

S · Moments

AQA · A-Level · Mathematics

Scope and prerequisites

AQA A-level Mathematics 7357, section S (Moments), version 1.3 (first assessment 2018). Original teaching notes.

Prerequisites: forces and Newton's laws from section R.

These are original teaching examples, not official AQA material.

Moments

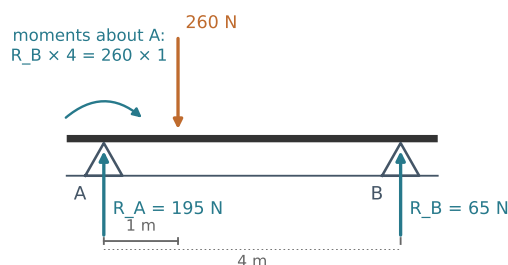
■ The question

- A long spanner loosens a tight nut that a short spanner cannot move, with exactly the same push. Where does the extra turning effect come from?
 - Before reading on, say what the turning effect of a force depends on, and why doubling the handle length doubles it.
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■ The method

- The **moment** of a force about a point is the force multiplied by the PERPENDICULAR distance from the point to the line of the force, in N m. A force through the point has zero distance and zero moment – it cannot turn the body about that point.
- A body is in **equilibrium** only when BOTH conditions hold: the forces sum to zero (no translation) AND the moments about every point sum to zero (no rotation). Taking moments about an unknown force's **pivot** removes it from the equation, because its distance is zero – one equation, one unknown.
- moment: Force $\times$ perpendicular distance from the point to the force's line of action, in N m; equilibrium: A state with zero resultant force and zero resultant moment, so no translation and no rotation; pivot: The point about which moments are taken or a body turns.

Equilibrium: upward forces balance,
moments about any point balance



Moments: the three working rules

- 1 moment = $F \times \text{perpendicular distance}$
units N m; distance from the pivot to the LINE of the force
- 2 equilibrium needs BOTH $\Sigma F = 0$ and $\Sigma M = 0$
no translation AND no rotation
- 3 take moments about an unknown force
its distance is zero, so it drops out of the equation

check: $R_A + R_B = 195 + 65 = 260 \text{ N}$ ✓ balances the load

Moments: original worked-case diagram

■ Checking your work

- Choose the point to take moments about so that the most awkward unknown passes through it. For a beam on two supports, take moments about one support to find the other reaction, then use $\Sigma F = 0$ for the first.
- The distance is measured to the LINE of the force, not to where the force is drawn: a force at 40° to a 3 m bar has perpendicular distance $3 \sin 40^\circ$, not 3 m.

■ A worked case

- Known: a light 4 m beam rests on supports A and B at its ends, with a 260 N load 1 m from A. Moments about A: $R_B \times 4 = 260 \times 1$, so $R_B = 65 \text{ N}$. Vertical forces: $R_A + R_B = 260$, so $R_A = 195 \text{ N}$. The check $R_A + R_B = 260$ ✓ and moments about B ($195 \times 4 = 260 \times 3 = 780$ ✓) both close.

Example:

A uniform 6 m plank of weight 200 N rests on supports at its ends, and a 400 N crate sits 2 m from one end. Find both reactions, using the same sequence: moments about one support $\rightarrow \Sigma F = 0 \rightarrow$ check about the other. (Answer: moments about the near support give $6R_{\text{far}} = 400 \times 2 + 200 \times 3 = 1,400$, so $R_{\text{far}} \approx 233 \text{ N}$ and $R_{\text{near}} = 600 - 233 \approx 367 \text{ N}$.)

■ Limits and traps

- Take moments about a SECOND point as a check: a sign slip in the first equation shows up immediately. Both reactions of a supported beam must come out positive – a negative reaction means the beam would lift off that support and the model has changed.

Warn:

Zero resultant force is enough for equilibrium. This claim is false: two equal opposite forces at different points (a couple) give zero resultant force but still spin the body. Equilibrium needs zero resultant moment as well – that is why the spanner’s length matters even when the push is unchanged.

Key:

Moments: moment = force $\times$ perpendicular distance in N m; equilibrium needs $\Sigma F = 0$ AND $\Sigma M = 0$; take moments about an unknown force to remove it, and check by re-taking moments about a second point.