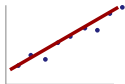


Exploring Two-Variable Data

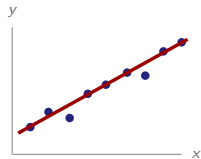
AP Statistics ■ Unit 2

AP Statistics



What we will cover

- 1 Related variables
- 2 Two categorical variables
- 3 Scatterplots and correlation
- 4 The regression line
- 5 Residuals and fit
- 6 Departures from linearity



1

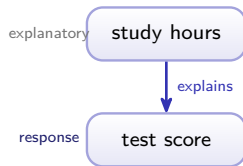
Related

Related variables

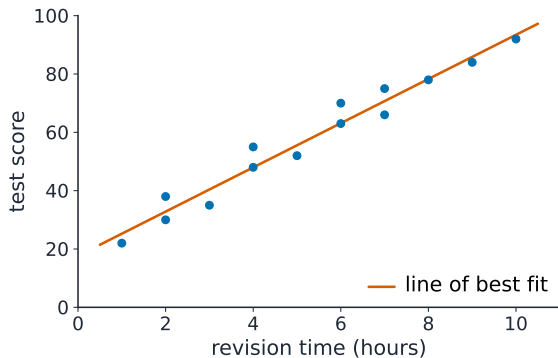
When two variables change together, they show an **association** 关联.

- **explanatory** 解释变量 variable predicts
- **response** 响应变量 variable is the outcome

An apparent pattern may be real – or a fluke of this one sample.



Scatterplots for Two Quantitative Variables



A line of best fit runs through the middle of the scattered points

2

Categorical

Two categorical variables

A **two-way table** 双向表 gives three relative frequencies:

- **joint** 联合相对频率: $\text{cell} \div \text{grand total}$
- **marginal** 边际相对频率: $\text{margin} \div \text{total}$
- **conditional** 条件相对频率: $\text{cell} \div \text{row/col}$

Different **conditional distributions**
条件分布 $\Rightarrow$ association – but never
causation 因果关系.

	pet	none
boy	30	20
girl	25	25

$P(\text{pet} \mid \text{boy})$
 $= 30/50 = 0.60.$

Reading a two-way table

Marginal distribution

A **marginal distribution** is one variable alone – a row total or a column total, as a percentage of the grand total.

Conditional distribution

A conditional distribution restricts to one row or column first, then takes percentages within it. That is what shows association.

Segmented bar charts

Segmented bar charts (or mosaic plots) draw the conditional distributions side by side: if the segments are the same height, there is no association.

Saying it

Compare conditional distributions in context – “68% of seniors, against 41% of juniors” – never raw counts.

Marginal answers “how many overall”; conditional answers “given that”. Almost every mistake in this unit is using the first when the question asked the second.

Comparing Groups with Conditional Distributions

A **conditional distribution** 条件分布 is the distribution of one variable *within* a fixed category of the other (found by dividing each cell by its row or column total).

If the conditional distributions differ across groups, the two variables are **associated**; if they are the same, there is no association.

3

Scatter

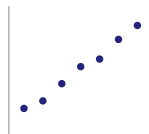
Scatterplots and correlation

Describe a **scatterplot** 散点图 by **direction** 方向, **form** 形态, **strength** 强度, outliers.

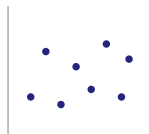
The **correlation** 相关系数 r measures *linear* strength:

$$-1 \leq r \leq 1$$

r near ± 1 is tight; near 0 is a cloud. r is **not resistant**, and never implies cause.



$r \approx 0.95$



$r \approx 0.1$

The regression line, term by term

Correlation coefficient

The **correlation coefficient** r measures the strength and direction of a *linear* relationship, between -1 and 1 – and it has no units.

Slope

The slope is the predicted change in y for each one-unit increase in x – state it in context, with the units.

y -intercept

The **y -intercept** is the predicted y when $x = 0$: meaningful only if $x = 0$ is inside the data's range.

Both from r

$b = r \frac{s_y}{s_x}$, and the line always passes through $(\bar{x}, \bar{y})$.

r describes the scatter; the line predicts from it. A high r still says nothing about causation.

