C · Coordinate geometry

AQA · A-Level · Mathematics

Scope and prerequisites

AQA A-level Mathematics 7357, section C (Coordinate geometry), version 1.3 (first assessment 2018). Original teaching notes.

Prerequisites: GCSE straight lines, Pythagoras and quadratic equations.

These are original teaching examples, not official AQA material.

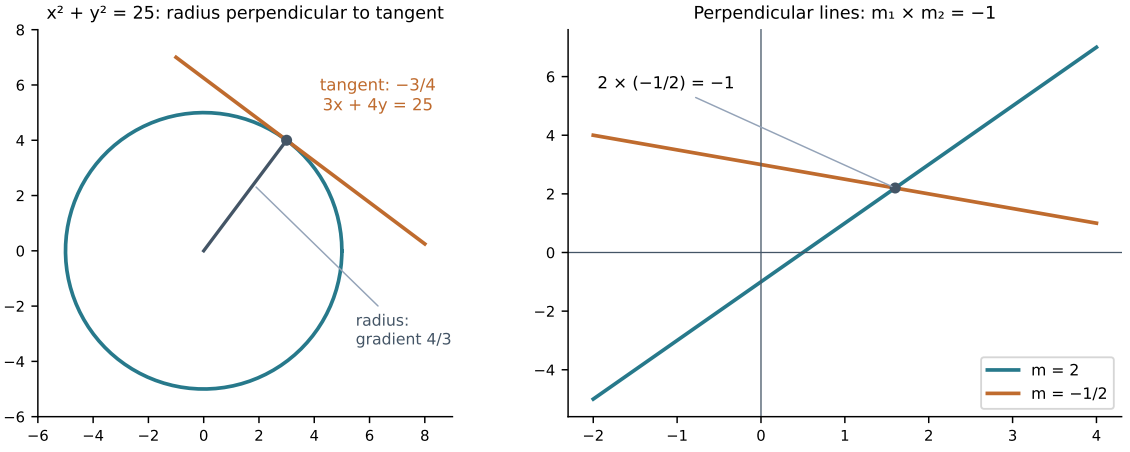
Coordinate geometry

The question

- A satellite dish's cross-section and a wheel's rim are both circles, but only the rim can be written $(x - a)^2 + (y - b)^2 = r^2$ with a centre you can read at a glance. What does that equation buy you?
- Before reading on, say what the parameters a, b and r each control, and how a tangent direction could be found without differentiation.

The method

- A line through (x_1, y_1) with **gradient** m is $y - y_1 = m(x - x_1)$. Parallel lines share m; perpendicular finite gradients satisfy $m_1 \times m_2 = -1$.
- A **circle** with centre (a, b) and radius r is $(x - a)^2 + (y - b)^2 = r^2$. The radius to a contact point is perpendicular to the **tangent** there. Where both gradients are finite, their product is -1; a horizontal radius has a vertical tangent, and a vertical radius has a horizontal tangent. A diameter subtends a right angle at the circumference.
- gradient: The change in y per unit change in x; circle: The set of points at a fixed distance from a centre; tangent: A line touching the circle at one point, perpendicular to the radius there.



Radius gradient 4/3 and tangent gradient −3/4 multiply to −1: perpendicular lines

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Checking your work

- Check that a proposed point lies on the circle by substituting into $(x - a)^2 + (y - b)^2 = r^2$ before building a tangent at it. Complete the square to rescue a centre from the expanded form $x^2 + y^2 + px + qy + s = 0$ – and adjust the right side too.
- Intersect a line with the circle by substitution: a repeated root means tangency, two roots a chord, and a negative discriminant no meeting.

A worked case

- Known: the circle $x^2 + y^2 = 25$ and the point (3, 4) on it ($3^2 + 4^2 = 25$ ✓). The radius from (0, 0) to (3, 4) has gradient 4/3, so the tangent gradient is $-3/4$ by $m_1 \times m_2 = -1$. The tangent is $y - 4 = -(3/4)(x - 3)$, which rearranges to $3x + 4y = 25$.

Example:

Find the centre and radius of $x^2 + y^2 - 4x + 2y - 20 = 0$ using the same sequence: group terms → complete each square → read the form. (Answer: $(x - 2)^2 + (y + 1)^2 = 25$, so the centre is $(2, -1)$ and the radius is 5; note the bracket signs reverse in the centre.)

Limits and traps

- The negative-reciprocal rule needs finite, nonzero gradients; a horizontal radius gives a vertical tangent, stated as $x =$ constant. A chord bisected by a centre-perpendicular is a right-triangle shortcut: half-chord, radius and centre distance obey Pythagoras.

Warn:

The tangent at a circle point has the same gradient as the radius to that point. This claim is false: the radius and tangent are perpendicular at the contact point, so their gradients multiply to -1 – at $(3, 4)$, the radius has $4/3$ and the tangent has $-3/4$.

Key:

Coordinate geometry: lines by point-gradient, circles by $(x - a)^2 + (y - b)^2 = r^2$. Tangents use the radius-perpendicular relation; intersections use substitution and the discriminant.