Inference for Proportions

AP Statistics ▪ Unit 6

AP Statistics

What we will cover

1. Confidence interval for p
2. Interpreting an interval
3. Setting up a test
4. p-values and conclusions
5. Type I and II errors
6. Comparing two proportions

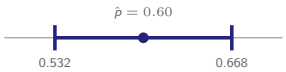
1 Interval

Confidence interval for a proportion

A **confidence interval** 置信区间 is estimate ± margin:

$$\hat{p} \pm z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

- **margin of error** 误差幅度 $= z^*$·**standard error** 标准误
- for 95%: **critical value** 临界值 $z^* = 1.96$



Check: random, $\leq 10\%$ of population, and large counts.

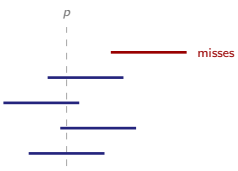
2 Interpret

Interpreting an interval

Two layers of meaning:

- *interval*: "95% confident the true p is in $(0.532, 0.668)$"
- *level*: "95% of such intervals capture p" – confidence is in the method

A value outside the interval is not plausible. Bigger $n \Rightarrow$ narrower; higher confidence $\Rightarrow$ wider.



Most intervals catch p; some miss.

3 Test setup

Setting up a test

A **significance test** 显著性检验 weighs evidence against a claim.

- **null** 原假设 $H_0 : p = p_0$ ("no effect")
- **alternative** 备择假设 $H_a : p >, <,$ or $\neq p_0$

The **test statistic** 检验统计量 (one-sample z-test), using p_0:

$$z = \frac{\hat{p} - p_0}{\sqrt{p_0(1-p_0)/n}}$$

> Check conditions with the assumed p_0, not $\hat{p}$.

A coin

58 heads in 100: $\hat{p} = 0.58$, $H_0 : p = 0.5$, $H_a : p > 0.5$.

$$z = \frac{0.58 - 0.50}{0.05} = 1.6$$

4 p-value

p-values and conclusions

The **p-value** p 值: *if* H_0 *were true*, the chance of a result at least this **extreme** 极端 – a tail area from z.

- $p \leq \alpha$: **reject** 拒绝 H_0
- $p > \alpha$: **fail to reject** 不拒绝 H_0

> Failing to reject is not proof of H_0 – just too little evidence.

5 Errors

Type I and Type II errors

A test on chance data can err two ways:

- **Type I** 第一类错误: reject a *true* H_0 (false alarm) – prob. α
- **Type II** 第二类错误: miss a *false* H_0 – prob. β

> **Power** 功效 $= 1 - \beta$ rises with larger n, a bigger true effect, or larger α.

	H_0 true	H_0 false
reject	Type I (α)	correct (power)
fail	correct	Type II (β)

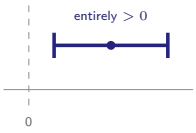
6 Two proportions

Comparing two proportions

A **two-sample** z**-interval** 双样本 z 区间:

$$(\hat{p}_1 - \hat{p}_2) \pm z^* \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$$

If the interval excludes 0, the groups truly differ. The test pools: $\hat{p}_c = \frac{x_1 + x_2}{n_1 + n_2}$.



Excludes 0 $\Rightarrow p_1 > p_2$.

7 Recap

Unit 6 – key takeaways

1 Interval

$\hat{p} \pm z^* \cdot \text{SE}$; 95% of intervals catch p

2 Test

$H_0 : p = p_0$; z uses p_0; small p rejects

3 Errors

Type I (α) false alarm; Type II (β); power $1 - \beta$

4 Two groups

interval excluding 0 $\Rightarrow$ real difference; pool for the test

Check yourself

1. What is z^* for a 95% confidence interval?
2. A small p-value is evidence for or against H_0?
3. Rejecting a true H_0 is which type of error?
4. What value must a difference interval exclude to show a real gap?

Answers **1.** 1.96 **2.** against H_0 **3.** Type I **4.** zero