H · Integration

AQA · A-Level · Mathematics

Scope and prerequisites

AQA A-level Mathematics 7357, section H (Integration), version 1.3 (first assessment 2018). Original teaching notes.

Prerequisites: differentiation from section G.

These are original teaching examples, not official AQA material.

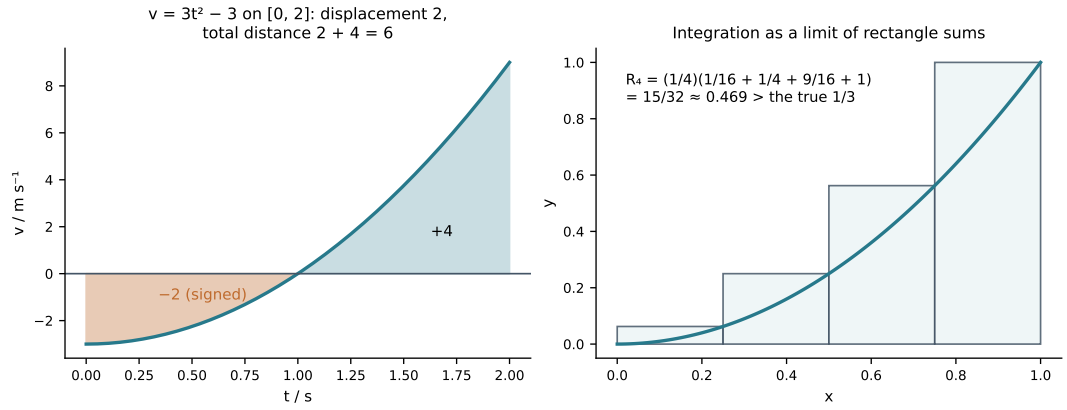
Integration

The question

- A runner's velocity graph dips below the time axis for a while, then climbs back. The "area under the curve" is nearly zero – did the runner go nowhere?
- Before reading on, say what a signed area means, and how total distance differs from displacement.

The method

- Integration reverses differentiation: an **antiderivative** of ax^n is $ax^{n+1}/(n+1) + C$ for $n \neq -1$, while $1/x$ integrates to $\ln|x| + C$. A **definite integral** from a to b is signed accumulation: $F(b) - F(a)$.
- Signed area cancels where the curve crosses the axis, so total positive area needs the integral SPLIT at each crossing, then the absolute values of the pieces added. The **Fundamental Theorem** connects both directions: differentiating an accumulation function returns the integrand.
- antiderivative: A function whose derivative is the given integrand; definite integral: The signed accumulation of a function between two limits; Fundamental Theorem: The two-way link that differentiating the accumulation function gives back the integrand.



Displacement = −2 + 4 = 2 m but distance = 2 + 4 = 6 m; rectangle sums approach the true area 1/3

Checking your work

- Verify every antiderivative by differentiating it back, and keep the constant C for indefinite integrals – a supplied point fixes it. The $n = -1$ case is separate: $1/x$ integrates to $\ln|x|$, never the power rule with division by zero.
- Before computing total area, find the sign changes of the integrand on the interval and split there; a velocity with a sign change makes displacement and total distance different answers.

A worked case

- Known: $v(t) = 3t^2 - 3$ for $0 \leq t \leq 2$. An antiderivative is $t^3 - 3t$. Displacement $= [t^3 - 3t]$ from 0 to $2 = (8 - 6) - 0 = 2$ m. But $v = 0$ at $t = 1$, so total distance $= -[t^3 - 3t]$ from 0 to $1 + [t^3 - 3t]$ from 1 to $2 = 2 + 4 = 6$ m.

Example:

Find the area between $y = x^2$ and the x-axis from 0 to 1, then the right-end rectangle estimate with 4 strips, using the same sequence. (Answer: exact $\int_0^1 x^2\,dx = 1/3 \approx 0.333$; the estimate is $(1/4)(1/16 + 1/4 + 9/16 + 1) = 15/32 \approx 0.469$, an overestimate for this increasing convex curve.)

Limits and traps

- Integration is the limit of rectangle sums; a finite strip count only estimates it, and the error depends on curvature. Do not integrate across a discontinuity such as $1/x$ at $x = 0$, and remember signed area can be zero while total area is positive.

Warn:

A definite integral always equals the total positive area between the curve and the axis. This claim is false: the integral is SIGNED – regions below the axis count negatively, so displacement can be small or zero while the total distance travelled is large.

Key:

Integration: antiderivatives verified by differentiating back, with C fixed by a supplied point; definite integrals are signed accumulation $F(b) - F(a)$; total area needs a split at every sign change, and the Fundamental Theorem links accumulation to the integrand.