra and functions), version 1.3 (first assessment 2018). Original teaching notes; objective-level mapping lives in [content/teaching/aqa_a_level_mathematics/objective-review.yaml](content/teaching/aqa_a_level_mathematics/objective-review.yaml).

Prerequisites: GCSE algebra: indices, surds, factorising and solving quadratics. Each lesson develops its own method before independent transfer.

These are original teaching examples, not official AQA material.

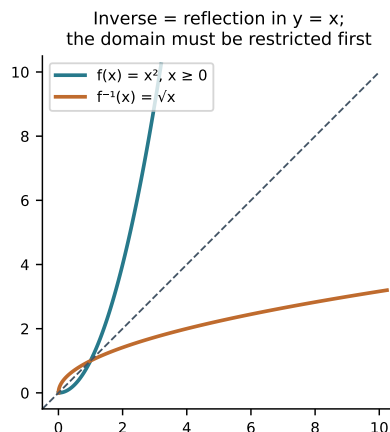
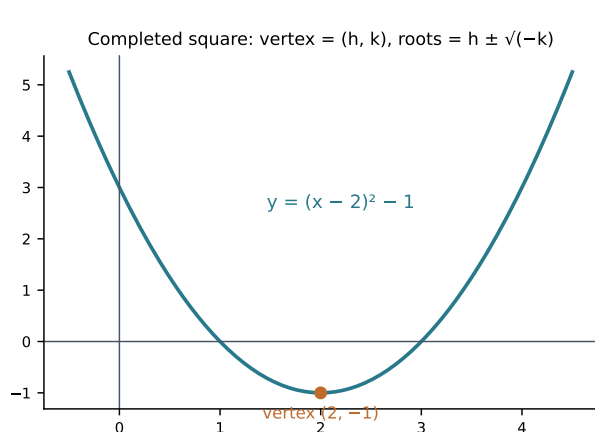
Algebra and functions

■ What would explain this observation?

- Two students solve $x^2 - 4x + 1 = 0$. One completes the square and gets exact answers; the other substitutes into the formula and rounds early, drifting off by a tenth. What separates an exact method from a drifting one?
 - Start with a prediction. State what information each solution method needs, and how you would check a final answer.
-

■ Build the model

- A quadratic can be solved by factorising, **completing the square** $(a(x - h)^2 + k$ giving vertex (h, k)) or the formula. The **discriminant** $b^2 - 4ac$ counts real roots before any solving.
- A **function** assigns each input of its **domain** exactly one output. An inverse exists only if the function is one-to-one on its domain – restrict x^2 to $x \geq 0$ first. Composition $fg(x) = f(g(x))$ applies g first, and fg and gf generally differ. Graph transformations read from the factored form $a \cdot f(b(x - h)) + k$: scale by $1/b$ and a , then shift by h and k .
- completing the square: Rewriting a quadratic as $a(x-h)^2+k$ to read the vertex and solve exactly; discriminant: The quantity b^2-4ac deciding how many real roots exist; function: A rule giving exactly one output for each allowed input; domain: The set of allowed inputs of a function.



■ Choose evidence that can test it

- Check every candidate solution in the ORIGINAL equation, especially after squaring (which can add false roots) or dividing by an expression (which can lose the zero case). Keep surds exact until the requested rounding; rounding mid-calculation is how a tenth is lost.
- For a claimed inverse, verify $f(f^{-1}(x)) = x$ on the inverse's domain, and state that domain explicitly – the inverse of x^2 exists only on a restricted branch.

■ Work from known quantities

- State the known values and their units. Choose the relation because its assumptions fit this case, then rearrange before substitution.
- Known: $x^2 - 4x + 1 = 0$. Complete the square: $(x - 2)^2 - 4 + 1 = 0$, so $(x - 2)^2 = 3$. Rearranged: $x - 2 = \pm\sqrt{3}$. Substitution gives $x = 2 + \sqrt{3} \approx 3.732$ or $x = 2 - \sqrt{3} \approx 0.268$. Check: $(2 - \sqrt{3})^2 - 4(2 - \sqrt{3}) + 1 = 7 - 4\sqrt{3} - 8 + 4\sqrt{3} + 1 = 0 \checkmark$.

Example:

For $f(x) = x^2 - 4$ with domain $x \geq 0$, find the inverse and its domain using the same sequence: known relation $\rightarrow$ rearrange $\rightarrow$ select branch $\rightarrow$ state domain and range. (Answer: from $y = x^2 - 4$, $x = \sqrt{y + 4}$ using the $x \geq 0$ branch, so $f^{-1}(x) = \sqrt{x + 4}$ with domain $x \geq -4$ and range $y \geq 0$.)

■ Check the conclusion and its limits

- The formula always works but exact forms are preferred when the roots are surds; a decimal is an approximation. Composition is order-sensitive, and a transformed graph must be checked by tracking one known point, not just the formula.
- Return to the original observation. Explain what the result supports, which conditions it assumes, and one way to test a competing explanation.

Warn:

The function $f(x) = x^2$ has an inverse over all real numbers. This claim is false: two inputs share each output (2 and -2 both give 4), so the reverse mapping is not a function – the domain must first be restricted, for example to $x \geq 0$, giving $f^{-1}(x) = \sqrt{x}$.

Key:

Algebra and functions: complete the square or use the formula with the discriminant as a root counter; check candidates in the original equation. Functions need stated domains; inverses need one-to-one branches, and graph transformations read from the factored form.

C · Coordinate geometry · supported focus

AQA · A-Level · Mathematics

Scope and prerequisites

AQA A-level Mathematics 7357, section C (Coordinate geometry), version 1.3 (first assessment 2018). Original teaching notes; objective-level mapping lives in [content/teaching/aqa_a_level_mathematics/objective-review.yaml](#).

Prerequisites: GCSE straight lines, Pythagoras and quadratic equations. Each lesson develops its own method before independent transfer.

These are original teaching examples, not official AQA material.

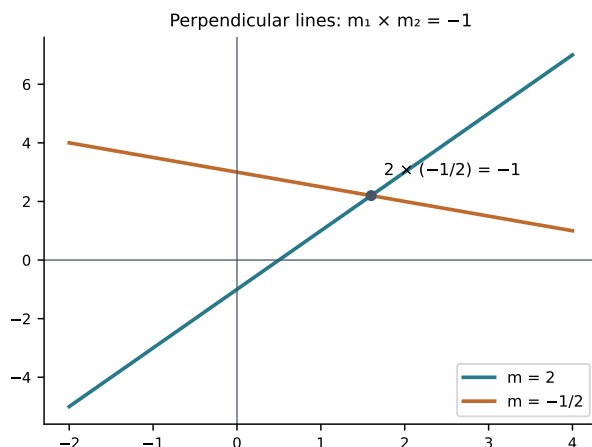
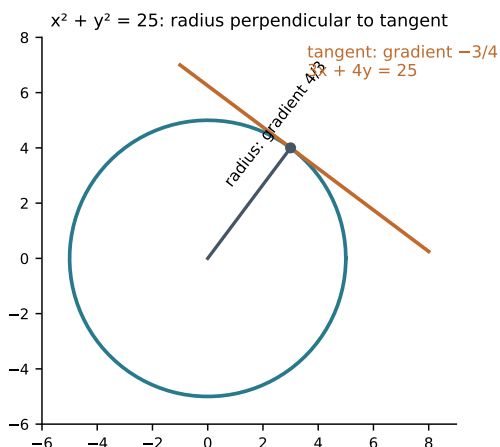
Coordinate geometry

■ What would explain this observation?

- A satellite dish's cross-section and a wheel's rim are both circles, but only the rim can be written $(x - a)^2 + (y - b)^2 = r^2$ with a centre you can read at a glance. What does that equation buy you?
 - Start with a prediction. State what the parameters a , b and r each control, and how a tangent direction could be found without differentiation.
-

■ Build the model

- A line through (x_1, y_1) with **gradient** m is $y - y_1 = m(x - x_1)$. Parallel lines share m ; perpendicular finite gradients satisfy $m_1 \times m_2 = -1$.
- A **circle** with centre (a, b) and radius r is $(x - a)^2 + (y - b)^2 = r^2$. The radius to a contact point is perpendicular to the **tangent** there, so the tangent's gradient is the negative reciprocal of the radius's. A diameter subtends a right angle at the circumference.
- gradient: The change in y per unit change in x ; circle: The set of points at a fixed distance from a centre; tangent: A line touching the circle at one point, perpendicular to the radius there.



■ Choose evidence that can test it

- Check that a proposed point lies on the circle by substituting into $(x - a)^2 + (y - b)^2 = r^2$ before building a tangent at it. Complete the square to rescue a centre from the expanded form $x^2 + y^2 + px + qy + s = 0$ – and adjust the right side too.
- Intersect a line with the circle by substitution: a repeated root means tangency, two roots a chord, and a negative discriminant no meeting.

■ Work from known quantities

- State the known values and their units. Choose the relation because its assumptions fit this case, then rearrange before substitution.
- Known: the circle $x^2 + y^2 = 25$ and the point (3, 4) on it ($3^2 + 4^2 = 25$ ✓). The radius from (0, 0) to (3, 4) has gradient $4/3$, so the tangent gradient is $-3/4$ by $m_1 \times m_2 = -1$. The tangent is $y - 4 = -(3/4)(x - 3)$, which rearranges to $3x + 4y = 25$.

Example:

Find the centre and radius of $x^2 + y^2 - 4x + 2y - 20 = 0$ using the same sequence: group terms → complete each square → read the form. (Answer: $(x - 2)^2 + (y + 1)^2 = 25$, so the centre is (2, -1) and the radius is 5; note the bracket signs reverse in the centre.)

■ Check the conclusion and its limits

- The negative-reciprocal rule needs finite, nonzero gradients; a horizontal radius gives a vertical tangent, stated as $x = \text{constant}$. A chord bisected by a centre-perpendicular is a right-triangle shortcut: half-chord, radius and centre distance obey Pythagoras.

- Return to the original observation. Explain what the result supports, which conditions it assumes, and one way to test a competing explanation.

Warn:

The tangent at a circle point has the same gradient as the radius to that point. This claim is false: the radius and tangent are perpendicular at the contact point, so their gradients multiply to -1 – at $(3, 4)$, the radius has $4/3$ and the tangent has $-3/4$.

Key:

Coordinate geometry: lines by point-gradient, circles by $(x - a)^2 + (y - b)^2 = r^2$. Tangents use the radius-perpendicular relation; intersections use substitution and the discriminant.

D · Sequences and series · supported focus

AQA · A-Level · Mathematics

Scope and prerequisites

AQA A-level Mathematics 7357, section D (Sequences and series), version 1.3 (first assessment 2018). Original teaching notes; objective-level mapping lives in content/teaching/aqa_a_level_mathematics/objective-review.yaml.

Prerequisites: GCSE arithmetic sequences and simultaneous equations. Each lesson develops its own method before independent transfer.

These are original teaching examples, not official AQA material.

