Sampling Distributions

AP Statistics

Why Two Samples Never Match: Sampling Variability

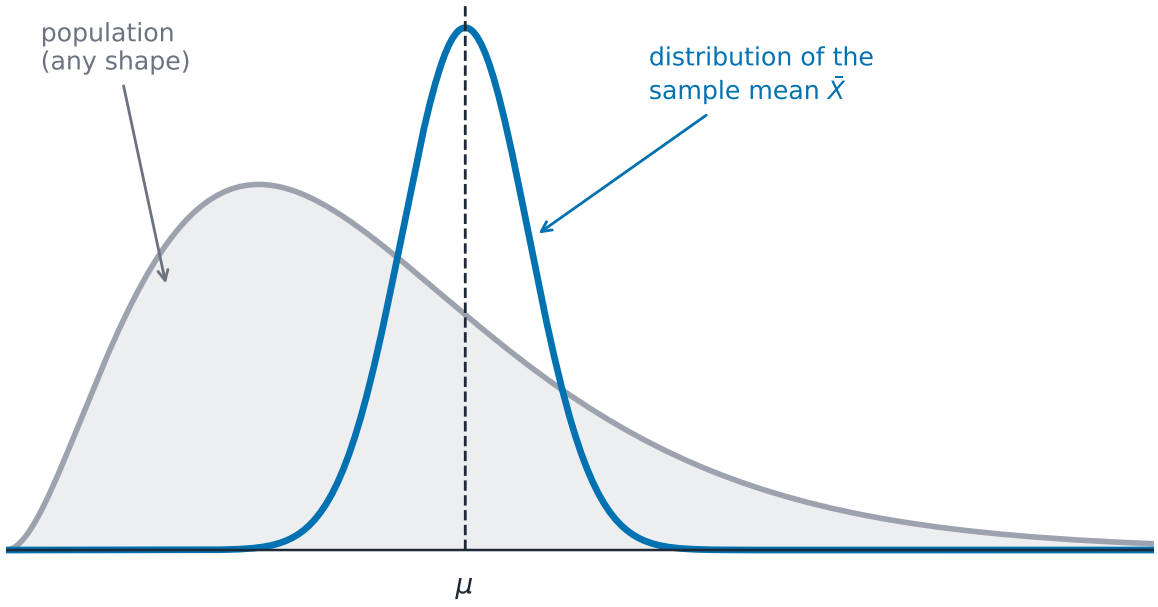
A **statistic** 统计量 (like a sample mean $\bar{x}$ or sample proportion $\hat{p}$) is computed from a sample and **varies** from sample to sample –this is **sampling variability** 抽样变异. A **parameter** 参数 (μ or p) is the fixed truth about the population. The **sampling distribution** 抽样分布 is the distribution of a statistic over all possible samples of a given size –it is the bridge from one sample to inference.

The Normal Curve as a Model for a Statistic

For large enough samples, many sampling distributions are approximately **normal**. That lets us describe a statistic by a **center** (its mean), a **spread** (its **standard error** 标准误), and a normal shape –and then compute how likely a given sample result is.

The Central Limit Theorem

The **Central Limit Theorem** 中心极限定理 (CLT): for a **sample mean**, if the sample size n is large enough (a common rule is $n \geq 30$), the sampling distribution of $\bar{x}$ is approximately **normal**, *regardless* of the population's shape. The larger n, the more normal and the tighter the distribution.



The sample mean is nearly normal whatever the shape of the population

Good Guesses and Bad Guesses: Bias

A statistic is **unbiased** 无偏 if the mean of its sampling distribution equals the parameter –it is correct **on average**. Bias is about the center being off; **variability** is about the spread. A good estimator is both unbiased (centered right) and low-variability (precise); larger samples reduce variability but do not fix bias from bad sampling.

The Sampling Distribution of a Sample Proportion

For a sample proportion $\hat{p}$ from an SRS: the mean is p (unbiased), and the standard deviation is

$$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}.$$

It is approximately **normal** when $np \geq 10$ and $n(1-p) \geq 10$ (the **Large Counts** condition), and the 10% condition ($n \leq 0.10N$) keeps the observations near-independent.

