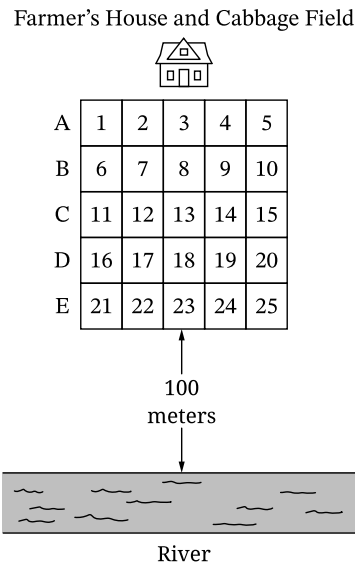


2. Aphids are tiny insects that feed on plants such as cabbage plants. A farmer wants to reduce the number of aphids in a cabbage field. A river is located 100 meters south of the cabbage field. The farmer divides the field into 25 regions of equal size, as shown in the diagram. Each region has approximately the same number of cabbage plants.



The farmer would like to estimate the proportion of cabbage plants in the field that are affected by aphids and believes that the extent of aphid damage is greater for the regions in the cabbage field closer to the river. To obtain the estimate, the farmer is considering three sampling methods.

- Sampling method I: Select region 3, which is closest to the farmer's house and farthest from the river. Examine every cabbage plant in the region for aphid damage.
- Sampling method II: Randomly select one row (A, B, C, D, or E). For every region in the selected row, examine every cabbage plant for aphid damage.
- Sampling method III: Randomly select one region from each of rows A, B, C, D, and E. For each selected region, examine every cabbage plant for aphid damage.

- A. Explain whether sampling method I is an appropriate sampling method for the farmer to use to estimate the proportion of cabbage plants in the field that are damaged by aphids.
- B. Using sampling method II, the farmer randomly selected row E and examined every cabbage plant in row E. If the farmer's belief is correct, determine whether the selection of row E is likely to provide an overestimate or an underestimate of the proportion of cabbage plants in the field that are damaged by aphids. Justify your answer.
- C. Using the information provided in the diagram of the cabbage field, describe how to implement sampling method III, which requires a random selection of one region from each of rows A, B, C, D, and E.

I. Descriptive Statistics

$$\bar{x} = \frac{1}{n} \sum x_i = \frac{\sum x_i}{n}$$
$$\hat{y} = a + bx$$
$$r = \frac{1}{n-1} \sum \left(\frac{x_i - \bar{x}}{s_x} \right) \left(\frac{y_i - \bar{y}}{s_y} \right)$$

$$s_x = \sqrt{\frac{1}{n-1} \sum (x_i - \bar{x})^2} = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n-1}}$$
$$\bar{y} = a + b\bar{x}$$
$$b = r \frac{s_y}{s_x}$$

II. Probability and Distributions

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Probability Distribution	Mean	Standard Deviation
Discrete random variable, X	$\mu_X = E(X) = \sum x_i P(x_i)$	$\sigma_X = \sqrt{\sum (x_i - \mu_X)^2 P(x_i)}$
If X has a binomial distribution with parameters n and p , then: $P(X = x) = \binom{n}{x} p^x (1 - p)^{n-x}$ where $x = 0, 1, 2, 3, \dots, n$	$\mu_X = np$	$\sigma_X = \sqrt{np(1 - p)}$
If X has a geometric distribution with parameter p , then: $P(X = x) = (1 - p)^{x-1} p$ where $x = 1, 2, 3, \dots$	$\mu_X = \frac{1}{p}$	$\sigma_X = \frac{\sqrt{1 - p}}{p}$

III. Sampling Distributions and Inferential Statistics

Standardized test statistic: $\frac{\text{statistic} - \text{parameter}}{\text{standard error of the statistic}}$
Confidence interval: $\text{statistic} \pm (\text{critical value})(\text{standard error of statistic})$
Chi-square statistic: $\chi^2 = \sum \frac{(\text{observed} - \text{expected})^2}{\text{expected}}$

Sampling distributions for proportions:

Random Variable	Parameters of Sampling Distribution		Standard Error* of Sample Statistic
For one population: $\hat{p}$	$\mu_{\hat{p}} = p$	$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$	$s_{\hat{p}} = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$
For two populations: $\hat{p}_1 - \hat{p}_2$	$\mu_{\hat{p}_1 - \hat{p}_2} = p_1 - p_2$	$\sigma_{\hat{p}_1 - \hat{p}_2} = \sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}}$	$s_{\hat{p}_1 - \hat{p}_2} = \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$ When $p_1 = p_2$ is assumed: $s_{\hat{p}_1 - \hat{p}_2} = \sqrt{\hat{p}_c(1-\hat{p}_c)\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}$ where $\hat{p}_c = \frac{X_1 + X_2}{n_1 + n_2}$

