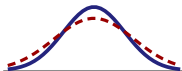


Inference for Means

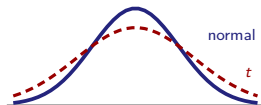
AP Statistics ■ Unit 7

AP Statistics



What we will cover

- 1 The t-distribution
- 2 Confidence interval for a mean
- 3 Test for a mean
- 4 Difference of two means
- 5 Two-mean test and paired data
- 6 Choosing a procedure



The t-distribution

We almost never know σ , so we use s_x instead. That extra guessing means the **t-distribution** t 分布, not the normal.

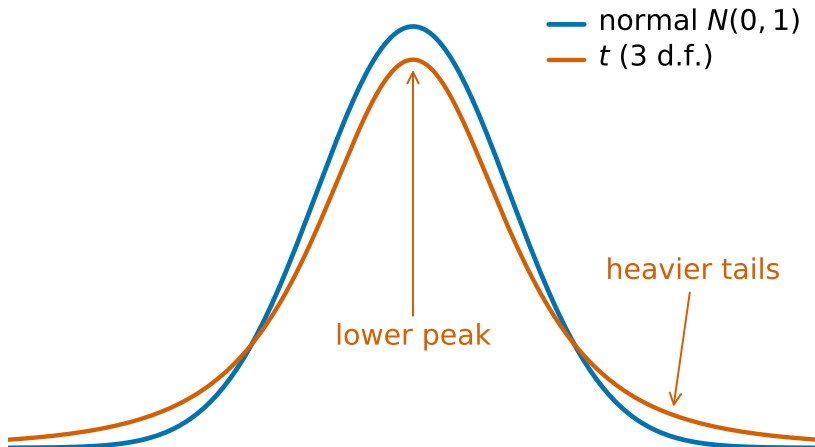
- heavier tails $\Rightarrow$ a little more caution
- **degrees of freedom** 自由度 $= n - 1$

Sampling variability 抽样变异性 is honest bounce – not a mistake a calculation can fix.



two group comparison

The t-distribution



Confidence interval for a mean

With σ unknown, use t^* with $df = n - 1$:

$$\bar{x} \pm t^* \frac{s_x}{\sqrt{n}}$$

- **standard error** 标准误 $= s_x / \sqrt{n}$
- **margin of error** 误差幅度 $= t^* \cdot \text{SE}$

Check: random, $\leq 10\%$ of population, roughly normal (or $n \geq 30$).

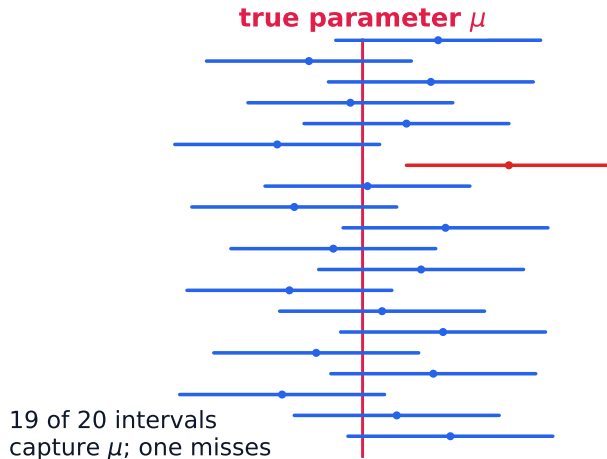
Batteries

$n = 25$, $\bar{x} = 50$, $s_x = 8$; $t^* = 2.064$ ($df = 24$):

$$50 \pm 2.064 \cdot \frac{8}{5} = 50 \pm 3.30$$

$= (46.70, 53.30)$ hours.

Confidence interval for a mean



Test for a mean

A **one-sample t-test** 单样本 t 检验:

- **null** 原假设 $H_0 : \mu = \mu_0$
- **alternative** 备择假设 $H_a : \mu >, <, \text{ or } \neq \mu_0$

$$t = \frac{\bar{x} - \mu_0}{s_x / \sqrt{n}}$$

Find the p-value from the t -distribution with $df = n - 1$; reject if $p \leq \alpha$.

Batteries

Claim $\mu = 50$; sample $\bar{x} = 53$, $s_x = 8$, $n = 25$, $H_a : \mu > 50$:

$$t = \frac{53 - 50}{8/\sqrt{5}} = 1.875$$

$p \approx 0.037 < 0.05$: reject.

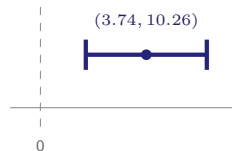
Difference of two means

The design decides the tool:

- **independent samples** 独立样本 $\Rightarrow$ **two-sample t-interval** 双样本 t 区间
- **paired** 配对 data $\Rightarrow$ one-sample t on the differences

$$(\bar{x}_1 - \bar{x}_2) \pm t^* \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

Variances **add** under the root. If the interval excludes 0, the means differ.



Excludes 0 $\Rightarrow$ Brand

1 longer.

Two-mean test and paired data

A **two-sample t-test** 双样本 t 检验 of $H_0 : \mu_1 = \mu_2$ - **do not pool** (unlike proportions):

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{s_1^2/n_1 + s_2^2/n_2}}$$

Same subjects measured twice? Use a **paired t-test** 配对 t 检验: reduce each pair to a difference d , then a one-sample t .

Brands

$\bar{x}_1 = 85, \bar{x}_2 = 78; SE = \sqrt{2}$:

$$t = \frac{7}{1.414} \approx 4.95$$

$df = 8, p \approx 0.001$: reject.

Choosing a procedure

Two questions point to the right tool:

Goal?

estimate $\Rightarrow$ interval;
weigh a claim $\Rightarrow$ test

How many groups?

one $\Rightarrow$ one-sample; two
 $\Rightarrow$ two-sample or paired

Mean or proportion?

mean $\Rightarrow$ t ; proportion $\Rightarrow$ z

Then **select** 选择 the formula, **implement** 实施 with conditions checked, and **communicate** 表达 in context.

Unit 7 – key takeaways

1 The t-model

unknown $\sigma \Rightarrow$ use s_x and t ; $df = n - 1$

2 One mean

$$\bar{x} \pm t^* s_x / \sqrt{n}; \quad t = (\bar{x} - \mu_0) / (s_x / \sqrt{n})$$

3 Two means

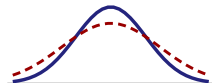
add variances; don't pool; paired
 $\rightarrow$ one-sample t

4 Choosing

goal + groups +
mean/proportion select the test

Check yourself

- 1 Why use t instead of z for a mean?
- 2 What are the degrees of freedom for a one-sample t ?
- 3 Do you pool for a two-sample t -test of means?
- 4 Before-and-after on the same subjects calls for which test?



Answers 1. σ is unknown (use s_x) 2. $n - 1$ 3. no 4. a paired t -test