Worked example. Suppose 40% of voters favor a measure ($p = 0.4$) and you sample $n = 100$. The standard error is $\sigma_{\hat{p}} = \sqrt{\frac{0.4(0.6)}{100}} = 0.049$. The chance a sample gives $\hat{p} > 0.5$ is $z = \frac{0.5 - 0.4}{0.049} = 2.04$, so $P(\hat{p} > 0.5) \approx 0.02$ –a majority in the sample would be surprising.

Comparing Two Groups: Difference of Sample Proportions

For $\hat{p}_1 - \hat{p}_2$ from two independent samples: the mean is $p_1 - p_2$, and because the samples are independent the **variances add**:

$$\sigma_{\hat{p}_1 - \hat{p}_2} = \sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}}.$$

It is approximately normal when the Large Counts condition holds in **both** samples.

The Sampling Distribution of a Sample Mean

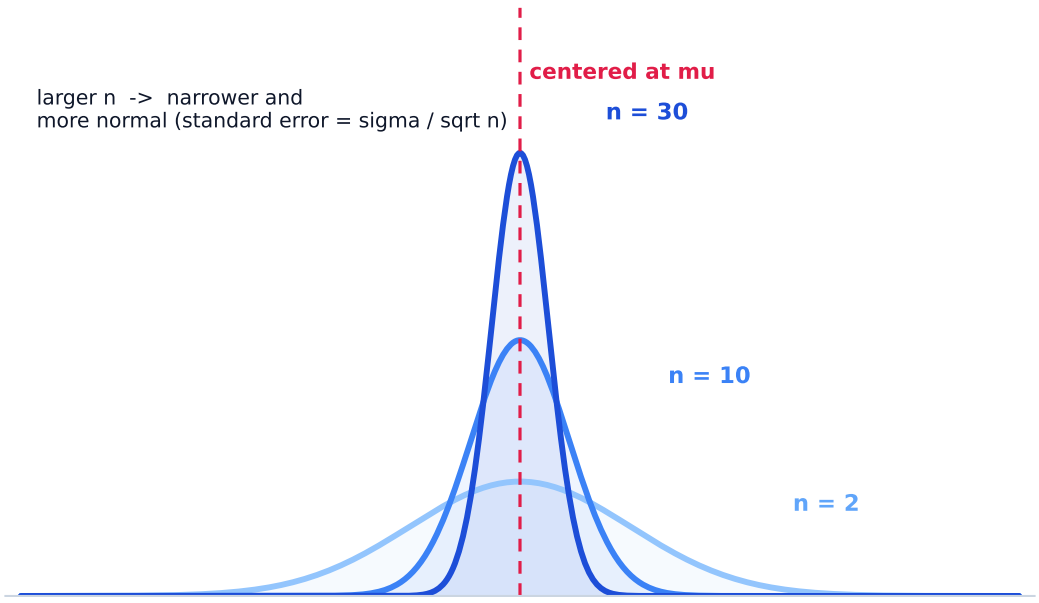
For a sample mean $\bar{x}$ from an SRS: the mean is μ (unbiased), and the standard deviation is

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}.$$

Its shape is normal if the population is normal, or approximately normal for large n by the CLT. Note the spread shrinks like $\sqrt{n}$ –quadrupling the sample halves the standard error.

Worked example. A population has $\mu = 70$ and $\sigma = 12$. For samples of $n = 36$, the sampling distribution of $\bar{x}$ is centered at 70 with standard error $\frac{12}{\sqrt{36}} = 2$. The chance a

sample mean exceeds 73 is $z = \dfrac{73 - 70}{2} = 1.5$, so $P(\bar{x} > 73) \approx 0.067$.



All three sampling distributions are centered at μ, but a larger n makes the standard error $\sigma/\sqrt{n}$ smaller, so the distribution of $\bar{x}$ is taller and narrower.

Comparing Two Groups: Difference of Sample Means

For $\bar{x}_1 - \bar{x}_2$ from two independent samples: the mean is $\mu_1 - \mu_2$, and (independent, so variances add)

$$\sigma_{\bar{x}_1 - \bar{x}_2} = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}.$$

This is the foundation for two-sample inference in the next units.

Exam tips

- A **sampling distribution** is the distribution of a statistic over many samples, **centered on the true parameter**.
- The **Central Limit Theorem**: for a large enough sample the sample mean is approximately normal, even if the population is not.
- Larger samples give **less** variability (a smaller standard error).
- Check the conditions (random, independent/10%, large enough) before using a normal model.
- Keep straight what varies —the statistic —versus the fixed parameter.