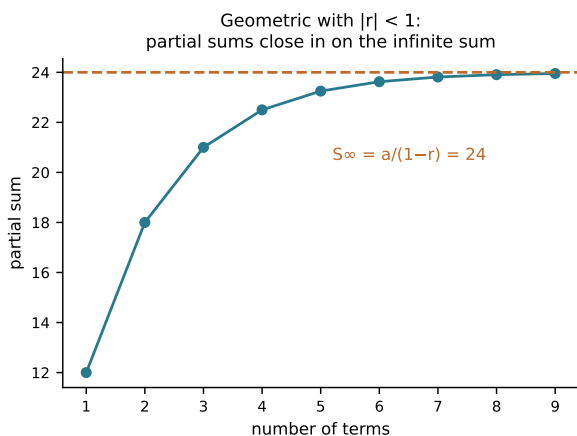
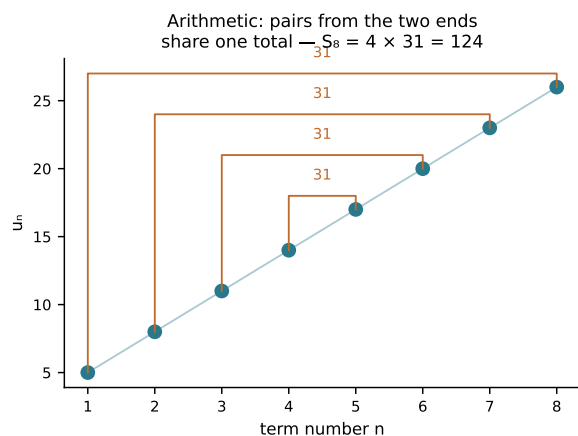
Sequences and series

■ What would explain this observation?

- A saver deposits the same amount every year while an investment grows by the same percentage every year. After eight years, which balance can you total exactly in one line?
 - Start with a prediction. State which of the two grows by addition and which by multiplication, and what each sum formula needs.
-

■ Build the model

- An **arithmetic sequence** adds a constant difference: $u_n = a + (n - 1)d$. Its series sums by pairing ends: $S_n = n(a + u_n)/2$. A **geometric sequence** multiplies by a constant ratio: $u_n = ar^{n-1}$, with $S_n = a(1 - r^n)/(1 - r)$ for $r \neq 1$.
- An infinite geometric sum exists only when $|r| < 1$, because the terms shrink towards zero: $S_\infty = a/(1 - r)$. A **recurrence relation** defines each term from the previous one and needs an initial value; a **periodic sequence** repeats after a fixed number of steps.
- arithmetic sequence: A sequence with a constant difference between terms; geometric sequence: A sequence with a constant ratio between terms; recurrence relation: A rule giving each term from the preceding value; periodic sequence: A sequence whose terms repeat after a fixed number of steps.



■ Choose evidence that can test it

- Recover a and d from two stated terms by simultaneous equations before using a formula: $u_3 = 10$ and $u_7 = 22$ give $4d = 12$, so $d = 3$ and $a = 4$. A term count must come out as a positive integer; a fraction or negative root is rejected, never rounded into place.
- Check a claimed sum against the first few terms directly, and verify an at-least target against the integers on BOTH sides of the crossing.

■ Work from known quantities

- State the known values and their units. Choose the relation because its assumptions fit this case, then rearrange before substitution.
- Known: $a = 5$, $d = 3$, $n = 8$. The eighth term is $u_8 = 5 + 7 \times 3 = 26$. The sum is $S_8 = n(a + u_n)/2 = 8(5 + 26)/2 = 124$. Each of the four end-pairs totals 31, and $4 \times 31 = 124$ ✓.

Example:

For $a = 12$, $r = 0.5$, find the infinite sum and the first n whose partial sum exceeds 23.9, using the same sequence: known $\rightarrow$ formula $\rightarrow$ substitution $\rightarrow$ integer check. (Answer: $S_\infty = 12/(1 - 0.5) = 24$. The partial sums are $S_n = 24(1 - 0.5^n)$: $S_7 = 24(1 - 1/128) = 23.8125 < 23.9$ and $S_8 = 24(1 - 1/256) = 23.90625 \geq 23.9$, so $n = 8$.)

■ Check the conclusion and its limits

- The infinite-sum formula is invalid at $r = 1$ or $r = -1$, where the terms do not shrink; a negative ratio converges if its MAGNITUDE is below 1. A recurrence needs the operation order preserved exactly: $B_{\text{next}} = 1.1B + 50$ differs from $1.1(B + 50)$.

- Return to the original observation. Explain what the result supports, which conditions it assumes, and one way to test a competing explanation.

Warn:

Every geometric series has a finite sum to infinity. This claim is false: the sum exists only when $|r| < 1$. At $r = 2$ the terms grow without bound, and at $r = -1$ the partial sums oscillate forever – neither converges.

Key:

Sequences and series: arithmetic adds (pair the ends to sum), geometric multiplies ($S_n = a(1-r^n)/(1-r)$, and $S_\infty = a/(1-r)$ only for $|r| < 1$). Recover parameters from stated terms, and demand integer answers for counts.

E · Trigonometry · supported focus

AQA · A-Level · Mathematics

Scope and prerequisites

AQA A-level Mathematics 7357, section E (Trigonometry), version 1.3 (first assessment 2018). Original teaching notes; objective-level mapping lives in `content/teaching/aqa_a_level_mathematics/objective-review.yaml`.

Prerequisites: GCSE right-triangle trigonometry and exact values. Each lesson develops its own method before independent transfer.

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Trigonometry

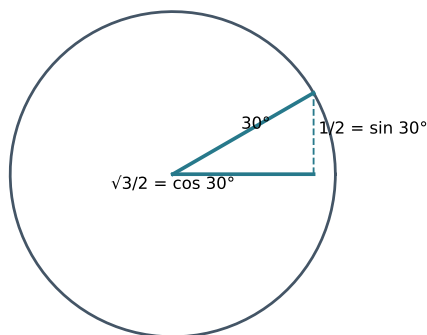
■ What would explain this observation?

- Two students are given $A = 30^\circ$, $a = 7$ and $b = 10$. One reports one triangle; the other insists there are two, and both are right. What information separates the cases?
 - Start with a prediction. State which rule pairs sides with opposite angles, and what extra check an inverse-sine answer needs.
-

■ Build the model

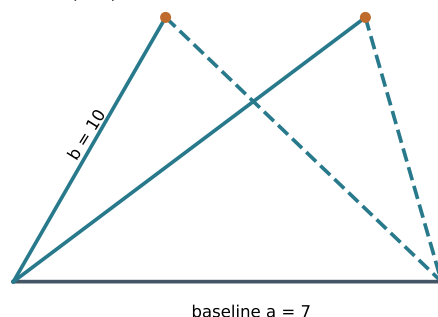
- On the **unit circle**, the point at angle θ is $(\cos \theta, \sin \theta)$, so signs follow the quadrant and exact values come from the 30-60-90 and 45-45-90 triangles. A **radian** is the angle whose arc equals the radius: $s = r\theta$ and sector area $= r^2\theta/2$ require radians.
- The **sine rule** pairs opposite sides and angles: $a/\sin A = b/\sin B$. The **cosine rule** $a^2 = b^2 + c^2 - 2bc \cos A$ uses the angle opposite a . In the **ambiguous case**, $\sin B = b \sin A / a$ can give B or its supplement $180^\circ - B$ – both must be tested against the remaining angle.
- unit circle: The circle of radius 1 on which $(\cos \theta, \sin \theta)$ is the point at angle θ ; radian: The angle subtended by an arc of length equal to the radius; sine rule: The rule $a/\sin A = b/\sin B$ pairing opposite sides and angles; cosine rule: The rule $a^2 = b^2 + c^2 - 2bc \cos A$; ambiguous case: The sine-rule situation where two triangles may fit the same data.

Unit circle: $(\cos \theta, \sin \theta)$ with reference triangles for exact values



Sine-rule ambiguous case:
B and $180^\circ - B$ can both work

two valid apex positions from the same $A = 30^\circ$, $a = 7$, $b = 10$



■ Choose evidence that can test it

- Choose the rule from the known information: a matching side-angle pair calls for the sine rule; two sides with the INCLUDED angle, or three sides, call for the cosine rule. For area, $ab \sin C / 2$ needs the angle BETWEEN a and b .
- Test each candidate angle: the remaining angle $C = 180^\circ - A - B$ must be positive, and every side positive. A calculator's inverse sine returns only the acute principal value – the supplement is a separate candidate.

■ Work from known quantities

- State the known values and their units. Choose the relation because its assumptions fit this case, then rearrange before substitution.
- Known: sides 6 and 8 enclose 60° . Use the cosine rule: $c^2 = 6^2 + 8^2 - 2 \times 6 \times 8 \times \cos 60^\circ = 36 + 64 - 96 \times 0.5 = 52$. So $c = \sqrt{52} = 2\sqrt{13} \approx 7.21$. The area is $(1/2) \times 6 \times 8 \times \sin 60^\circ = 24 \times (\sqrt{3}/2) = 12\sqrt{3}$.

Example:

For $A = 30^\circ$, $a = 7$, $b = 10$, find the possible values of B using the same sequence: known $\rightarrow$ sine rule $\rightarrow$ both branches $\rightarrow$ validity check. (Answer: $\sin B = 10 \times \sin 30^\circ / 7 = 5/7$, so $B \approx 45.6^\circ$ or $B \approx 180^\circ - 45.6^\circ = 134.4^\circ$. Both leave a positive third angle, so TWO triangles fit.)

■ Check the conclusion and its limits

- The ambiguous case needs the count: with $h = b \sin A$ and acute A , $a < h$ gives none, $a = h$ one right triangle, $h < a < b$ gives two, and $a \geq b$ gives one. Trig equations on an interval must list every solution: $\sin x = 1/2$ on $[0, 2\pi)$ has both $\pi/6$ and $5\pi/6$.

- Return to the original observation. Explain what the result supports, which conditions it assumes, and one way to test a competing explanation.

Warn:

The calculator's inverse-sine value is the only possible angle in a sine-rule problem. This claim is false: sine is positive in two quadrants, so the supplement $180^\circ - B$ is a second candidate and must be checked, which is exactly the ambiguous case.

Key:

Trigonometry: unit circle for values and signs, radians for arcs and sectors, sine rule with its ambiguous case, cosine rule for included angles, and $ab \sin C / 2$ for area – always with a validity check on the remaining angle.

F · Exponentials and logarithms · supported focus

AQA · A-Level · Mathematics

Scope and prerequisites

AQA A-level Mathematics 7357, section F (Exponentials and logarithms), version 1.3 (first assessment 2018). Original teaching notes; objective-level mapping lives in [content/teaching/aqa_a_level_mathematics/objective-review.yaml](https://www.aqa.org.uk/content/teaching/aqa_a_level_mathematics/objective-review.yaml).

Prerequisites: index laws and scientific calculators from GCSE. Each lesson develops its own method before independent transfer.

These are original teaching examples, not official AQA material.