Sampling distributions for means:

Random Variable	Parameters of Sampling Distribution		Standard Error* of Sample Statistic
For one population: $\bar{X}$	$\mu_{\bar{X}} = \mu$	$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}}$	$s_{\bar{X}} = \frac{s}{\sqrt{n}}$
For two populations: $\bar{X}_1 - \bar{X}_2$	$\mu_{\bar{X}_1 - \bar{X}_2} = \mu_1 - \mu_2$	$\sigma_{\bar{X}_1 - \bar{X}_2} = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$	$s_{\bar{X}_1 - \bar{X}_2} = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$

Sampling distributions for simple linear regression:

Random Variable	Parameters of Sampling Distribution		Standard Error* of Sample Statistic
For slope: b	$\mu_b = \beta$	$\sigma_b = \frac{\sigma}{\sigma_x \sqrt{n}}$ where $\sigma_x = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n}}$	$s_b = \frac{s}{s_x \sqrt{n-1}}$ where $s = \sqrt{\frac{\sum (y_i - \hat{y}_i)^2}{n-2}}$ and $s_x = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n-1}}$

*Standard deviation is a measurement of variability from the theoretical population. Standard error is the estimate of the standard deviation. If the standard deviation of the statistic is assumed to be known, then the standard deviation should be used instead of the standard error.

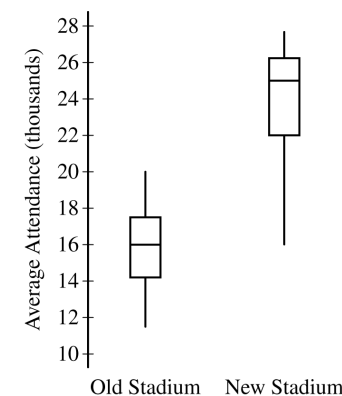
Part B

Question 6

Spend about 25 minutes on this part of the exam.

Directions: Show all your work. Indicate clearly the methods you use, because you will be scored on the correctness of your methods as well as on the accuracy and completeness of your results and explanations.

5. Attendance at games for a certain baseball team is being investigated by the team owner. The following boxplots summarize the attendance, measured as average number of attendees per game, for 47 years of the team's existence. The boxplots include the 30 years of games played in the old stadium and the 17 years played in the new stadium.

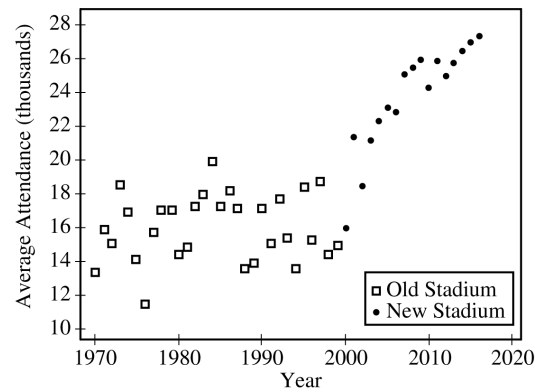


- (a) Compare the distributions of average attendance between the old and new stadiums.

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The following scatterplot shows average attendance versus year.

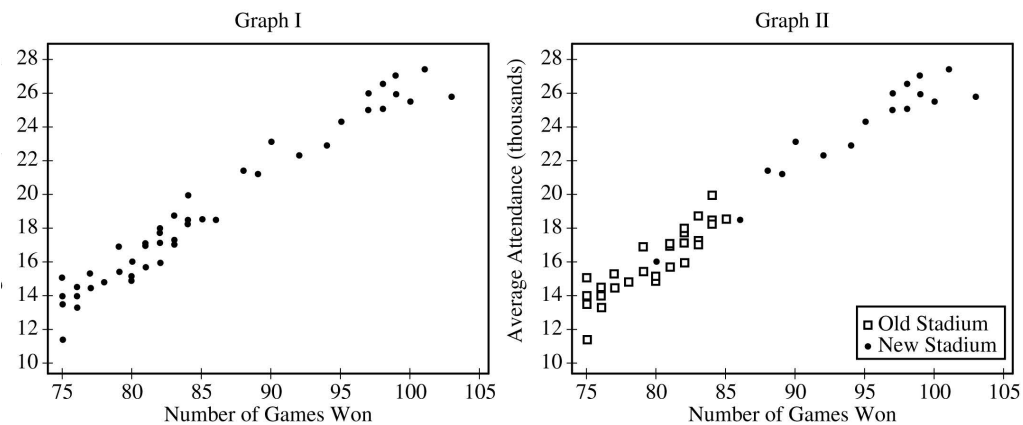


- (b) Compare the trends in average attendance over time between the old and new stadium.

