

Question 5: Multi-Focus

4 points

General Scoring Notes

- Each part of the question (indicated by a letter) is initially scored by determining if it meets the criteria for essentially correct (E), partially correct (P), or incorrect (I). The response is then categorized based on the scores assigned to each letter part and awarded an integer score between 0 and 4 (see the table at the end of the question).
- The model solution represents an ideal response to each part of the question and the scoring criteria identify the specific components of the model solution that are used to determine the score.

Model Solution	Scoring
A i. Let random variable X represent the number of bedrooms in a randomly selected newly built house in the 2024 sample from Country B. The probability that a randomly selected house from the 2024 sample had fewer than 3 bedrooms is the probability that the house had either 1 or 2 bedrooms, which is $P(X < 3) = P(X = 1) + P(X = 2)$ $P(X < 3) = 0.12 + 0.22$ $P(X < 3) = 0.34 .$ ii. The average number of bedrooms per house for the sample of newly built houses in 2024 is $E(X) = 1(0.12) + 2(0.22) + 3(0.28) + 4(0.22) + 5(0.14) + 6(0.02)$ $E(X) = 3.10 \text{ bedrooms.}$	Essentially correct (E) if the response satisfies at least three of the following four components: - In part A (i) the response correctly calculates the probability of 0.34. - In part A (i) the response provides supporting work or justification for component 1. - In part A (ii) the response correctly calculates the mean of 3.10. - In part A (ii) the response provides supporting work or justification for component 3. Partially correct (P) if the response satisfies only two of the components required for E. Incorrect (I) if the response does not meet the criteria for E or P.

Scoring Notes:

- An arithmetic or transcription error in a response can be ignored if correct work is shown.
- Supporting work for finding the expected value must include at least two of the terms in the equation to show the pattern, such as $1(0.12) + 2(0.22) + \dots$.
- A response to part A (ii) that indicates use of the appropriate formula for the mean of a discrete random variable in words (e.g., “the sum of x times p of x for all values of x ”) or describes an appropriate method (e.g., using the given distribution to create a hypothetical sample of 100 and then calculating the mean) may satisfy component 4.

Model Solution	Scoring
B i. Let μ = the population mean number of bedrooms in newly built houses in 2024 from Country B. The null hypothesis is $H_0: \mu = 2.9$ and the alternative hypothesis is $H_a: \mu \neq 2.9$. ii. A Type I error would be determining that the population mean number of bedrooms in newly built houses in 2024 from Country B is not equal to 2.9 when it is in fact 2.9.	Essentially correct (E) if the response satisfies the following four components: 1. In part B (i) the response states the correct equality for the null hypothesis with the value 2.9. 2. In part B (i) the response states the correct two-sided alternative hypothesis consistent with the null hypothesis. 3. In part B (i) the response provides sufficient context for the parameter by including a reference to the population mean AND the response variable (bedrooms) AND the sampling units (newly built houses). 4. In part B (ii) the response correctly defines a Type I error in context. Partially correct (P) if the response satisfies only three of the four components required for E. Incorrect (I) if the response does not meet the criteria for E or P.

Scoring Notes:

- Components 1 and 2 may be satisfied without regard to the symbol (or lack of symbol) used to represent the population parameter.
- Neither context nor the concept of the population is required to satisfy components 1 or 2.
- A response that states the hypotheses in words (e.g., “the null hypothesis is that the mean is 2.9, and the alternative hypothesis is that the mean is not equal to 2.9”) may satisfy components 1 and 2.
- A response that states the hypotheses in words AND refers to the population in context (e.g., “the null hypothesis is that the population mean number of bedrooms in newly built houses in 2024 from Country B is equal to 2.9, and the alternative hypothesis is that the population mean number of bedrooms in newly built houses in 2024 from Country B is not equal to 2.9”) may satisfy components 1, 2, and 3.
- If the response clearly refers to the **sample** mean instead of the **population** mean using words or a symbol (e.g., $\bar{x}$ or $\hat{\mu}$), then component 3 is not satisfied unless the symbol used is defined as the **population** mean.
- The phrase “mean number of bedrooms” or “mean number of bedrooms for houses” is not sufficient for the population aspect of component 3.
- A response may satisfy the population aspect of component 3 by the following:
 - Referring to the population by using words such as “population,” “all,” or “true” when defining the parameter.
 - Using notation such as μ when defining the hypothesis statements.
- A response that refers to “bedrooms” OR “rooms” may satisfy context for component 4.

Model Solution	Scoring
C Because the value 2.9 is not contained within the 97% confidence interval, the null hypothesis should be rejected. Therefore, there is convincing statistical evidence, at the $\alpha = 0.03$ level of significance, that the population mean number of bedrooms in newly built houses in 2024 from Country B is not equal to 2.9 (or is different than that in 2017).	Essentially correct (E) if the response satisfies the following two components: 1. States a conclusion consistent with and in terms of the alternative hypothesis using nondefinitive language 2. Provides a justification for the conclusion by indicating that 2.9 is not contained within the confidence interval Partially correct (P) if the response satisfies only one of the two components required for E. Incorrect (I) if the response does not meet the criteria for E or P.

Scoring Notes:

- A response that only provides an interpretation of the given confidence interval is scored I.
- A response may satisfy the conclusion aspect of component 1 by using words such as “there is evidence to support the alternative,” “there is statistical evidence that H_a is true,” “I am 97% confident there is a difference in means,” or “because 2.9 is not included in our interval, it is not a plausible value for the mean.”
- If an explicit decision is stated and the conclusion is inconsistent with the decision, component 1 is not satisfied. A response that incorrectly indicates that 2.9 is contained in the confidence interval and then provides an otherwise correct conclusion based on 2.9 being contained in the interval satisfies component 1 but not component 2.
- A response that provides a conclusion that is consistent with an incorrect alternative hypothesis identified in part B satisfies component 1.
- A definitive response that states that the average number of bedrooms is 3.10 (the value of the sample mean or any other number) does not satisfy component 1, even if the response makes additional correct statements about the alternative hypothesis.
- A response that draws a conclusion that is clearly about the sample mean (e.g., a statement about “average number of bedrooms for newly built houses in the study”) does not satisfy component 1.
- If the conclusion includes a definitive statement (e.g., “this proves that we have enough evidence that the mean number of bedrooms in newly built homes in 2024 is 2.9” or “Rodney is correct; the mean number of bedrooms in newly built homes in 2004 is 2.9”), then component 1 is not satisfied.
- A response that reverse engineers the given confidence interval to obtain a standard error using a reasonable critical value for a 97% confidence interval and then uses the standard error to compute a t -statistic and p -value to reach a correct conclusion satisfies component 1 but does not satisfy component 2.

Scoring for Question 5	Score
Complete Response Three parts essentially correct	4
Substantial Response Two parts essentially correct and one part partially correct	3
Developing Response Two parts essentially correct and no part partially correct <i>OR</i> One part essentially correct and one or two parts partially correct <i>OR</i> Three parts partially correct	2
Minimal Response One part essentially correct and no parts partially correct <i>OR</i> No part essentially correct and two parts partially correct	1