Exponentials and logarithms

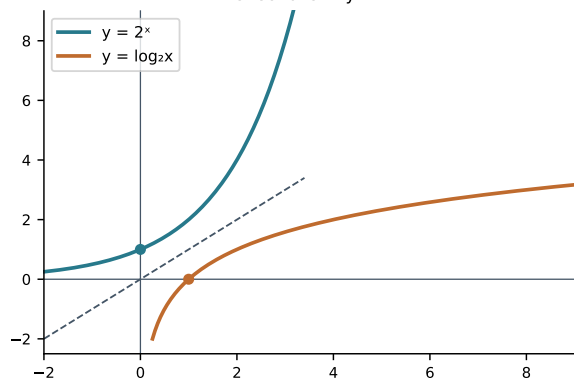
■ What would explain this observation?

- A sample halves every 3.5 hours, but the laboratory needs the level at 5 hours – no table gives it. How does one formula replace the missing table?
 - Start with a prediction. State what the model $y = Ae^{kt}$ needs as input, and how a log plot could recover k from data.
-

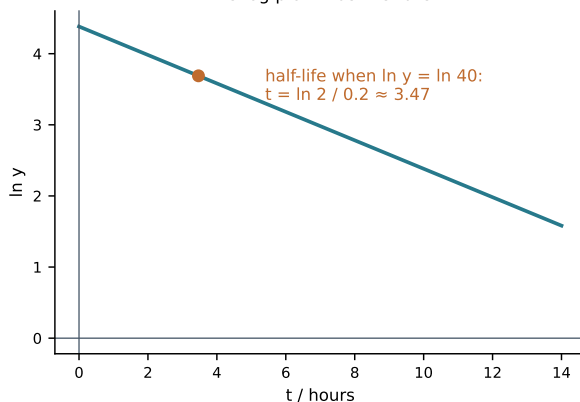
■ Build the model

- In $y = Ae^{kt}$, A is the initial amount and k is the proportional rate: positive for growth, negative for decay. The **half-life** is $\ln 2/|k|$; doubling time is $\ln 2/k$.
- The **logarithm** is the inverse of the exponential: $y = a^x$ means $x = \log_a y$, defined only for positive y . The log laws follow from the index laws: $\log(uv) = \log u + \log v$, $\log(u/v) = \log u - \log v$, $\log(u^k) = k \log u$. Solving $a^x = b$ gives $x = \ln b / \ln a$. Plotting $\ln y$ against t turns $y = Ae^{kt}$ into the straight line $\ln y = \ln A + kt$.
- half-life: The time for a decaying quantity to fall to half its initial value; logarithm: The exponent to which a stated base must be raised to produce a given positive number.

Exponential and logarithm are inverses:
reflections in $y = x$



$\ln y = \ln 80 - 0.2t$ is a straight line:
the log plot finds the rate



■ Choose evidence that can test it

- A fitted exponential is a model: check its stated time unit and range before predicting, and compare a prediction with a fresh measurement (a residual) before trusting it. A negative k means decay; a negative starting amount usually contradicts the context.
- In log equations, record the original domain first: every argument of every log must be positive, so candidates produced after combining logs must be checked against it.

■ Work from known quantities

- State the known values and their units. Choose the relation because its assumptions fit this case, then rearrange before substitution.
- Known: $y = 80e^{(-0.2t)}$, y in grams, t in hours. Half the amount means $40 = 80e^{(-0.2t)}$, so $e^{(-0.2t)} = 1/2$. Taking $\ln$: $-0.2t = -\ln 2$, so $t = \ln 2 / 0.2 \approx 3.466$ hours. The same k gives the half-life for any starting amount.

Example:

Solve $3^x = 20$ using the same sequence: take logs $\rightarrow$ apply the power law $\rightarrow$ divide $\rightarrow$ interpret. (Answer: $x \ln 3 = \ln 20$, so $x = \ln 20 / \ln 3 \approx 2.727$. Check: $3^{2.727} \approx 20$.)

■ Check the conclusion and its limits

- Log laws apply only to positive arguments; $\ln(x + y)$ has NO sum law ($\ln 2 + \ln 3 = \ln 6 \neq \ln 5$). The exponential model needs its unit of time matched to k ; an effective yearly rate of 10% means $k = \ln 1.1$, not $k = 0.1$.
- Return to the original observation. Explain what the result supports, which conditions it assumes, and one way to test a competing explanation.

Warn:

For positive x and y , $\ln(x + y) = \ln x + \ln y$. This claim is false: the product law is $\ln(xy) = \ln x + \ln y$; there is no law for a sum inside a logarithm, as $x = y = 1$ shows ($\ln 2 \approx 0.69$ but $\ln 1 + \ln 1 = 0$).

Key:

Exponentials and logarithms: $y = Ae^{kt}$ with half-life $\ln 2/|k|$; logs invert exponentials, follow the product/quotient/power laws, and turn an exponential fit into a straight line. Check domains and units before solving.

G · Differentiation · supported focus

AQA · A-Level · Mathematics

Scope and prerequisites

AQA A-level Mathematics 7357, section G (Differentiation), version 1.3 (first assessment 2018). Original teaching notes; objective-level mapping lives in `content/teaching/aqa_a_level_mathematics/objective-review.yaml`.

Prerequisites: algebraic manipulation and function notation from section B. Each lesson develops its own method before independent transfer.

These are original teaching examples, not official AQA material.

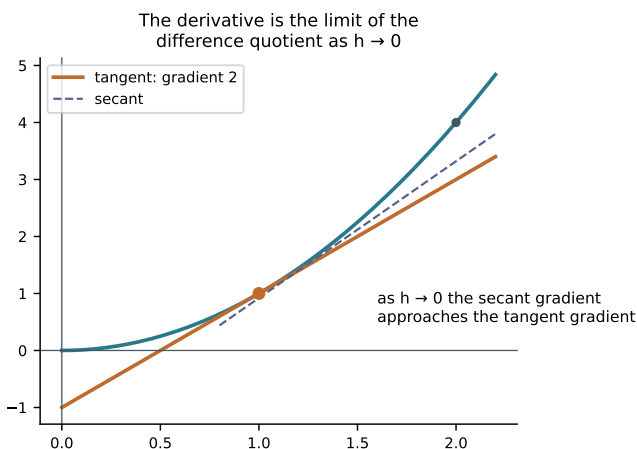
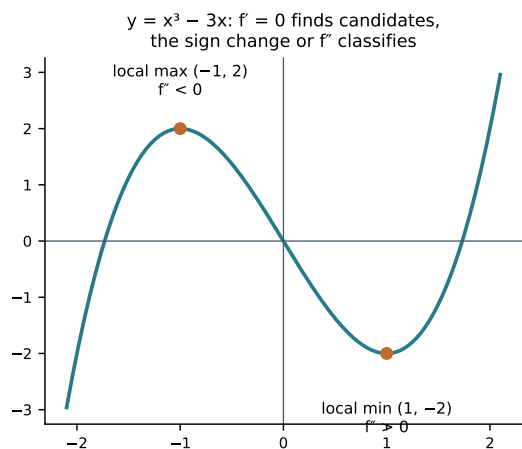
Differentiation

■ What would explain this observation?

- A car's distance-time curve keeps bending, so no single "speed" fits the whole journey – yet the speedometer shows one number now. What does that number mean geometrically?
 - Start with a prediction. State how a tangent gradient differs from an average speed between two times.
-

■ Build the model

- The **derivative** is the limit of the difference quotient $[f(x+h) - f(x)]/h$ as $h \rightarrow 0$: the tangent gradient and instantaneous rate. For $y = ax^n$, $dy/dx = anx^{n-1}$; trigonometric and exponential forms follow: $(\sin x)' = \cos x$, $(\cos x)' = -\sin x$, $(e^x)' = e^x$, $(\ln x)' = 1/x$, in radians.
- A **stationary point** has $dy/dx = 0$; classify it by the sign change of the derivative or the sign of the **second derivative** (positive: local minimum; negative: local maximum). The chain, product and quotient rules handle composite forms, with every inner-rate factor retained.
- derivative: The instantaneous rate of change, the tangent gradient; stationary point: A point where the derivative is zero; second derivative: The derivative of the derivative, measuring how the gradient changes.



■ Choose evidence that can test it

- A zero derivative is only a candidate: $y = x^3$ is stationary at 0 yet keeps rising, so classification needs the sign change or f'' . On a restricted interval, compare boundary values too – the global extreme can sit at an endpoint.
- After differentiating a product such as xe^x , check at an easy input: the product rule gives $e^x + xe^x$, which at $x = 0$ is 1, not 0. Multiplying the separate derivatives would give the wrong value there.

■ Work from known quantities

- State the known values and their units. Choose the relation because its assumptions fit this case, then rearrange before substitution.
- Known: $y = x^3 - 3x$, so $dy/dx = 3x^2 - 3$. Stationary where $3x^2 - 3 = 0$: $x = \pm 1$. The second derivative is $6x$: at $x = 1$ it is $6 > 0$, so $(1, -2)$ is a local minimum; at $x = -1$ it is $-6 < 0$, so $(-1, 2)$ is a local maximum.

Example:

For $y = x^3 - 3x$ at $x = 2$, find the tangent and normal equations using the same sequence: point from $f \rightarrow$ gradient from $f' \rightarrow$ line equations. (Answer: $y = 2$ at $x = 2$ and the gradient is 9, so the tangent is $y - 2 = 9(x - 2)$ and the normal is $y - 2 = -(1/9)(x - 2)$.)

■ Check the conclusion and its limits

- The derivative supplies the gradient, not the y-coordinate – the point always comes from the original curve. A horizontal tangent (gradient 0) has a vertical normal, stated as $x = \text{constant}$, since the negative reciprocal of zero does not exist.
- Return to the original observation. Explain what the result supports, which conditions it assumes, and one way to test a competing explanation.

Warn:

Every point with zero derivative is a local maximum or minimum. This claim is false: $y = x^3$ has $dy/dx = 0$ at $x = 0$ but the derivative $3x^2$ stays positive on both sides – a stationary point of inflection, with no extremum at all.

Key:

Differentiation: the derivative is the limit of the secant gradient, giving the tangent gradient and rate. Stationary candidates come from $f' = 0$ and are classified by the sign change or f'' ; tangents and normals use the point from f and the gradient from f' .

H · Integration · supported focus

AQA · A-Level · Mathematics

Scope and prerequisites

AQA A-level Mathematics 7357, section H (Integration), version 1.3 (first assessment 2018). Original teaching notes; objective-level mapping lives in `content/teaching/aqa_a_level_mathematics/objective-review.yaml`.

Prerequisites: differentiation from section G. Each lesson develops its own method before independent transfer.

These are original teaching examples, not official AQA material.