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- (c) Consider the following scatterplots.



- (i) Graph I shows the average attendance versus number of games won for each year. Describe the relationship between the variables.
- (ii) Graph II shows the same information as Graph I, but also indicates the old and new stadiums. Does Graph II suggest that the rate at which attendance changes as number of games won increases is different in the new stadium compared to the old stadium? Explain your reasoning.

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Name:

AP Statistics: 2021

- (d) Consider the three variables: number of games won, year, and stadium. Based on the graphs, explain how one of those variables could be a confounding variable in the relationship between average attendance and the other variables.

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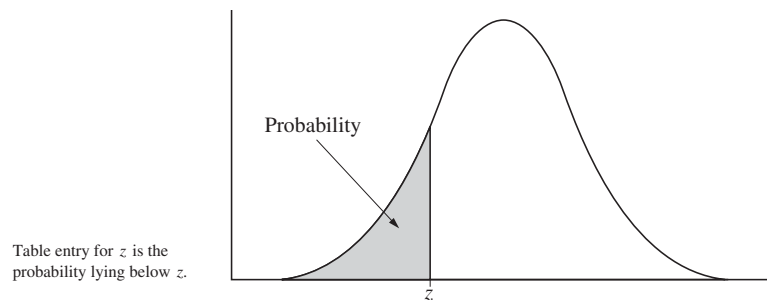
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AP Statistics: 2021

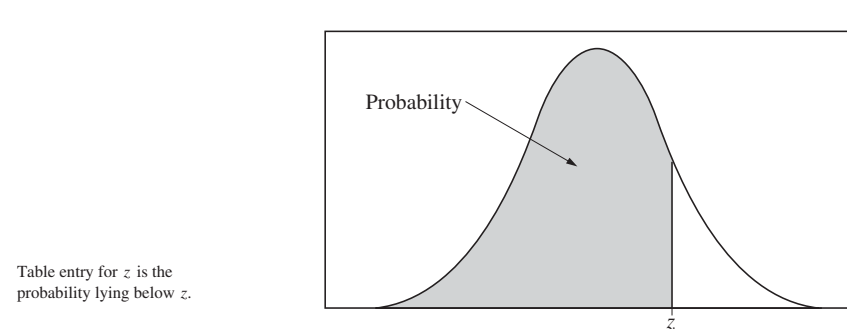
STOP

END OF EXAM

Use a pencil or pen with black or dark blue ink only. Do NOT write your name. Do NOT write outside the box.

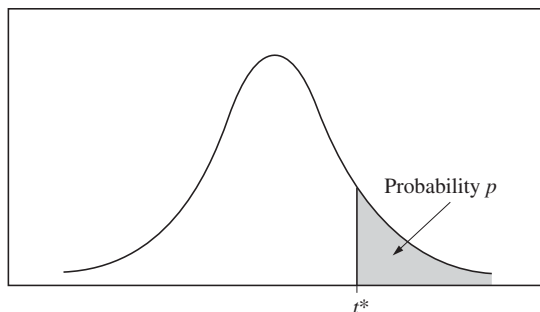

Table A Standard normal probabilities

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
-3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
-3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
-3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
-3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
-2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
-2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
-2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
-2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
-2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
-2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
-2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
-2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
-2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
-2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
-1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
-1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
-1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
-1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
-1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
-1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
-1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
-1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
-1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
-1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
-0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
-0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
-0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
-0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
-0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
-0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
-0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
-0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
-0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
-0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641


Table A (Continued)

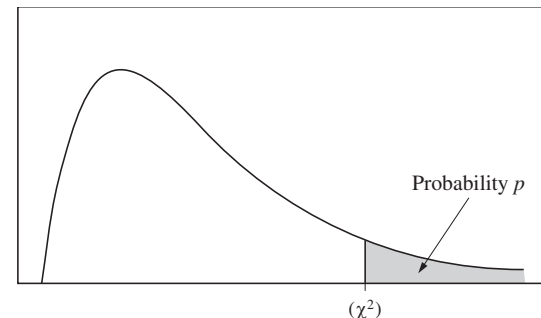
z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

Table entry for p and C is the point t^* with probability p lying above it and probability C lying between $-t^*$ and t^* .

