

## Question 5: Multi-Focus

4 points

## General Scoring Notes

- Each part of the question (indicated by a letter) is initially scored by determining if it meets the criteria for essentially correct (E), partially correct (P), or incorrect (I). The response is then categorized based on the scores assigned to each letter part and awarded an integer score between 0 and 4 (see the table at the end of the question).
- The model solution represents an ideal response to each section of the question, and the scoring criteria identify the specific components of the model solution that are used to determine the score.

| Model Solution                                                                                                                                                                                                                          | Scoring                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            |
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| (a) The scatterplot reveals a strong, positive, roughly linear association between the chest circumference and weight of tule elk. There are no points that seriously deviate from the straight-line pattern of the points in the plot. | <p><b>Essentially correct (E)</b> if the response provides a description that includes at least three of components 1-4 and component 5:</p> <ol style="list-style-type: none"> <li>Direction of association (positive or increasing)</li> <li>Strength of association (strong)</li> <li>Form of association (linear or approximately linear)</li> <li>Unusual features (no points with large discrepancies from the straight-line pattern exhibited by most of the points on the plot)</li> <li>Context (association between chest circumference and weight of tule elk)</li> </ol> <p><b>Partially correct (P)</b> if the response satisfies only one or two components out of components 1-4 and component 5<br/> OR<br/> if the response satisfies at least two out of components 1-4 but does not satisfy component 5.</p> <p><b>Incorrect (I)</b> if the response does not meet the criteria for E or P.</p> |

## Additional Notes:

- To satisfy component 4 it is sufficient to simply indicate that there are no unusual features or no outliers.
- To satisfy component 5 it is minimally sufficient for the response to refer to the association or relationship between chest circumference or measurement and weight without explicitly mentioning tule elk.
- The quality of communication in part (a) should be considered if holistic scoring is required.

| Model Solution                                                                                                                                                                                                                                                                                                                | Scoring                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |
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| <p>(b) (i) The predicted weight of a male tule elk with a chest circumference of 145.9 cm is <math>-350.3 + 3.7455(145.9) \approx 196.17</math> kg.</p> <p>(ii) The residual for a male tule elk with a chest circumference of 145.9 cm with an actual weight of 204.3 kg is <math>204.3 - 196.17 \approx 8.13</math> kg.</p> | <p><b>Essentially correct (E)</b> if the response satisfies the following two components:</p> <ol style="list-style-type: none"> <li>Provides the correct value of 196.17 kg in part (b-i) with work shown</li> <li>Provides the correct value of 8.13 kg in part (b-ii), or a value consistent with the predicted weight calculated for component 1, with work shown</li> </ol> <p><b>Partially correct (P)</b> if the response satisfies only one of the two components required for an E<br/> OR<br/> the response gives both correct values with no work shown.</p> <p><b>Incorrect (I)</b> if the response does not meet the criteria for E or P.</p> |

## Additional Notes:

- An arithmetic or transcription error in a response can be ignored if correct work is shown.
- A response that includes an undefined or incorrect variable to identify work shown (e.g.,  $x$  or  $\hat{p}$ ) does not satisfy component 1.

| Model Solution                                                                                                                                                                                                                      | Scoring                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                |
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| <p>(c) The value of the slope of the least-squares regression line is 3.7455. This value indicates that the predicted weight of a tule elk increases by 3.7455 kilograms for each additional centimeter of chest circumference.</p> | <p><b>Essentially correct (E)</b> if the response satisfies the following three components:</p> <ol style="list-style-type: none"> <li>1. Identifies the value of the slope as 3.7455</li> <li>2. Provides an interpretation that references an increase of a number of kilograms of weight for each one-centimeter increase in chest circumference</li> <li>3. Indicates that the slope represents a change in a prediction using non-deterministic language such as “predicted,” “estimated,” “expected,” or “average”</li> </ol> <p><b>Partially correct (P)</b> if the response satisfies only two of the three components.</p> <p><b>Incorrect (I)</b> if the response does not meet the criteria for E or P.</p> |