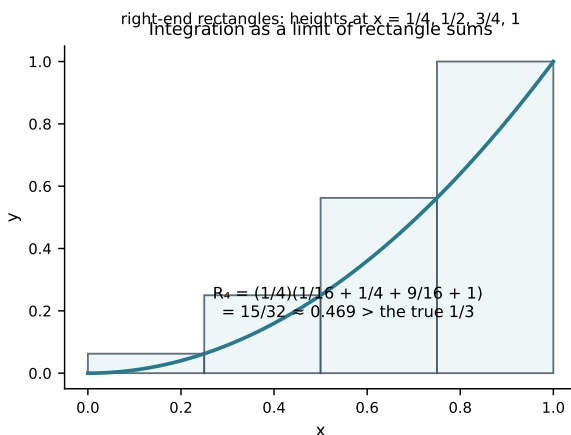
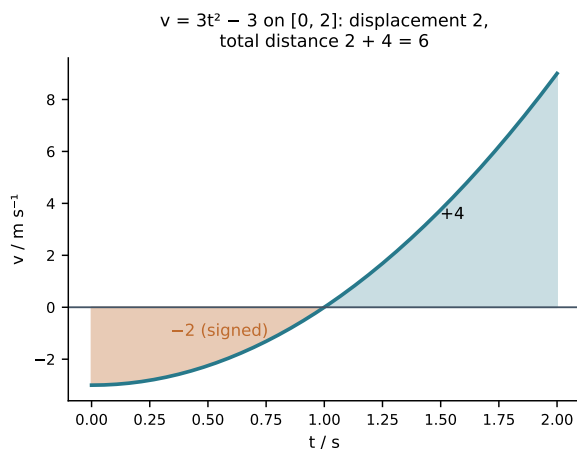
Integration

■ What would explain this observation?

- A runner's velocity graph dips below the time axis for a while, then climbs back. The "area under the curve" is nearly zero – did the runner go nowhere?
 - Start with a prediction. State what a signed area means, and how total distance differs from displacement.
-

■ Build the model

- Integration reverses differentiation: an **antiderivative** of ax^n is $ax^{n+1}/(n+1) + C$ for $n \neq -1$, while $1/x$ integrates to $\ln|x| + C$. A **definite integral** from a to b is signed accumulation: $F(b) - F(a)$.
- Signed area cancels where the curve crosses the axis, so total positive area needs the integral SPLIT at each crossing. The **Fundamental Theorem** connects both directions: differentiating an accumulation function returns the integrand.
- antiderivative: A function whose derivative is the given integrand; definite integral: The signed accumulation of a function between two limits; Fundamental Theorem: The two-way link that differentiating the accumulation function gives back the integrand.



■ Choose evidence that can test it

- Verify every antiderivative by differentiating it back, and keep the constant C for indefinite integrals – a supplied point fixes it. The $n = -1$ case is separate: $1/x$ integrates to $\ln|x|$, never the power rule with division by zero.
- Before computing total area, find the sign changes of the integrand on the interval and split there; a velocity with a sign change makes displacement and total distance different answers.

■ Work from known quantities

- State the known values and their units. Choose the relation because its assumptions fit this case, then rearrange before substitution.
- Known: $v(t) = 3t^2 - 3$ for $0 \leq t \leq 2$. An antiderivative is $t^3 - 3t$. Displacement = $[t^3 - 3t]$ from 0 to 2 = $(8 - 6) - 0 = 2$ m. But $v = 0$ at $t = 1$, so total distance = $-[t^3 - 3t]$ from 0 to 1 + $[t^3 - 3t]$ from 1 to 2 = $2 + 4 = 6$ m.

Example:

Find the area between $y = x^2$ and the x -axis from 0 to 1, then the right-end rectangle estimate with 4 strips, using the same sequence. (Answer: exact $\int_0^1 x^2 dx = 1/3 \approx 0.333$; the estimate is $(1/4)(1/16 + 1/4 + 9/16 + 1) = 15/32 \approx 0.469$, an overestimate for this increasing convex curve.)

■ Check the conclusion and its limits

- Integration is the limit of rectangle sums; a finite strip count only estimates it, and the error depends on curvature. Do not integrate across a discontinuity such as $1/x$ at $x = 0$, and remember signed area can be zero while total area is positive.
- Return to the original observation. Explain what the result supports, which conditions it assumes, and one way to test a competing explanation.

Warn:

A definite integral always equals the total positive area between the curve and the axis. This claim is false: the integral is SIGNED – regions below the axis count negatively, so displacement can be small or zero while the total distance travelled is large.

Key:

Integration: antiderivatives verified by differentiating back, with C fixed by a supplied point; definite integrals are signed accumulation $F(b) - F(a)$; total area needs a split at every sign change, and the Fundamental Theorem links accumulation to the integrand.

I · Numerical methods · supported focus

AQA · A-Level · Mathematics

Scope and prerequisites

AQA A-level Mathematics 7357, section I (Numerical methods), version 1.3 (first assessment 2018). Original teaching notes; objective-level mapping lives in [content/teaching/aqa_a_level_mathematics/objective-review.yaml](#).

Prerequisites: function evaluation and sign analysis from sections B and G. Each lesson develops its own method before independent transfer.

These are original teaching examples, not official AQA material.

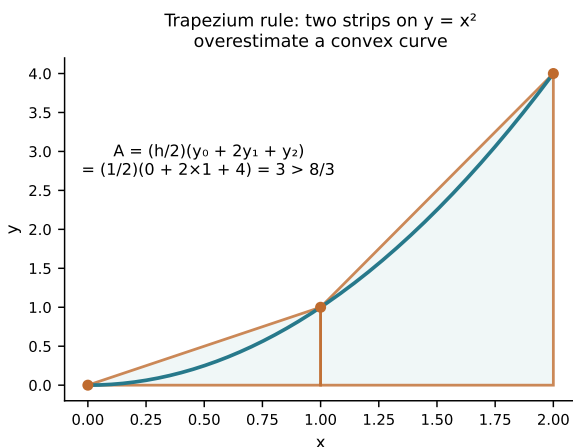
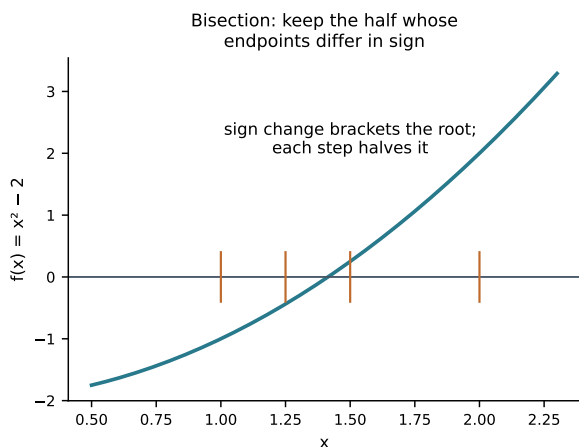
Numerical methods

■ What would explain this observation?

- The equation $x^3 - x - 1 = 0$ has no tidy algebraic solution, yet an engineer needs the root to four decimal places. How can an approximate answer be PROVED correct to a stated accuracy?
 - Start with a prediction. State what a sign change shows, and what could make a decimal answer trustworthy.
-

■ Build the model

- A continuous function with opposite signs at two endpoints has a root between them. **Bisection** halves the interval, keeping the half whose endpoints still differ in sign.
- **Iteration** $x_{\text{next}} = g(x)$ refines an estimate from a rearrangement; a limit must satisfy $L = g(L)$ and be checked in the original equation. **Newton–Raphson** uses $x_{\text{next}} = x - f(x)/f'(x)$ with a nonzero derivative. The **trapezium rule** estimates a definite integral: $(h/2)(y_0 + 2y_1 + \dots + 2y_{n-1} + y_n)$.
- bisection: Halving a root bracket, keeping the half with a sign change; iteration: Repeating an update $x_{\text{next}} = g(x)$ from the previous estimate; Newton–Raphson: The root update $x_{\text{next}} = x - f(x)/f'(x)$; trapezium rule: Estimating an integral by adding trapezium strips.



■ Choose evidence that can test it

- A sign change proves a root only where the function is CONTINUOUS: $1/x$ changes sign across $x = 0$ with no root there. Report a rounded root with its bracket, such as $[1.3245, 1.3255]$ for four decimal places, so the accuracy is provable.
- Iteration can diverge or cycle: check the iterates, the domain and the original equation. The trapezium rule over- or under-estimates depending on curvature, not on whether the function merely increases.

■ Work from known quantities

- State the known values and their units. Choose the relation because its assumptions fit this case, then rearrange before substitution.
- Known: $f(x) = x^2 - 2$ and $x_0 = 1.5$ for Newton–Raphson. $f'(x) = 2x$, so $x_1 = x_0 - f(x_0)/f'(x_0) = 1.5 - (2.25 - 2)/3 = 1.5 - 0.0833... = 1.4166667$. A second step: $f(1.4166667) \approx 0.0069444$, $f' \approx 2.8333333$, so $x_2 \approx 1.4142157$, closing on $\sqrt{2}$.

Example:

Estimate $\int_0^2 x^2 dx$ with two equal trapezium strips and compare with the exact value, using the same sequence. (Answer: $h = 1$, heights 0, 1, 4, so $A = (1/2)(0 + 2 \times 1 + 4) = 3$. The exact value is $8/3 \approx 2.667$ – the convex curve makes the estimate an overestimate.)

■ Check the conclusion and its limits

- Each method has conditions: bisection needs a continuous sign change, Newton needs a nonzero derivative and can fail from a poor start, and a rearranged iteration converges only near a fixed point with $|g'| < 1$. More strips improve a trapezium estimate but never make it exact by assertion.

- Return to the original observation. Explain what the result supports, which conditions it assumes, and one way to test a competing explanation.

Warn:

Every sign change proves a root, including one across a discontinuity. This claim is false: the intermediate-value reasoning needs continuity on the whole interval – across an asymptote the function can jump from negative to positive with no root at all.

Key:

Numerical methods: bracket a root with a continuous sign change and halve with bisection; iterate with checked convergence; Newton–Raphson uses the tangent; the trapezium rule estimates integrals with curvature deciding the error direction.

J · Vectors · supported focus

AQA · A-Level · Mathematics

Scope and prerequisites

AQA A-level Mathematics 7357, section J (Vectors), version 1.3 (first assessment 2018). Original teaching notes; objective-level mapping lives in `content/teaching/aqa_a_level_mathematics/objective-review.yaml`.

Prerequisites: coordinate geometry from section C and Pythagoras. Each lesson develops its own method before independent transfer.

These are original teaching examples, not official AQA material.

Vectors

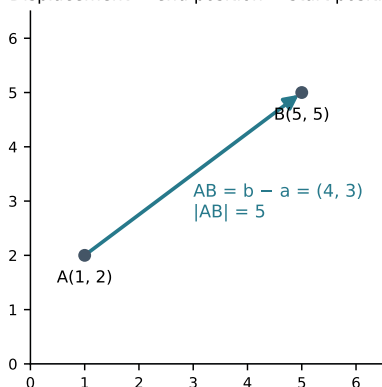
■ What would explain this observation?

