

Inference for Proportions

AP Statistics ■ Unit 6

AP Statistics



What we will cover

- 1 Confidence interval for p
- 2 Interpreting an interval
- 3 Setting up a test
- 4 p-values and conclusions
- 5 Type I and II errors
- 6 Comparing two proportions



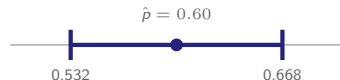
Confidence interval for a proportion

A **confidence interval** 置信区间 is estimate $\pm$ margin:

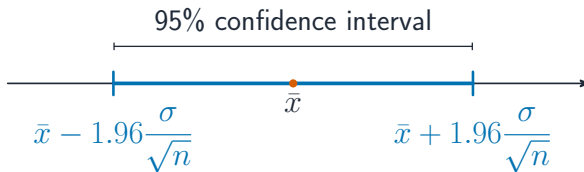
$$\hat{p} \pm z^* \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}}$$

- **margin of error** 误差幅度
= $z^* \cdot$ **standard error** 标准误
- for 95%: **critical value** 临界值
 $z^* = 1.96$

Check: random, $\leq 10\%$ of population,
and large counts.



Confidence interval for a proportion

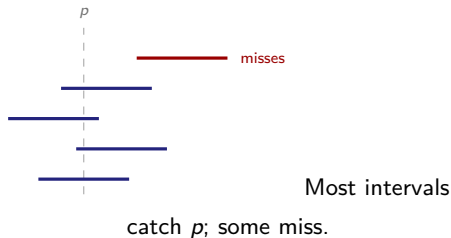


Interpreting an interval

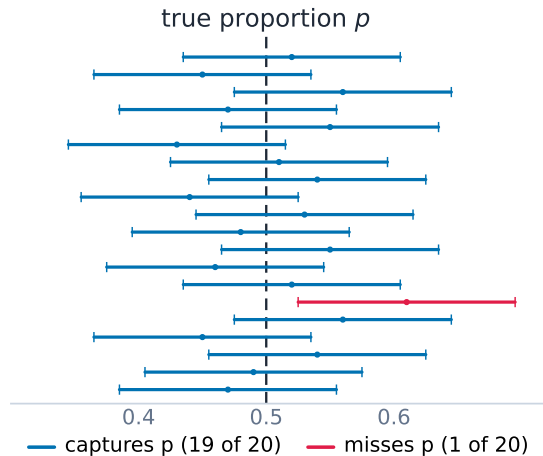
Two layers of meaning:

- *interval*: “95% confident the true p is in $(0.532, 0.668)$ ”
- *level*: “95% of such intervals capture p ” – confidence is in the **method**

A value outside the interval is not plausible. Bigger $n \Rightarrow$ narrower; higher confidence $\Rightarrow$ wider.



Interpreting an interval



Setting up a test

A **significance test** 显著性检验 weighs evidence against a claim.

- **null** 原假设 $H_0 : p = p_0$ (“no effect”)
- **alternative** 备择假设 $H_a : p >, <, \text{ or } \neq p_0$

The **test statistic** 检验统计量 (one-sample z-test), using p_0 :

$$z = \frac{\hat{p} - p_0}{\sqrt{p_0(1 - p_0)/n}}$$

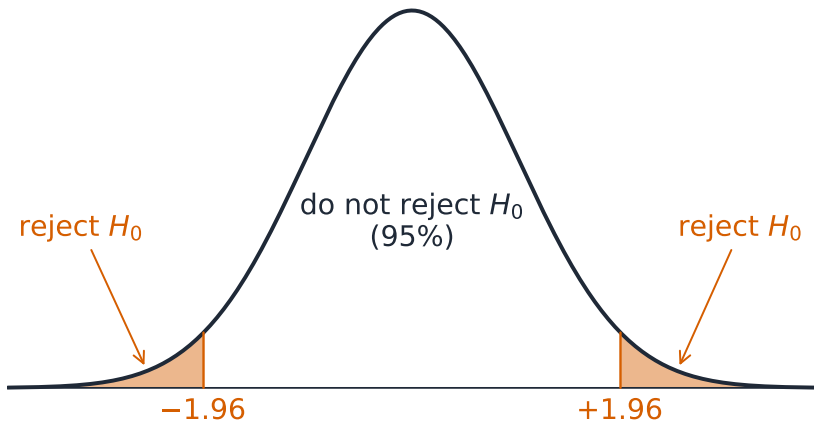
Check conditions with the **assumed** p_0 , not $\hat{p}$.

A coin

58 heads in 100: $\hat{p} = 0.58$, $H_0 : p = 0.5$, $H_a : p > 0.5$.

$$z = \frac{0.58 - 0.50}{0.05} = 1.6$$

Setting up a test

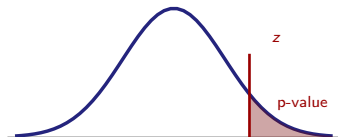


p-values and conclusions

The **p-value** p 值: if H_0 were true, the chance of a result at least this **extreme** 极端 – a tail area from z .

- $p \leq \alpha$: **reject** 拒绝 H_0
- $p > \alpha$: **fail to reject** 不拒绝 H_0

Failing to reject is **not** proof of H_0 – just too little evidence.



Type I and Type II errors

A test on chance data can err two ways:

- **Type I** 第一类错误: reject a *true* H_0 (false alarm) – prob. α
- **Type II** 第二类错误: miss a *false* H_0 – prob. β

Power 功效 = $1 - \beta$ rises with larger n , a bigger true effect, or larger α .

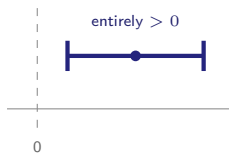
	H_0 true	H_0 false
reject	Type I (α)	correct (power)
fail	correct	Type II (β)

Comparing two proportions

A **two-sample z-interval** 双样本 z 区间:

$$(\hat{p}_1 - \hat{p}_2) \pm z^* \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$$

If the interval excludes 0, the groups truly differ. The test pools: $\hat{p}_c = \frac{x_1 + x_2}{n_1 + n_2}$.



Excludes 0 $\Rightarrow p_1 > p_2$.

Unit 6 – key takeaways

1 Interval

$\hat{p} \pm z^* \cdot \text{SE}$; 95% of intervals catch p

2 Test

$H_0 : p = p_0$; z uses p_0 ; small p rejects

3 Errors

Type I (α) false alarm; Type II (β); power $1 - \beta$

4 Two groups

interval excluding 0 $\Rightarrow$ real difference; pool for the test

Check yourself

- 1 What is z^* for a 95% confidence interval?
- 2 A small p-value is evidence for or against H_0 ?
- 3 Rejecting a true H_0 is which type of error?
- 4 What value must a difference interval exclude to show a real gap?



Answers 1. 1.96 2. against H_0 3. Type I 4. zero