D · Sequences and series

AQA · A-Level · Mathematics

Scope and prerequisites

AQA A-level Mathematics 7357, section D (Sequences and series), version 1.3 (first assessment 2018). Original teaching notes.

Prerequisites: GCSE arithmetic sequences and simultaneous equations.

These are original teaching examples, not official AQA material.

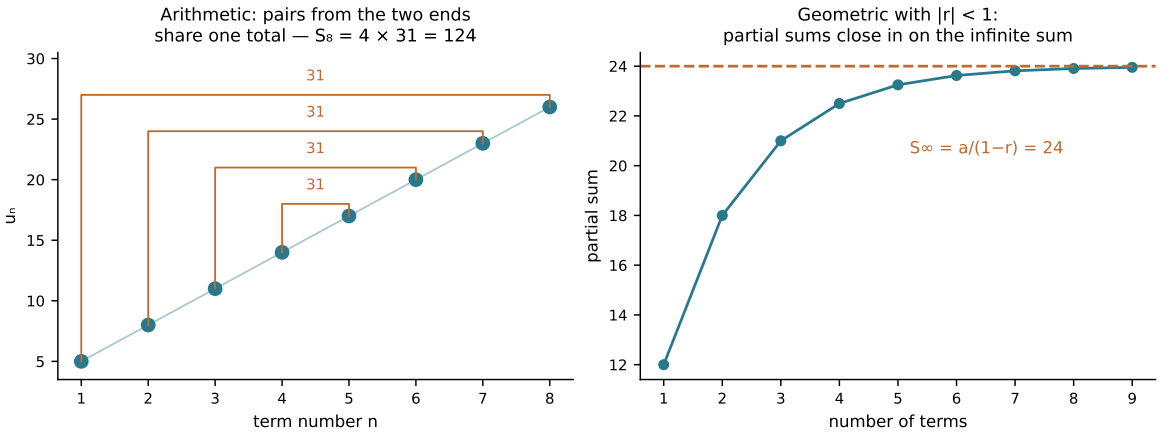
Sequences and series

■ The question

- A saver deposits the same amount every year while an investment grows by the same percentage every year. After eight years, which balance can you total exactly in one line?
- Before reading on, say which of the two grows by addition and which by multiplication, and what each sum formula needs.

■ The method

- An **arithmetic sequence** adds a constant difference: $u_n = a + (n - 1)d$. Its series sums by pairing ends: $S_n = n(a + u_n)/2$. A **geometric sequence** multiplies by a constant ratio: $u_n = ar^{n-1}$, with $S_n = a(1 - r^n)/(1 - r)$ for $r \neq 1$.
- An infinite geometric sum exists only when $|r| < 1$, because the terms shrink towards zero: $S\infty = a/(1 - r)$. A **recurrence relation** defines each term from the previous one and needs an initial value; a **periodic sequence** repeats after a fixed number of steps.
- arithmetic sequence: A sequence with a constant difference between terms; geometric sequence: A sequence with a constant ratio between terms; recurrence relation: A rule giving each term from the preceding value; periodic sequence: A sequence whose terms repeat after a fixed number of steps.



Sequences and series: original worked-case diagram

■ Checking your work

- Recover a and d from two stated terms by simultaneous equations before using a formula: $u_3 = 10$ and $u_7 = 22$ give $4d = 12$, so $d = 3$ and $a = 4$. A term count must come out as a positive integer; a fraction or negative root is rejected, never rounded into place.
- Check a claimed sum against the first few terms directly, and verify an at-least target against the integers on BOTH sides of the crossing.

■ A worked case

- Known: $a = 5$, $d = 3$, $n = 8$. The eighth term is $u_8 = 5 + 7 \times 3 = 26$. The sum is $S_8 = n(a + u_n)/2 = 8(5 + 26)/2 = 124$. Each of the four end-pairs totals 31, and $4 \times 31 = 124$ ✓.

Example:

For $a = 12$, $r = 0.5$, find the infinite sum and the first n whose partial sum exceeds 23.9, using the same sequence: known → formula → substitution → integer check. (Answer: $S\infty = 12/(1 - 0.5) = 24$. The partial sums are $S_n = 24(1 - 0.5^n)$: $S_7 = 24(1 - 1/128) = 23.8125 < 23.9$ and $S_8 = 24(1 - 1/256) = 23.90625 \geq 23.9$, so $n = 8$.)

■ Limits and traps

- The infinite-sum formula is invalid at $r = 1$ or $r = -1$, where the terms do not shrink; a negative ratio converges if its MAGNITUDE is below 1. A recurrence needs the operation order preserved exactly: B next = 1.1B + 50 differs from 1.1(B + 50).

Warn:

Every geometric series has a finite sum to infinity. This claim is false: the sum exists only when $|r| < 1$. At $r = 2$ the terms grow without bound, and at $r = -1$ the partial sums oscillate forever – neither converges.

Key:

Sequences and series: arithmetic adds (pair the ends to sum), geometric multiplies ($S_n = a(1-r^n)/(1-r)$, and $S\infty = a/(1-r)$ only for $|r| < 1$). Recover parameters from stated terms, and demand integer answers for counts.