- A drone flies from A(1, 2) to B(5, 5), but the mission software asks for the return leg too. Which single formula gives both the outward displacement and its exact reverse?
 - Start with a prediction. State what a displacement vector needs beyond a distance, and how the return leg differs.
-

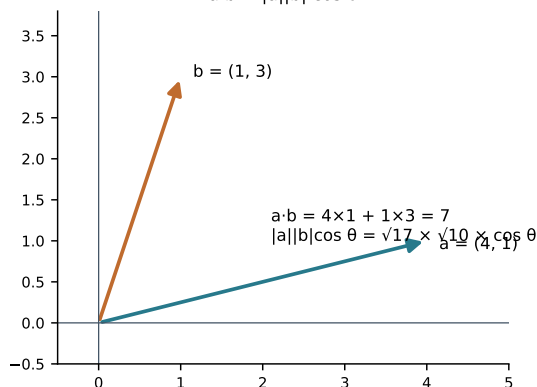
■ Build the model

- A **vector** carries magnitude and direction. Position vectors locate points from one origin; the **displacement** from A to B is $\mathbf{AB} = \mathbf{b} - \mathbf{a}$, and $\mathbf{BA} = -\mathbf{AB}$ reverses it. Add and subtract components; a scalar multiple keeps a vector parallel.
- The **scalar product** $\mathbf{a} \cdot \mathbf{b} = a_1b_1 + a_2b_2 = |\mathbf{a}||\mathbf{b}| \cos \theta$ measures the angle: perpendicular vectors have $\mathbf{a} \cdot \mathbf{b} = 0$, and $\mathbf{a} \cdot \mathbf{a} = |\mathbf{a}|^2$. Magnitude comes from Pythagoras: $|(4, 3)| = 5$.
- vector: A quantity with magnitude and direction; displacement: The vector from one point to another, $\mathbf{b} - \mathbf{a}$; scalar product: The sum of component products, equal to $|\mathbf{a}||\mathbf{b}| \cos \theta$.

Displacement = end position – start position



The scalar product measures the angle:
 $a \cdot b = |a||b| \cos \theta$



■ Choose evidence that can test it

- Keep the order of subtraction: $BA = -AB$, so swapping endpoints reverses every component. A claimed parallel needs a scalar-multiple relation between the vectors; a sketch alone is not a proof.
- For collinearity of three points, show two displacements are parallel AND share a common point; parallel vectors at different locations prove nothing about one line.

■ Work from known quantities

- State the known values and their units. Choose the relation because its assumptions fit this case, then rearrange before substitution.
- Known: $a = (4, 1)$ and $b = (1, 3)$. The scalar product is $a \cdot b = 4 \times 1 + 1 \times 3 = 7$. The magnitudes are $|a| = \sqrt{17}$ and $|b| = \sqrt{10}$, so $\cos \theta = 7/(\sqrt{17} \times \sqrt{10}) \approx 0.537$, giving $\theta \approx 57.5^\circ$. Since $a \cdot b \neq 0$, the vectors are not perpendicular.

Example:

With $A(1, 2)$ and $B(5, 5)$, find the displacement AB , its magnitude and the return displacement, using the same sequence. (Answer: $AB = (5, 5) - (1, 2) = (4, 3)$; $|AB| = \sqrt{4^2 + 3^2} = 5$; $BA = (-4, -3)$, the same magnitude in the opposite direction.)

■ Check the conclusion and its limits

- A vector answers “how far AND which way”; a distance is only a scalar. The scalar product sees direction: it is largest when the vectors align, zero when perpendicular, and negative when they oppose.
- Return to the original observation. Explain what the result supports, which conditions it assumes, and one way to test a competing explanation.

Warn:

AB and BA have the same components. This claim is false: $BA = -AB$ reverses every

component – from B back to A is $(-4, -3)$, not $(4, 3)$. Only the magnitudes agree.

Key:

Vectors: displacement = end – start, reversed by the return leg; components add; scalar multiples stay parallel; the scalar product gives the angle, with perpendicularity exactly at $\mathbf{a} \cdot \mathbf{b} = 0$.

K · Statistical sampling · supported focus

AQA · A-Level · Mathematics

Scope and prerequisites

AQA A-level Mathematics 7357, section K (Statistical sampling), version 1.3 (first assessment 2018). Original teaching notes; objective-level mapping lives in content/teaching/aqa_a_level_mathematics/objective-review.yaml.

Prerequisites: percentages and proportional reasoning from GCSE. Each lesson develops its own method before independent transfer.

These are original teaching examples, not official AQA material.

Statistical sampling

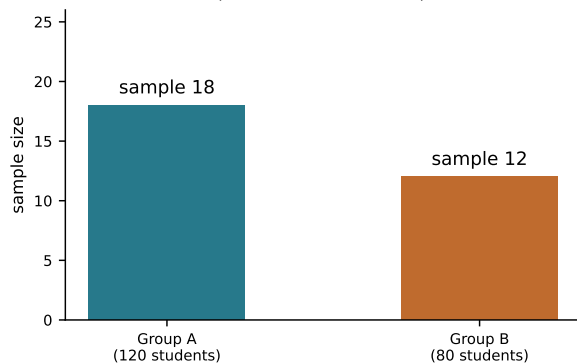
■ What would explain this observation?

- A school with 120 Year 12 students and 80 Year 13 students wants 30 opinions. A committee of 30 all comes from Year 12 “because they were easiest to find.” What is wrong with the arithmetic AND the method?
 - Start with a prediction. State what a representative sample requires, and whether a bigger convenience sample would fix it.
-

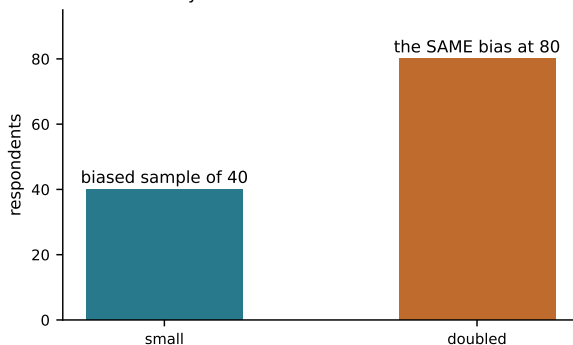
■ Build the model

- A **population** is the whole group the claim is about; a **sample** is the subset measured. A **sampling frame** is the list the sample is drawn from. Random, systematic, stratified, quota and opportunity sampling are different methods with different bias risks.
- A **stratified sample** allocates in proportion to group size, then selects randomly within each group. **Bias** comes from the selection method, not the size: doubling a biased sample repeats the same flaw at scale. Missing data is not zero; check units, dates and variable definitions before comparing.
- population: The complete group about which a statistical claim is made; sample: The subset actually measured; sampling frame: The list or population definition the sample is selected from; stratified sample: A sample allocated in proportion to each group’s size; bias: A systematic distortion from an unfair selection method.

Stratified sample of 30, in proportion:
 $30 \times 120/200 = 18$ and $30 \times 80/200 = 12$



A larger biased sample stays biased:
 only the selection METHOD can fix it



■ Choose evidence that can test it

- Name the population, sampling unit and frame before collecting. Stratification is not “whoever is available from each group” – random selection is still needed within each stratum.
- A large biased sample remains biased: a voluntary online poll over-represents strong opinions however many respond. AQA large-data-set familiarity needs the actual supplied data and its metadata, not invented values.

■ Work from known quantities

- State the known values and their units. Choose the relation because its assumptions fit this case, then rearrange before substitution.
- Known: 120 students in group A and 80 in group B; a proportional stratified sample of 30. Total population = 200. Group A allocation = $30 \times 120/200 = 18$; group B allocation = $30 \times 80/200 = 12$. Each group is then sampled randomly at that size.

Example:

A sample includes 45 of 300 people. Find the sample percentage, and the allocation for a stratified sample of 40 from groups of 150 and 150, using the same sequence. (Answer: $45/300 \times 100 = 15\%$. Equal groups split evenly: 20 and 20.)

■ Check the conclusion and its limits

- A proportion applied to a population gives an ESTIMATE with sampling variability, not an exact count. A census measures everyone; any sample, however large, inherits the method’s selection effects.
- Return to the original observation. Explain what the result supports, which conditions it assumes, and one way to test a competing explanation.

Warn:

Doubling a biased sample automatically removes its selection bias. This claim is false: bias lives in HOW members are chosen, not how many – surveying twice as many volunteers still over-represents people with strong opinions; only changing the method helps.

Key:

Statistical sampling: define the population and frame; stratified sampling allocates proportionally with random selection inside groups; bias comes from the method, not the size, and estimates carry sampling variability.

L · Data presentation and interpretation · supported focus

AQA · A-Level · Mathematics

Scope and prerequisites

AQA A-level Mathematics 7357, section L (Data presentation and interpretation), version 1.3 (first assessment 2018). Original teaching notes; objective-level mapping lives in content/teaching/aqa_a_level_mathematics/objective-review.yaml.

Prerequisites: GCSE averages, frequency tables and box plots. Each lesson develops its own method before independent transfer.

These are original teaching examples, not official AQA material.

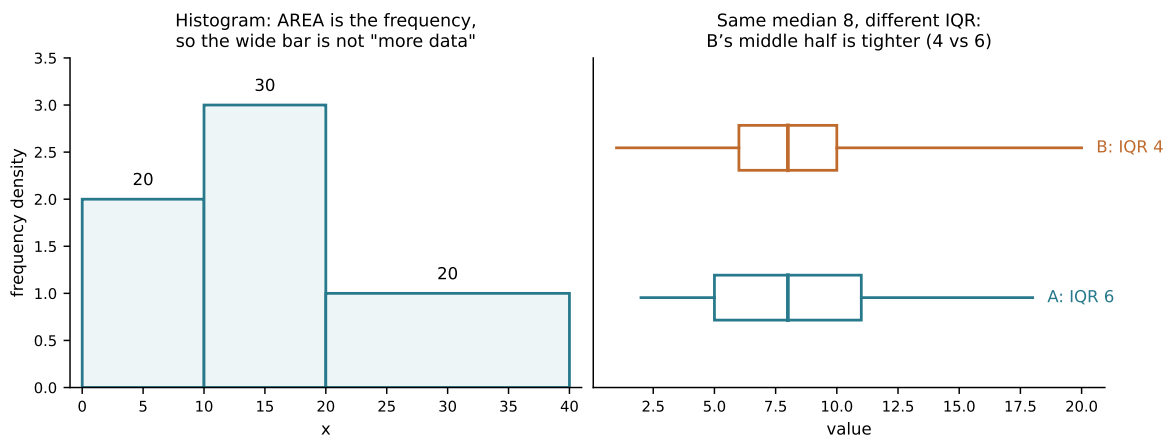
Data presentation and interpretation

■ What would explain this observation?

- The tallest bar of a histogram contains FEWER observations than a shorter, wider one next to it. How can the picture be honest and the reading still be wrong?
 - Start with a prediction. State what the AREA of a histogram bar means, and what height alone tells you.
-

■ Build the model

- A histogram uses AREA for frequency, so with unequal class widths the height is the **frequency density** : $\text{frequency} \div \text{class width}$. Grouped means and medians are estimates – values inside a class are represented by its midpoint or interpolated position.
- A **box plot** shows minimum, quartiles, median and maximum. Compare the median for the typical value and the **interquartile range** ($\text{IQR} = Q_3 - Q_1$) for consistency. An outlier rule such as $1.5 \times \text{IQR}$ beyond the quartiles is a stated convention, not a law.
- frequency density: Frequency divided by class width, used as histogram height; box plot: A display of the five-number summary for comparing distributions; interquartile range: The upper quartile minus the lower quartile, the spread of the middle half.