4

Regression

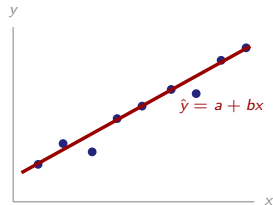
The regression line

The **least-squares regression line** 最小二乘回归线:

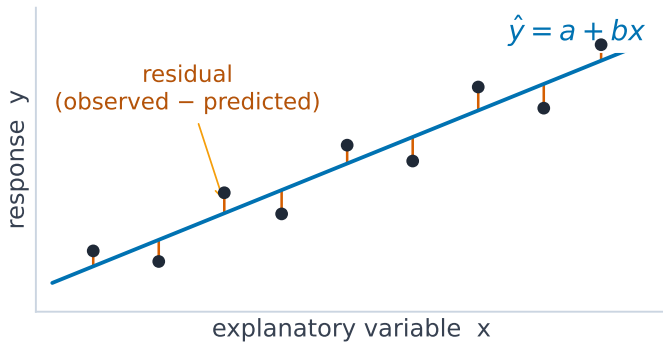
$$\hat{y} = a + bx$$

- **slope** 斜率 b : predicted change in y per unit x
- **intercept** 纵截距 a : predicted y at $x = 0$

Use it to **predict** 预测, but never **extrapolate** 外推 far beyond the data.



Least-Squares Regression and Its Fit



residual = $y - \hat{y}$ · least squares minimises $\sum (y - \hat{y})^2$

The least-squares line minimizes the sum of squared residuals

5

Residuals

Residuals and fit

A **residual** 残差 is the miss: $y - \hat{y}$.

- random **residual plot** 残差图 $\Rightarrow$ a line fits
- a curve $\Rightarrow$ line is wrong
- s : the **standard deviation of the residuals** – the typical prediction error, in y 's own units
- r^2 : the **coefficient of determination** 决定系数

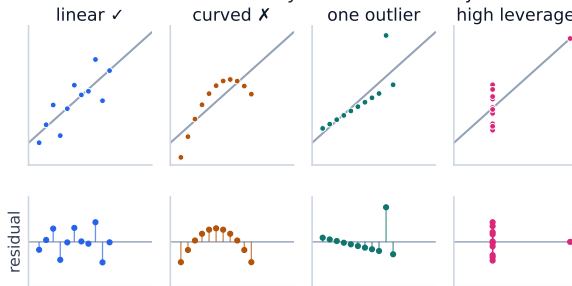


r^2 is the fraction of y 's variation the line explains – report it **in context**;

$$b = r \frac{s_y}{s_x}.$$

A caution about r and the line

every panel has the same $r = 0.82$ and the same line $\hat{y} = 3.0 + 0.5x$ — only the residual plot tells them apart



Anscombe: same stats, different plots — always graph data

All four datasets share $r = 0.82$ and the line $\hat{y} = 3.0 + 0.5x$, yet only the first is genuinely linear — always plot the data before you trust a summary.

6

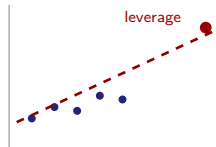
Departures

Departures from linearity

An **influential point** 影响点 swings the line when removed:

- a **y-outlier** sits far vertically
- a **high-leverage** 高杠杆 point sits at extreme x

If the pattern curves, a **transformation** 变换 (e.g. $\log y$) can straighten it for a line.



Correlation



Positive correlation rises together; negative correlation moves in opposite directions

Exam tips

- On a scatterplot describe **direction, form, strength, and outliers**; r ranges -1 to 1 .
- **Correlation is not causation** – a lurking variable can drive both.
- Interpret the slope of the least-squares line in context (“per one unit of x , predicted y changes by b ”).
- Check a **residual plot**: no pattern means a line fits; a curve means it does not. Avoid extrapolation.
- r^2 is the fraction of variation in y explained by the model.

7

Recap

Worked example – using a regression line

$\hat{y} = 3 + 2x$ with $r = 0.85$. Predict y at $x = 10$; the actual value is 25. Find the residual and interpret r^2 .

- $\hat{y} = 3 + 2(10) = 23$
- Residual = actual – predicted
 $= 25 - 23 = +2$ (the line under-predicted).
- $r^2 = 0.85^2 = 0.7225 \Rightarrow$ about 72% of the variation in y is explained by x .
- Slope: each 1-unit rise in x predicts a 2-unit rise in y on average.

Residual = actual – predicted (positive means above the line). Interpret in context with “on average” – and never claim causation from r alone.

Unit 2 – key takeaways

1

Association

explanatory vs response; conditionals show it

2

Correlation

$-1 \leq r \leq 1$; linear only; not resistant; not cause

3

Regression

$\hat{y} = a + bx$; read slope + intercept; don't extrapolate

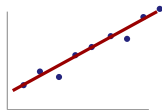
4

Fit

residual plot checks linearity; r^2 explained; leverage

Check yourself

- 1 What range must the correlation r fall in?
- 2 In $\hat{y} = a + bx$, what does the slope b mean?
- 3 What does a curved residual plot tell you?
- 4 Does a high r prove causation?



Answers 1. $-1 \leq r \leq 1$ 2. predicted change in y per unit x 3. a line is the wrong model 4. no