Table B t distribution critical values

df	Tail probability p										
	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001
1	1.000	1.376	1.963	3.078	6.314	12.71	15.89	31.82	63.66	127.3	318.3
2	.816	1.061	1.386	1.886	2.920	4.303	4.849	6.965	9.925	14.09	22.33
3	.765	.978	1.250	1.638	2.353	3.182	3.482	4.541	5.841	7.453	10.21
4	.741	.941	1.190	1.533	2.132	2.776	2.999	3.747	4.604	5.598	7.173
5	.727	.920	1.156	1.476	2.015	2.571	2.757	3.365	4.032	4.773	5.893
6	.718	.906	1.134	1.440	1.943	2.447	2.612	3.143	3.707	4.317	5.208
7	.711	.896	1.119	1.415	1.895	2.365	2.517	2.998	3.499	4.029	4.785
8	.706	.889	1.108	1.397	1.860	2.306	2.449	2.896	3.355	3.833	4.501
9	.703	.883	1.100	1.383	1.833	2.262	2.398	2.821	3.250	3.690	4.297
10	.700	.879	1.093	1.372	1.812	2.228	2.359	2.764	3.169	3.581	4.144
11	.697	.876	1.088	1.363	1.796	2.201	2.328	2.718	3.106	3.497	4.025
12	.695	.873	1.083	1.356	1.782	2.179	2.303	2.681	3.055	3.428	3.930
13	.694	.870	1.079	1.350	1.771	2.160	2.282	2.650	3.012	3.372	3.852
14	.692	.868	1.076	1.345	1.761	2.145	2.264	2.624	2.977	3.326	3.787
15	.691	.866	1.074	1.341	1.753	2.131	2.249	2.602	2.947	3.286	3.733
16	.690	.865	1.071	1.337	1.746	2.120	2.235	2.583	2.921	3.252	3.686
17	.689	.863	1.069	1.333	1.740	2.110	2.224	2.567	2.898	3.222	3.646
18	.688	.862	1.067	1.330	1.734	2.101	2.214	2.552	2.878	3.197	3.611
19	.688	.861	1.066	1.328	1.729	2.093	2.205	2.539	2.861	3.174	3.579
20	.687	.860	1.064	1.325	1.725	2.086	2.197	2.528	2.845	3.153	3.552
21	.686	.859	1.063	1.323	1.721	2.080	2.189	2.518	2.831	3.135	3.527
22	.686	.858	1.061	1.321	1.717	2.074	2.183	2.508	2.819	3.119	3.505
23	.685	.858	1.060	1.319	1.714	2.069	2.177	2.500	2.807	3.104	3.485
24	.685	.857	1.059	1.318	1.711	2.064	2.172	2.492	2.797	3.091	3.467
25	.684	.856	1.058	1.316	1.708	2.060	2.167	2.485	2.787	3.078	3.450
26	.684	.856	1.058	1.315	1.706	2.056	2.162	2.479	2.779	3.067	3.435
27	.684	.855	1.057	1.314	1.703	2.052	2.158	2.473	2.771	3.057	3.421
28	.683	.855	1.056	1.313	1.701	2.048	2.154	2.467	2.763	3.047	3.408
29	.683	.854	1.055	1.311	1.699	2.045	2.150	2.462	2.756	3.038	3.396
30	.683	.854	1.055	1.310	1.697	2.042	2.147	2.457	2.750	3.030	3.385
40	.681	.851	1.050	1.303	1.684	2.021	2.123	2.423	2.704	2.971	3.307
50	.679	.849	1.047	1.299	1.676	2.009	2.109	2.403	2.678	2.937	3.261
60	.679	.848	1.045	1.296	1.671	2.000	2.099	2.390	2.660	2.915	3.232
80	.678	.846	1.043	1.292	1.664	1.990	2.088	2.374	2.639	2.887	3.195
100	.677	.845	1.042	1.290	1.660	1.984	2.081	2.364	2.626	2.871	3.174
1000	.675	.842	1.037	1.282	1.646	1.962	2.056	2.330	2.581	2.813	3.098
∞	.674	.841	1.036	1.282	1.645	1.960	2.054	2.326	2.576	2.807	3.091
50% 60% 70% 80% 90% 95% 96% 98% 99% 99.5% 99.8% 99.9%											
Confidence level C											

Table entry for p is the point (χ^2) with probability p lying above it.