■ Choose evidence that can test it

- Before comparing displays, read the axis quantities, units and scale: a truncated axis exaggerates, and a grouped mean is an estimate because the true values inside each class are unknown.
- Compare both a centre and a spread in context: equal medians do not mean equal distributions, and the IQR describes only the middle half – a larger full range can still accompany a tighter box.

■ Work from known quantities

- State the known values and their units. Choose the relation because its assumptions fit this case, then rearrange before substitution.
- Known: classes 0–10, 10–20, 20–40 with frequencies 20, 30, 20. Widths are 10, 10, 20, so densities are 2, 3, 1. The last bar is shortest despite 20 observations because its width is double; the areas 20, 30, 20 recover the frequencies exactly.

Example:

For those grouped data, estimate the mean using midpoints, using the same sequence: midpoints $\rightarrow$ weighted sum $\rightarrow$ divide. (Answer: midpoints 5, 15, 30 give estimated sum $5 \times 20 + 15 \times 30 + 30 \times 20 = 1,150$; estimated mean $= 1,150/70 \approx 16.4$. This is an estimate, since each class is represented by its midpoint.)

■ Check the conclusion and its limits

- The tallest bar is not automatically the most frequent: check widths first. A cumulative-frequency graph plots running totals at UPPER class boundaries, and quartiles read from it are interpolation estimates, not exact measurements.
- Return to the original observation. Explain what the result supports, which conditions it assumes, and one way to test a competing explanation.

Warn:

A histogram bar's height always equals its frequency, even with unequal class widths. This claim is false: height is frequency DENSITY; only the area equals the frequency. The wide 20-frequency bar with density 1 holds exactly as much data as the narrow one with density 2.

Key:

Data presentation and interpretation: histogram area carries frequency, grouped statistics are midpoint estimates, box plots compare median and IQR in context, and every comparison starts from the axis units and scale.

M · Probability · supported focus

AQA · A-Level · Mathematics

Scope and prerequisites

AQA A-level Mathematics 7357, section M (Probability), version 1.3 (first assessment 2018). Original teaching notes; objective-level mapping lives in content/teaching/aqa_a_level_mathematics/objective-review.yaml.

Prerequisites: GCSE probability trees and Venn diagrams. Each lesson develops its own method before independent transfer.

These are original teaching examples, not official AQA material.

Probability

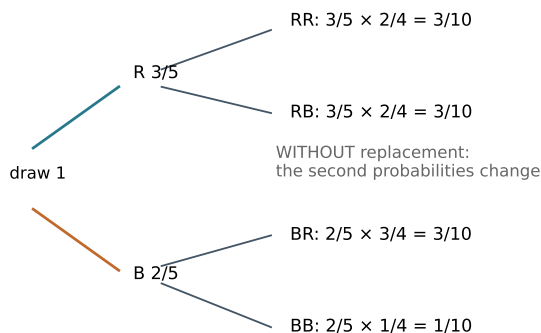
■ What would explain this observation?

- Two counters leave a bag of 3 red and 2 blue, one after another. The second draw's probabilities are different from the first – is that an error or the point?
 - Start with a prediction. State what “without replacement” changes, and how a tree shows it.
-

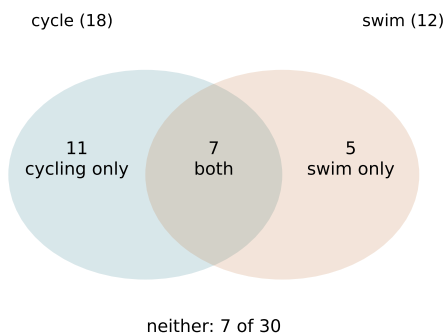
■ Build the model

- Multiply along a tree branch and add disjoint branches for one outcome. **Conditional probability** restricts the sample space: $P(A \text{ given } B) = P(A \cap B)/P(B)$ for $P(B) > 0$. Without replacement, the composition changes for the next draw.
- **Mutually exclusive** means no overlap; **independent** means knowing one event does not change the other's probability. Two disjoint events with positive probability are never independent: if A happens, $P(B)$ drops to 0.
- conditional probability: The probability of an event within the group selected by given information; mutually exclusive: Events that cannot happen together; independent: Events where knowing one leaves the other's probability unchanged.

Multiply along a branch, add the branches of one outcome: $P(\text{one of each}) = 6/10$



Fill the overlap first:
at least one = $11 + 7 + 5 = 23$



■ Choose evidence that can test it

- Fill a Venn diagram from the OVERLAP outward: with 18 cycling, 12 swimming and 7 doing both, the only-regions are 11 and 5, so at least one is 23 and neither is 7 of 30. Adding $18 + 12$ double-counts the 7.
- A conditional probability uses the conditioned group's total as its denominator: $P(\text{swim given cycle}) = 7/18$, not $7/30$. Reversing the condition usually changes the value.

■ Work from known quantities

- State the known values and their units. Choose the relation because its assumptions fit this case, then rearrange before substitution.
- Known: 3 red and 2 blue, two draws without replacement. $P(\text{two red}) = 3/5 \times 2/4 = 3/10$. $P(\text{one of each}) = 3/5 \times 2/4 + 2/5 \times 3/4 = 3/10 + 3/10 = 6/10$. The branches multiply and the disjoint routes add.

Example:

Given $P(A \cap B) = 0.12$ and $P(B) = 0.3$, find $P(A \text{ given } B)$ using the same sequence: identify the conditioned group $\rightarrow$ divide $\rightarrow$ interpret. (Answer: $P(A \text{ given } B) = 0.12/0.3 = 0.4$. The denominator is B's probability, not the whole space.)

■ Check the conclusion and its limits

- Independence and exclusivity answer different questions: disjoint events with positive probability are always dependent. Expected counts (np) are long-run averages, not guarantees for a single run.
- Return to the original observation. Explain what the result supports, which conditions it assumes, and one way to test a competing explanation.

Warn:

Mutually exclusive events with positive probabilities must be independent. This claim is false: if A and B cannot happen together, then $P(B \text{ given } A) = 0 \neq P(B)$ – knowing A happened changes B's probability completely, the opposite of independence.

Key:

Probability: multiply along branches, add disjoint routes; conditional probability divides by the conditioned group; without replacement the probabilities evolve; disjoint with positive probability means dependent, never independent.

N · Statistical distributions · supported focus

AQA · A-Level · Mathematics

Scope and prerequisites

AQA A-level Mathematics 7357, section N (Statistical distributions), version 1.3 (first assessment 2018). Original teaching notes; objective-level mapping lives in `content/teaching/aqa_a_level_mathematics/objective-review.yaml`.

Prerequisites: probability from section M. Each lesson develops its own method before independent transfer.

These are original teaching examples, not official AQA material.

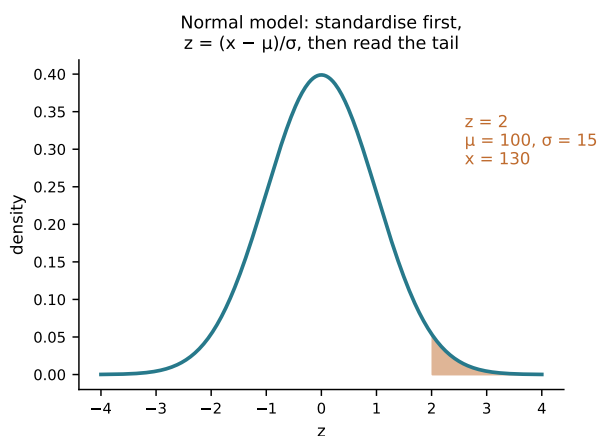
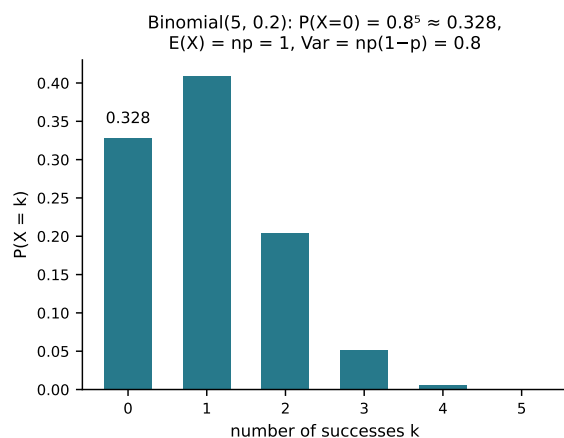
Statistical distributions

■ What would explain this observation?

- A machine fills bottles with a little scatter around 500 ml, while a machine-count of defects is always a whole number. One model uses areas under a smooth curve; the other counts discrete probabilities. Which fits which?
 - Start with a prediction. State what makes a count binomial, and what a continuous model needs.
-

■ Build the model

- A **binomial distribution** needs fixed n , independent trials, two outcomes and a constant p : $P(X = k) = C(n, k) p^k (1 - p)^{n-k}$, with $E(X) = np$ and $\text{Var}(X) = np(1 - p)$. The **expected value** is the probability-weighted mean.
- A continuous quantity follows a density curve: probability is AREA, and $P(X = x) = 0$ for any exact value. The **normal distribution** standardises with $z = (x - \mu)/\sigma$ before reading a tail probability. A Poisson model counts events at a constant rate, with mean equal to variance.
- binomial distribution: The count of successes in n independent two-outcome trials with constant probability p ; expected value: The probability-weighted mean of a random variable; normal distribution: The symmetric bell-shaped model for continuous quantities, standardised by $z = (x - \mu)/\sigma$.



■ Choose evidence that can test it

- Check the binomial conditions before using the formula: sampling without replacement makes trials dependent and changes p , so the model fails. Not every count is binomial.
- For a continuous variable, never ask $P(X = \text{exactly } 130)$: compute a tail or an interval. State the event before calculating, and draw the shaded region first.

■ Work from known quantities

- State the known values and their units. Choose the relation because its assumptions fit this case, then rearrange before substitution.
- Known: $X \sim \text{binomial}(5, 0.2)$. $P(X = 0) = 0.8^5 = 0.32768$. $E(X) = np = 1$ and $\text{Var}(X) = np(1 - p) = 0.8$. For a normal quantity with $\mu = 100$, $\sigma = 15$, the value 130 has $z = (130 - 100)/15 = 2$, so it lies two standard deviations above the mean.

Example:

For $X \sim \text{binomial}(5, 0.2)$, find $P(X = 1)$ using the same sequence: choose the model $\rightarrow$ formula $\rightarrow$ substitution. (Answer: $P(X = 1) = C(5,1) \times 0.2 \times 0.8^4 = 5 \times 0.2 \times 0.4096 = 0.4096$.)

■ Check the conclusion and its limits

- Binomial moments follow only when all four conditions hold. A normal model needs roughly symmetric data; a continuity correction matters when approximating a discrete distribution by a normal one.
- Return to the original observation. Explain what the result supports, which conditions it assumes, and one way to test a competing explanation.

Warn:

Every count of successes has a binomial distribution regardless of dependence. This

claim is false: the model requires independent trials with constant p – sampling without replacement or a changing success rate breaks it, so the formula would report a false precision.

