

R · Forces and Newton's laws

AQA · A-Level · Mathematics

Scope and prerequisites

AQA A-level Mathematics 7357, section R (Forces and Newton's laws), version 1.3 (first assessment 2018). Original teaching notes.

Prerequisites: vectors from section J and kinematics from section Q.

These are original teaching examples, not official AQA material.

Forces and Newton's laws

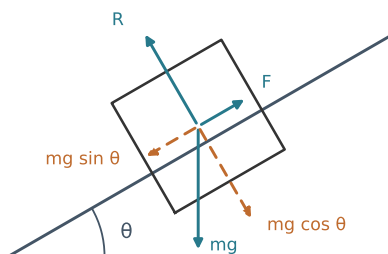
■ The question

- A lift passenger feels heavier while the lift speeds up upwards, and lighter while it slows down – yet the passenger's mass never changes. What is actually changing?
 - Before reading on, say which forces act on the passenger, and which one the floor's scale really measures.
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■ The method

- A **force** is a push or pull that can change motion or shape, measured in newtons. Newton's first law: with zero **resultant force**, velocity stays constant – rest, or a straight line at steady speed. Newton's second law: the resultant force equals mass $\times$ acceleration, $F = ma$, so forces must be added as vectors BEFORE the equation is used.
- Newton's third law: paired forces are equal and opposite but act on DIFFERENT bodies, so they never cancel on one free-body diagram. On a slope, weight mg resolves into $mg \sin \theta$ along the plane and $mg \cos \theta$ into it; the reaction balances only the perpendicular part, and **friction** opposes sliding up to a limit of μR .
- force: A push or pull that can change motion or shape, measured in newtons;
resultant force: The single vector sum of all forces on a body, which equals ma ;
friction: The contact force opposing relative sliding, at most μR .

Resolve weight into components:
parallel $mg \sin \theta$, perpendicular $mg \cos \theta$



Newton's laws: what each one really says

N1

no resultant force $\Leftrightarrow$ velocity is constant
rest or straight line at steady speed

N2

$F = ma$ is the RESULTANT force
add every force as a vector first

N3

action-reaction pairs are equal and opposite
...but act on DIFFERENT bodies

on the slope: $R = mg \cos \theta$, acceleration from $mg \sin \theta - F = ma$

On a slope: $R = mg \cos \theta$, and the resultant along the slope is $mg \sin \theta - F = ma$

■ Checking your work

- Draw the free-body diagram before any algebra: one arrow per force, touching the body, nothing else. If an arrow cannot name its source (weight, reaction, tension, friction, thrust), it does not belong.
- The lift scale reads the reaction R , not the weight: accelerating upwards, $R - mg = ma$ gives $R = m(g + a) > mg$ – the “heavier” feeling is a larger reaction, matching the observation.

■ A worked case

- Known: a 3 kg block sits on a 30° slope held by friction, $g = 9.8 \text{ m/s}^2$. Along the plane: $mg \sin \theta = F$, so $F = 3 \times 9.8 \times 0.5 = 14.7 \text{ N}$. Perpendicular: $R = mg \cos \theta = 3 \times 9.8 \times (\sqrt{3}/2) \approx 25.5 \text{ N}$. The block needs a friction coefficient of at least $F/R = \tan 30^\circ \approx 0.577$ to stay put.

Example:

A 60 kg passenger stands in a lift accelerating upwards at 1.2 m/s^2 . Find the scale reading, using the same sequence: free-body diagram $\rightarrow R - mg = ma \rightarrow$ substitute. (Answer: $R = 60 \times (9.8 + 1.2) = 660 \text{ N}$.)

■ Limits and traps

- Every $F = ma$ line should name its direction and be consistent with the diagram's signs; a resultant force pointing the wrong way flips the acceleration's direction. Check magnitudes: a reaction smaller than zero means the body has left the surface – the model, not the arithmetic, has broken.

Warn:

The normal reaction always equals the weight. This claim is false: on a slope $R = mg$

$\cos \theta < mg$, and in an accelerating lift $R = m(g + a)$. Reaction equals weight only on a horizontal surface with no vertical acceleration – assuming it everywhere silently breaks every slope and lift calculation.

Key:

Forces and Newton's laws: draw the free-body diagram first; the resultant of all forces equals ma ; reaction pairs act on different bodies; on a slope resolve weight into $mg \sin \theta$ and $mg \cos \theta$, and the reaction is NOT always the weight.