

F · Exponentials and logarithms

AQA · A-Level · Mathematics

Scope and prerequisites

AQA A-level Mathematics 7357, section F (Exponentials and logarithms), version 1.3 (first assessment 2018). Original teaching notes.

Prerequisites: index laws and scientific calculators from GCSE.

These are original teaching examples, not official AQA material.

Exponentials and logarithms

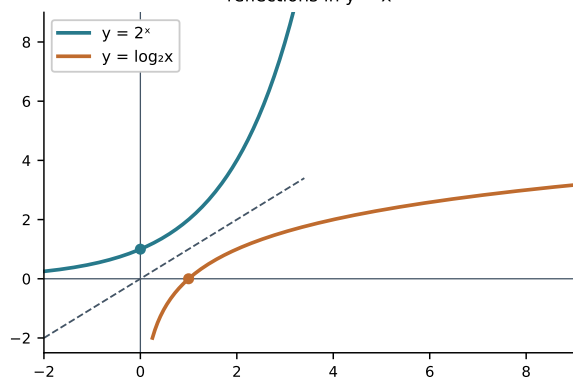
■ The question

- A sample halves every 3.5 hours, but the laboratory needs the level at 5 hours – no table gives it. How does one formula replace the missing table?
 - Before reading on, say what the model $y = Ae^{(kt)}$ needs as input, and how a log plot could recover k from data.
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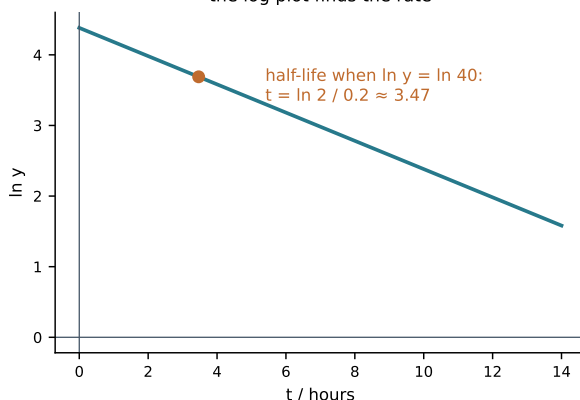
■ The method

- In $y = Ae^{(kt)}$, A is the initial amount and k is the proportional rate: positive for growth, negative for decay. The **half-life** is $\ln 2/|k|$; doubling time is $\ln 2/k$.
- The **logarithm** is the inverse of the exponential: $y = a^x$ means $x = \log_a y$, defined only for positive y . The log laws follow from the index laws: $\log(uv) = \log u + \log v$, $\log(u/v) = \log u - \log v$, $\log(u^k) = k \log u$. Solving $a^x = b$ gives $x = \ln b / \ln a$. Plotting $\ln y$ against t turns $y = Ae^{(kt)}$ into the straight line $\ln y = \ln A + kt$.
- half-life: The time for a decaying quantity to fall to half its initial value; logarithm: The exponent to which a stated base must be raised to produce a given positive number.

Exponential and logarithm are inverses:
reflections in $y = x$



$\ln y = \ln 80 - 0.2t$ is a straight line:
the log plot finds the rate



Exponentials and logarithms: original worked-case diagram

■ Checking your work

- A fitted exponential is a model: check its stated time unit and range before predicting, and compare a prediction with a fresh measurement (a residual) before trusting it. A negative k means decay; a negative starting amount usually contradicts the context.
- In log equations, record the original domain first: every argument of every log must be positive, so candidates produced after combining logs must be checked against it.

■ A worked case

- Known: $y = 80e^{(-0.2t)}$, y in grams, t in hours. Half the amount means $40 = 80e^{(-0.2t)}$, so $e^{(-0.2t)} = 1/2$. Taking $\ln$: $-0.2t = -\ln 2$, so $t = \ln 2 / 0.2 \approx 3.466$ hours. The same k gives the half-life for any starting amount.

Example:

Solve $3^x = 20$ using the same sequence: take logs $\rightarrow$ apply the power law $\rightarrow$ divide $\rightarrow$ interpret. (Answer: $x \ln 3 = \ln 20$, so $x = \ln 20 / \ln 3 \approx 2.727$. Check: $3^{2.727} \approx 20$.)

■ Limits and traps

- Log laws apply only to positive arguments; $\ln(x + y)$ has NO sum law ($\ln 2 + \ln 3 = \ln 6 \neq \ln 5$). The exponential model needs its unit of time matched to k ; an effective yearly rate of 10% means $k = \ln 1.1$, not $k = 0.1$.

Warn:

For positive x and y , $\ln(x + y) = \ln x + \ln y$. This claim is false: the product law is

$\ln(xy) = \ln x + \ln y$; there is no law for a sum inside a logarithm, as $x = y = 1$ shows ($\ln 2 \approx 0.69$ but $\ln 1 + \ln 1 = 0$).

Key:

Exponentials and logarithms: $y = Ae^{kt}$ with half-life $\ln 2/|k|$; logs invert exponentials, follow the product/quotient/power laws, and turn an exponential fit into a straight line. Check domains and units before solving.