Key:

Statistical distributions: binomial for fixed- n independent counts ($E = np$, $\text{Var} = np(1 - p)$); normal for continuous quantities via $z = (x - \mu)/\sigma$; probabilities are areas for continuous models and exact terms for discrete ones.

O · Statistical hypothesis testing · supported focus

AQA · A-Level · Mathematics

Scope and prerequisites

AQA A-level Mathematics 7357, section O (Statistical hypothesis testing), version 1.3 (first assessment 2018). Original teaching notes; objective-level mapping lives in `content/teaching/aqa_a_level_mathematics/objective-review.yaml`.

Prerequisites: the binomial and normal models from section N. Each lesson develops its own method before independent transfer.

These are original teaching examples, not official AQA material.

Statistical hypothesis testing

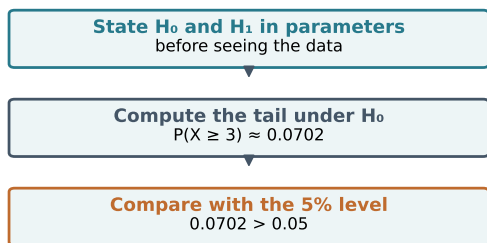
■ What would explain this observation?

- A factory claims a 10% defect rate, and a sample of 10 has 3 defects. A student says “3 defects proves the rate is above 10%.” What is the right way to test that claim?
 - Start with a prediction. State the default claim, the alternative, and what probability the evidence needs to be judged against.
-

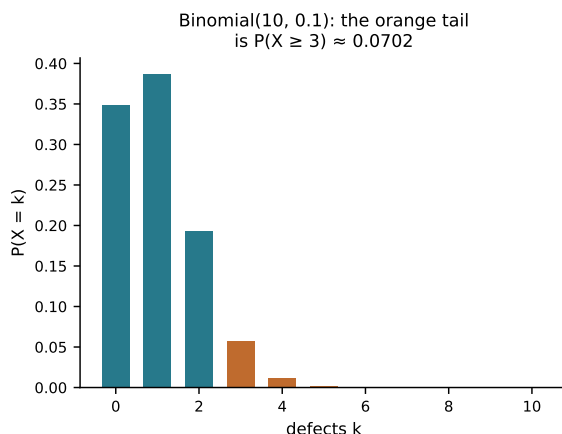
■ Build the model

- State the **null hypothesis** H_0 and the **alternative hypothesis** H_1 in population parameters BEFORE inspecting the outcome. Compute the probability, under H_0 , of a result at least as extreme as the observed one – the tail probability.
- Compare it with the chosen **significance level** (such as 5%). If the tail meets the level, reject H_0 ; otherwise say the evidence is INSUFFICIENT, never that H_0 is proven. Statistical significance says nothing about the practical SIZE of an effect.
- null hypothesis: The default claim about a population parameter, tested against the evidence; alternative hypothesis: The claim accepted if the evidence against H_0 is strong enough; significance level: The chosen probability threshold for rejecting the null hypothesis.

Test the claim by the probability of evidence THIS extreme



insufficient evidence ≠ proof that H_0 is true



■ Choose evidence that can test it

- Choose the tail from H_1 , not from whichever tail gives a small number: $H_1: p > 0.1$ needs $P(X \geq \text{observed})$, an upper tail. Two-tailed tests double the tail when the claim is “different”, not “greater”.
- The hypotheses are about the PARAMETER (the true defect rate), not the sample; and they are fixed before the data are seen, otherwise the test is rigged.

■ Work from known quantities

- State the known values and their units. Choose the relation because its assumptions fit this case, then rearrange before substitution.
- Known: $H_0: p = 0.1$, $H_1: p > 0.1$, $n = 10$, and 3 defects observed. Under H_0 , $X \sim \text{binomial}(10, 0.1)$: $P(X \geq 3) = 1 - (0.9^{10} + 10 \times 0.1 \times 0.9^9 + 45 \times 0.01 \times 0.9^8) \approx 1 - 0.9298 = 0.0702$. At the 5% level, $0.0702 > 0.05$: insufficient evidence that the defect rate exceeds 10%.

Example:

A correct one-tailed p-value is 0.03 at the 5% level. State the conclusion using the same sequence: compare with the level → conclude in context. (Answer: $0.03 < 0.05$, so reject H_0 : the evidence supports the alternative claim at the 5% level.)

■ Check the conclusion and its limits

- Failing to reject H_0 is not proof it is true – the sample may simply be too small to detect a real difference. A 5% level means 1 in 20 innocent claims will be “rejected” by chance over many tests; significance does not measure importance.
- Return to the original observation. Explain what the result supports, which conditions it assumes, and one way to test a competing explanation.

Warn:

Failing to reject a null hypothesis proves it is true. This claim is false: the test only measures whether the evidence against H_0 clears the threshold. Weak evidence leaves H_0 unproven, not proven – “insufficient evidence” and “proven true” are entirely different conclusions.

Key:

Statistical hypothesis testing: state H_0 and H_1 in parameters first, compute the tail probability under H_0 , compare with the pre-set significance level, and conclude in context – never claim the null is proven when it survives.

P · Quantities and units in mechanics · supported focus

AQA · A-Level · Mathematics

Scope and prerequisites

AQA A-level Mathematics 7357, section P (Quantities and units in mechanics), version 1.3 (first assessment 2018). Original teaching notes; objective-level mapping lives in [content/teaching/aqa_a_level_mathematics/objective-review.yaml](#).

Prerequisites: scientific notation and unit conversions from GCSE. Each lesson develops its own method before independent transfer.

These are original teaching examples, not official AQA material.

Quantities and units in mechanics

■ What would explain this observation?

- A report says a cyclist did “300 N of work pedalling 50 metres.” A marker crosses it out without checking any numbers. What did the marker see?
 - Start with a prediction. State what is wrong with that sentence dimensionally, and what units work, force and power each need.
-

■ Build the model

- Every mechanics quantity is built from the base units kilogram, metre and second: velocity is m/s , acceleration is m/s^2 , force is the newton (kg m/s^2), work is the joule (N m) and power is the watt (J/s).
- **Work** is energy transferred by a force through a displacement: $W = Fs$ in the direction of the force. **Power** is the rate of doing work: $P = W/t$. Units are checked on every line – a correct equation gives the same unit on both sides, so a dimensional mismatch proves a slip before any number is trusted.
- work: Energy transferred by a force through a displacement, $W = Fs$, in joules;
power: The rate of energy transfer, $P = W/t$, in watts.

Every mechanics quantity is built from kg, m and s



Check units on every line: $W = Fs$ gives $N \cdot m = J$,
 $F = ma$ gives $kg \times m/s^2 = N$

■ Choose evidence that can test it

- Convert before combining: speeds in km/h must become m/s (divide by 3.6) before meeting forces or energies. A value's unit is part of its meaning; a bare number is not an answer in mechanics.
- Check a formula dimensionally on the DATA line, not just the result: if $F = ma$ is to give newtons, m must be in kilograms and a in m/s^2 .

■ Work from known quantities

- State the known values and their units. Choose the relation because its assumptions fit this case, then rearrange before substitution.
- Known: a cyclist applies 80 N over 50 m in 20 s. $Work = Fs = 80 \times 50 = 4,000 \text{ J}$. $Power = W/t = 4,000/20 = 200 \text{ W}$. The “300 N of work” sentence fails dimensionally: newtons are force, and work is measured in joules.

Example:

A 60 kg student climbs 4 m of stairs in 10 s. Using $g = 9.8 \text{ m/s}^2$, find the work against gravity and the average power, using the same sequence: known $\rightarrow W = mgh \rightarrow P = W/t$. (Answer: $W = 60 \times 9.8 \times 4 = 2,352 \text{ J}$; $P = 2,352/10 \approx 235 \text{ W}$.)

■ Check the conclusion and its limits

- A dimensional check catches wrong formulas but cannot catch a wrong NUMBER inside a right-shaped formula; units pass, then arithmetic must still be verified. “Powerful” is not a technical term – compare watts, not adjectives.
- Return to the original observation. Explain what the result supports, which conditions it assumes, and one way to test a competing explanation.

Warn:

Work, force and power can be quoted in the same units because they all involve force. This claim is false: force is in newtons, work in joules ($N \cdot m$) and power in watts (J/s)

– different dimensions, and confusing them produces nonsense like “300 N of work.”

Key:

Quantities and units in mechanics: everything builds from kg, m, s; work is $W = Fs$ in joules, power is W/t in watts, and a dimensional check on every line catches formula slips before the arithmetic is trusted.

Q · Kinematics · supported focus

AQA · A-Level · Mathematics

Scope and prerequisites

AQA A-level Mathematics 7357, section Q (Kinematics), version 1.3 (first assessment 2018). Original teaching notes; objective-level mapping lives in content/teaching/aqa_a_level_mathematics/objective-review.yaml.

Prerequisites: gradients and areas from GCSE graphs, and the quantities and units of section P. Each lesson develops its own method before independent transfer.

These are original teaching examples, not official AQA material.

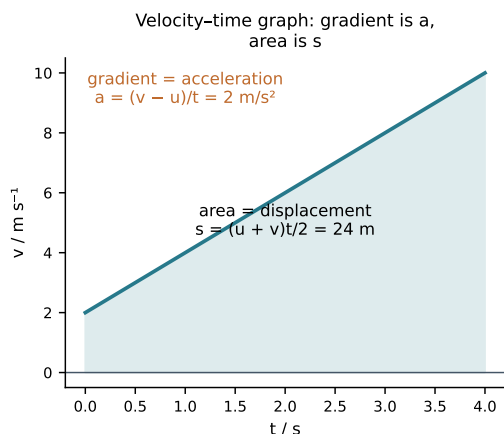
Kinematics

■ What would explain this observation?

- Two cars cover the same 24 metres in the same 4 seconds, yet their speed–time graphs look completely different – one is a sloping line, one is flat. How can both trips be 24 m?
 - Start with a prediction. State what the gradient and the area of a velocity–time graph each measure, and why a flat graph and a sloping graph can enclose the same area.
-

■ Build the model

- **Kinematics** describes motion without asking what causes it. **Displacement** s is position change in metres; velocity v is the rate of change of displacement; **acceleration** a is the rate of change of velocity, in m/s^2 .
- On a velocity–time graph the gradient is the acceleration and the area under the graph is the displacement. When acceleration is constant, five quantities s , u , v , a , t are linked by the suvat equations, each omitting one quantity: $v = u + at$, $s = ut + at^2/2$, $s = (u + v)t/2$, $v^2 = u^2 + 2as$, $s = vt - at^2/2$.
- kinematics: The description of motion through displacement, velocity, acceleration and time, without reference to causes; displacement: Position change in a stated direction, in metres; acceleration: Rate of change of velocity, in m/s^2 .



suvat: each equation omits one quantity — choose by what is missing

$$v = u + at$$

lacks no s

$$s = ut + at^2/2$$

lacks no v

$$v^2 = u^2 + 2as$$

lacks no t

valid only for constant acceleration

■ Choose evidence that can test it

- Choose the suvat equation by what is MISSING, not by what is given: list the four quantities involved in the question, find which of the five is absent, and pick the equation that omits it.
- Vertical motion under gravity is the same model with $a = -g \approx -9.8 \text{ m/s}^2$ when up is positive: a ball thrown up keeps the same downward acceleration on the way up, at the top, and on the way down.