Table C χ^2 critical values

df	Tail probability p										
	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001
1	1.32	1.64	2.07	2.71	3.84	5.02	5.41	6.63	7.88	9.14	10.83
2	2.77	3.22	3.79	4.61	5.99	7.38	7.82	9.21	10.60	11.98	13.82
3	4.11	4.64	5.32	6.25	7.81	9.35	9.84	11.34	12.84	14.32	16.27
4	5.39	5.99	6.74	7.78	9.49	11.14	11.67	13.28	14.86	16.42	18.47
5	6.63	7.29	8.12	9.24	11.07	12.83	13.39	15.09	16.75	18.39	20.51
6	7.84	8.56	9.45	10.64	12.59	14.45	15.03	16.81	18.55	20.25	22.46
7	9.04	9.80	10.75	12.02	14.07	16.01	16.62	18.48	20.28	22.04	24.32
8	10.22	11.03	12.03	13.36	15.51	17.53	18.17	20.09	21.95	23.77	26.12
9	11.39	12.24	13.29	14.68	16.92	19.02	19.68	21.67	23.59	25.46	27.88
10	12.55	13.44	14.53	15.99	18.31	20.48	21.16	23.21	25.19	27.11	29.59
11	13.70	14.63	15.77	17.28	19.68	21.92	22.62	24.72	26.76	28.73	31.26
12	14.85	15.81	16.99	18.55	21.03	23.34	24.05	26.22	28.30	30.32	32.91
13	15.98	16.98	18.20	19.81	22.36	24.74	25.47	27.69	29.82	31.88	34.53
14	17.12	18.15	19.41	21.06	23.68	26.12	26.87	29.14	31.32	33.43	36.12
15	18.25	19.31	20.60	22.31	25.00	27.49	28.26	30.58	32.80	34.95	37.70
16	19.37	20.47	21.79	23.54	26.30	28.85	29.63	32.00	34.27	36.46	39.25
17	20.49	21.61	22.98	24.77	27.59	30.19	31.00	33.41	35.72	37.95	40.79
18	21.60	22.76	24.16	25.99	28.87	31.53	32.35	34.81	37.16	39.42	42.31
19	22.72	23.90	25.33	27.20	30.14	32.85	33.69	36.19	38.58	40.88	43.82
20	23.83	25.04	26.50	28.41	31.41	34.17	35.02	37.57	40.00	42.34	45.31
21	24.93	26.17	27.66	29.62	32.67	35.48	36.34	38.93	41.40	43.78	46.80
22	26.04	27.30	28.82	30.81	33.92	36.78	37.66	40.29	42.80	45.20	48.27
23	27.14	28.43	29.98	32.01	35.17	38.08	38.97	41.64	44.18	46.62	49.73
24	28.24	29.55	31.13	33.20	36.42	39.36	40.27	42.98	45.56	48.03	51.18
25	29.34	30.68	32.28	34.38	37.65	40.65	41.57	44.31	46.93	49.44	52.62
26	30.43	31.79	33.43	35.56	38.89	41.92	42.86	45.64	48.29	50.83	54.05
27	31.53	32.91	34.57	36.74	40.11	43.19	44.14	46.96	49.64	52.22	55.48
28	32.62	34.03	35.71	37.92	41.34	44.46	45.42	48.28	50.99	53.59	56.89
29	33.71	35.14	36.85	39.09	42.56	45.72	46.69	49.59	52.34	54.97	58.30
30	34.80	36.25	37.99	40.26	43.77	46.98	47.96	50.89	53.67	56.33	59.70
40	45.62	47.27	49.24	51.81	55.76	59.34	60.44	63.69	66.77	69.70	73.40
50	56.33	58.16	60.35	63.17	67.50	71.42	72.61	76.15	79.49	82.66	86.66
60	66.98	68.97	71.34	74.40	79.08	83.30	84.58	88.38	91.95	95.34	99.61
80	88.13	90.41	93.11	96.58	101.9	106.6	108.1	112.3	116.3	120.1	124.8
100	109.1	111.7	114.7	118.5	124.3	129.6	131.1	135.8	140.2	144.3	149.4