**Additional Notes**

- The value of the slope, 3.7455, may be rounded to 3.746, 3.75, or 3.7, but not to 3 or 4, to satisfy the numerical requirement in component 1.
- A response that only contains 3.7455 in the interpretation satisfies component 1.
- A calculation of slope may satisfy component 1 provided that two points from the line are used in the calculation.
- Units of measurements must be correctly specified for both weight and length to satisfy component 2.
- It is not required to refer specifically to the “least-squares regression line.”

| Model Solution                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        | Scoring                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            |
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| <p>(d) (i) The degrees of freedom for the test of slope are <math>n - 2 = 30 - 2 = 28</math>. The <math>t</math>-table shows that for 28 degrees of freedom, the <math>p</math>-value for a one-sided test would be 0.001. Because this is a two-sided test, the <math>p</math>-value is <math>(2)(0.001) = 0.002</math>.</p> <p>(ii) Because the <math>p</math>-value = 0.002 is less than <math>\alpha = 0.05</math>, reject the null hypothesis. There is sufficient statistical evidence that the population slope for the linear regression of weight vs. chest circumference for male tule elk is different from 4.5 kg/cm.</p> | <p><b>Essentially correct (E)</b> if the response satisfies the following three components:</p> <ol style="list-style-type: none"> <li>1. Gives a correct <math>p</math>-value of 0.002, consistent with the two-sided alternative hypothesis</li> <li>2. Provides correct comparison of the <math>p</math>-value to alpha (<math>p</math>-value is less than/greater than alpha) AND provides a correct decision about the null and/or alternative hypothesis</li> <li>3. Provides a conclusion statement in context, consistent with, and in terms of, the alternative hypothesis using non-deterministic language</li> </ol> <p><b>Partially correct (P)</b> if the response satisfies only two of the three components.</p> <p><b>Incorrect (I)</b> if the response does not meet the criteria for E or P.</p> |

**Additional Notes**

- The response need not make an explicit decision about the null hypothesis (reject  $H_0$  or fail to reject  $H_0$ ).
- If the conclusion and justification are consistent with an incorrect  $p$ -value (or an incorrect value of the test statistic, or an incorrect confidence interval, or an incorrect critical value), the response may satisfy component 2.
- To satisfy the  $p$ -value comparison in component 2, the response can compare the value of the test statistic to the appropriate critical value (e.g., 2.048 or -2.048).
- If an explicit decision is stated and the conclusion statement is inconsistent with the decision, component 2 is not satisfied.
- The decision part of component 2 may be satisfied by implying the decision within the conclusion statement (sufficient evidence/insufficient evidence for the alternative hypothesis).
- If the response includes a statement that is equivalent to accepting the null hypothesis (e.g., “we conclude that the mean population slope is 4.5”), then component 2 is not satisfied.
- A student who doesn’t give a  $p$ -value, but gives a correct conclusion based on a  $p$ -value  $< \alpha$  with a consistent conclusion OR a  $p$ -value  $> \alpha$  with a consistent conclusion may be scored P, e.g.,
  - If  $p$  were less than alpha, then reject  $H_0$  and say there is sufficient evidence that the population slope for the linear regression of weight vs. chest circumference for male tule elk is different than 4.5 kg/cm.
  - If  $p$  were greater than alpha, then fail to reject  $H_0$  and say there is not sufficient evidence that the population slope for the linear regression of weight vs. chest circumference for male tule elk is different than 4.5 kg/cm.
- If the response includes an incorrect interpretation of the  $p$ -value (e.g., “this is the probability that the null hypothesis is true”), then component 2 is not satisfied.
- The quality of communication in part (d) should be considered if holistic scoring is required.

**Scoring for Question 5**

Each essentially correct (E) part counts as 1 point, and each partially correct (P) part counts as  $\frac{1}{2}$  point.

**Score****Complete Response****4****Substantial Response****3****Developing Response****2****Minimal Response****1**

If a response is between two scores (for example, 2  $\frac{1}{2}$  points), use a holistic approach to decide whether to score up or down, depending on the strength of the response and quality of the communication.