■ Work from known quantities

- State the known values and their units. Choose the relation because its assumptions fit this case, then rearrange before substitution.
- Known: a cyclist starts at $u = 2 \text{ m/s}$ and accelerates at $a = 2 \text{ m/s}^2$ for $t = 4 \text{ s}$. Final velocity $v = u + at = 2 + 2 \times 4 = 10 \text{ m/s}$. Displacement $s = (u + v)t/2 = (2 + 10) \times 4/2 = 24 \text{ m}$ — the trapezium area under the graph, matching the flat-graph car at 6 m/s for 4 s .

Example:

A stone is dropped from rest and falls for 3 s . Using $g = 9.8 \text{ m/s}^2$, find its speed and the distance fallen, using the same sequence: known $\rightarrow v = u + at \rightarrow s = ut + at^2/2$. (Answer: $v = 0 + 9.8 \times 3 = 29.4 \text{ m/s}$; $s = 0 + 9.8 \times 9/2 = 44.1 \text{ m}$.)

■ Check the conclusion and its limits

- Check the answer against the graph: the numerical displacement must equal the area, and the sign of the acceleration must match the slope. A negative displacement means the object finished behind its start, not that it “moved backwards in time.”

- Return to the original observation. Explain what the result supports, which conditions it assumes, and one way to test a competing explanation.

Warn:

The suvat equations apply to any motion. This claim is false: they assume CONSTANT acceleration. On a curved velocity–time graph the gradient and area still give acceleration and displacement, but $v = u + at$ and its siblings do not apply – and real falling objects eventually reach terminal velocity, where acceleration is zero.

Key:

Kinematics: on a velocity–time graph gradient is acceleration and area is displacement; for constant acceleration pick the suvat equation that omits the missing quantity; vertical motion is the same model with $a = -g$.

R · Forces and Newton's laws · supported focus

AQA · A-Level · Mathematics

Scope and prerequisites

AQA A-level Mathematics 7357, section R (Forces and Newton's laws), version 1.3 (first assessment 2018). Original teaching notes; objective-level mapping lives in content/teaching/aqa_a_level_mathematics/objective-review.yaml.

Prerequisites: vectors from section J and kinematics from section Q. Each lesson develops its own method before independent transfer.

These are original teaching examples, not official AQA material.

Forces and Newton's laws

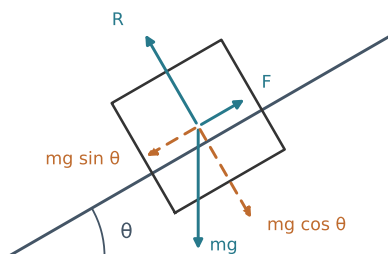
■ What would explain this observation?

- A lift passenger feels heavier while the lift speeds up upwards, and lighter while it slows down – yet the passenger's mass never changes. What is actually changing?
 - Start with a prediction. State which forces act on the passenger, and which one the floor's scale really measures.
-

■ Build the model

- A **force** is a push or pull that changes motion, in newtons. Newton's first law: with zero **resultant force**, velocity stays constant – rest, or a straight line at steady speed. Newton's second law: the resultant force equals mass $\times$ acceleration, $F = ma$, so forces must be added as vectors BEFORE the equation is used.
- Newton's third law: paired forces are equal and opposite but act on DIFFERENT bodies, so they never cancel on one free-body diagram. On a slope, weight mg resolves into $mg \sin \theta$ along the plane and $mg \cos \theta$ into it; the reaction balances only the perpendicular part, and **friction** opposes sliding up to a limit of μR .
- force: A push or pull that changes motion, in newtons; resultant force: The single vector sum of all forces on a body, which equals ma ; friction: The contact force opposing relative sliding, at most μR .

Resolve weight into components:
parallel $mg \sin \theta$, perpendicular $mg \cos \theta$



Newton's laws: what each one really says
no resultant force $\Leftrightarrow$ velocity is constant
rest or straight line at steady speed

N1

$F = ma$ is the RESULTANT force
add every force as a vector first

N2

action-reaction pairs are equal and opposite
...but act on DIFFERENT bodies

N3

on the slope: $R = mg \cos \theta$, acceleration from $mg \sin \theta - F = ma$

■ Choose evidence that can test it

- Draw the free-body diagram before any algebra: one arrow per force, touching the body, nothing else. If an arrow cannot name its source (weight, reaction, tension, friction, thrust), it does not belong.
- The lift scale reads the reaction R , not the weight: accelerating upwards, $R - mg = ma$ gives $R = m(g + a) > mg$ – the “heavier” feeling is a larger reaction, matching the observation.

■ Work from known quantities

- State the known values and their units. Choose the relation because its assumptions fit this case, then rearrange before substitution.
- Known: a 3 kg block sits on a 30° slope held by friction, $g = 9.8 \text{ m/s}^2$. Along the plane: $mg \sin \theta = F$, so $F = 3 \times 9.8 \times 0.5 = 14.7 \text{ N}$. Perpendicular: $R = mg \cos \theta = 3 \times 9.8 \times (\sqrt{3}/2) \approx 25.5 \text{ N}$. The block needs a friction coefficient of at least $F/R = \tan 30^\circ \approx 0.577$ to stay put.

Example:

A 60 kg passenger stands in a lift accelerating upwards at 1.2 m/s^2 . Find the scale reading, using the same sequence: free-body diagram $\rightarrow R - mg = ma \rightarrow$ substitute.
(Answer: $R = 60 \times (9.8 + 1.2) = 660 \text{ N}$.)

■ Check the conclusion and its limits

- Every $F = ma$ line should name its direction and be consistent with the diagram's signs; a resultant force pointing the wrong way flips the acceleration's direction. Check magnitudes: a reaction smaller than zero means the body has left the surface – the model, not the arithmetic, has broken.

- Return to the original observation. Explain what the result supports, which conditions it assumes, and one way to test a competing explanation.

Warn:

The normal reaction always equals the weight. This claim is false: on a slope $R = mg \cos \theta < mg$, and in an accelerating lift $R = m(g + a)$. Reaction equals weight only on a horizontal surface with no vertical acceleration – assuming it everywhere silently breaks every slope and lift calculation.

Key:

Forces and Newton's laws: draw the free-body diagram first; the resultant of all forces equals ma ; reaction pairs act on different bodies; on a slope resolve weight into $mg \sin \theta$ and $mg \cos \theta$, and the reaction is NOT always the weight.

S · Moments · supported focus

AQA · A-Level · Mathematics

Scope and prerequisites

AQA A-level Mathematics 7357, section S (Moments), version 1.3 (first assessment 2018). Original teaching notes; objective-level mapping lives in `content/teaching/aqa_a_level_mathematics/objective-review.yaml`.

Prerequisites: forces and Newton's laws from section R. Each lesson develops its own method before independent transfer.

These are original teaching examples, not official AQA material.

Moments

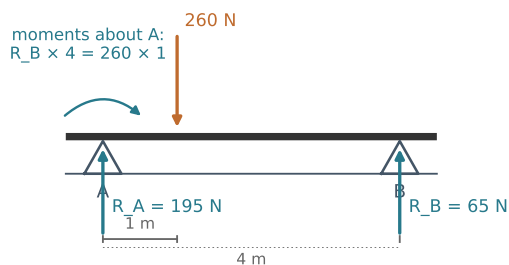
■ What would explain this observation?

- A long spanner loosens a tight nut that a short spanner cannot move, with exactly the same push. Where does the extra turning effect come from?
 - Start with a prediction. State what the turning effect of a force depends on, and why doubling the handle length doubles it.
-

■ Build the model

- The **moment** of a force about a point is the force multiplied by the PERPENDICULAR distance from the point to the line of the force, in N m. A force through the point has zero distance and zero moment – it cannot turn the body about that point.
- A body is in **equilibrium** only when BOTH conditions hold: the forces sum to zero (no translation) AND the moments about every point sum to zero (no rotation). Taking moments about an unknown force's **pivot** removes it from the equation, because its distance is zero – one equation, one unknown.
- moment: Force $\times$ perpendicular distance from the point to the force's line of action, in N m; equilibrium: A state with zero resultant force and zero resultant moment, so no translation and no rotation; pivot: The point about which moments are taken or a body turns.

Equilibrium: upward forces balance,
moments about any point balance



Moments: the three working rules

- 1 moment = $F \times \text{perpendicular distance}$
units N m; distance from the pivot to the **LINE** of the force
- 2 equilibrium needs BOTH $\Sigma F = 0$ and $\Sigma M = 0$
no translation **AND** no rotation
- 3 take moments about an unknown force
its distance is zero, so it drops out of the equation

check: $R_A + R_B = 195 + 65 = 260 \text{ N}$ ✓ balances the load

■ Choose evidence that can test it

- Choose the point to take moments about so that the most awkward unknown passes through it. For a beam on two supports, take moments about one support to find the other reaction, then use $\Sigma F = 0$ for the first.
- The distance is measured to the **LINE** of the force, not to where the force is drawn: a force at 40° to a 3 m bar has perpendicular distance $3 \sin 40^\circ$, not 3 m.

■ Work from known quantities

- State the known values and their units. Choose the relation because its assumptions fit this case, then rearrange before substitution.
- Known: a light 4 m beam rests on supports A and B at its ends, with a 260 N load 1 m from A. Moments about A: $R_B \times 4 = 260 \times 1$, so $R_B = 65 \text{ N}$. Vertical forces: $R_A + R_B = 260$, so $R_A = 195 \text{ N}$. The check $R_A + R_B = 260$ ✓ and moments about B ($195 \times 4 = 260 \times 3 = 780$ ✓) both close.

Example:

A uniform 6 m plank of weight 200 N rests on supports at its ends, and a 400 N crate sits 2 m from one end. Find both reactions, using the same sequence: moments about one support $\rightarrow \Sigma F = 0 \rightarrow$ check about the other. (Answer: moments about the near support give $6R_{\text{far}} = 400 \times 2 + 200 \times 3 = 1,400$, so $R_{\text{far}} \approx 233 \text{ N}$ and $R_{\text{near}} = 600 - 233 \approx 367 \text{ N}$.)

■ Check the conclusion and its limits

- Take moments about a **SECOND** point as a check: a sign slip in the first equation shows up immediately. Both reactions of a supported beam must come out positive – a negative reaction means the beam would lift off that support and the model has changed.

- Return to the original observation. Explain what the result supports, which conditions it assumes, and one way to test a competing explanation.

Warn:

Zero resultant force is enough for equilibrium. This claim is false: two equal opposite forces at different points (a couple) give zero resultant force but still spin the body. Equilibrium needs zero resultant moment as well – that is why the spanner's length matters even when the push is unchanged.

Key:

Moments: $\text{moment} = \text{force} \times \text{perpendicular distance}$ in N m; equilibrium needs $\Sigma F = 0$ AND $\Sigma M = 0$; take moments about an unknown force to remove it, and check by re-taking moments about a second point.