

Question 6: Investigative Task

4 points

General Scoring Notes

- Each part of the question (indicated by a letter) is initially scored by determining if it meets the criteria for essentially correct (E), partially correct (P), or incorrect (I). The response is then categorized based on the scores assigned to each letter part and awarded an integer score between 0 and 4 (see the table at the end of the question).
- The model solution represents an ideal response to each part of the question, and the scoring criteria identify the specific components of the model solution that are used to determine the score.

Model Solution	Scoring
(a) (i) The inference procedure that should be used to estimate the mean price, in dollars (\$), of this type of whistle at all stores that sell the whistle is a one-sample t-interval for a population mean. (ii) The parameter of interest is the mean whistle price, in dollars (\$), of this type of whistle at all stores that sell the whistle.	Essentially correct (E) if the response satisfies the following three components: - In part (a-i) the response identifies a one-sample t-interval for a population mean by name (e.g., “one sample t-interval”) - In part (a-ii) the response identifies the correct parameter by including reference to the mean - In part (a-ii) the response provides sufficient context for the parameter by including reference to the population (all stores that sell the whistle) and the variable of interest (the price of this type of whistle) Partially correct (P) if the response only satisfies one or two of the three components required for E. Incorrect (I) if the response does not meet the criteria for E or P.

Additional Notes:

- In part (a-i) a response that refers to a “test” does not satisfy component 1.
- In part (a-i) a response that refers to “a mean,” singular, satisfies the one-sample aspect of component 1. If the response states “one-sample,” it is not required to also include “for a mean” to satisfy component 1.
- A response that addresses “mean” in part (a-i) does not satisfy component 2 unless it is also addressed in part (a-ii).
- In part (a-ii) a response may satisfy the variable of interest aspect of component 3 with a minimum of “price” and “whistle.”
- In part (a-ii) a response may satisfy the population aspect of component 3 by using words such as: “population,” “all,” or “true,” or by using the symbol μ .
- In part (a-ii) if the response clearly refers to the sample mean instead of the population mean using words or symbols (e.g., $\bar{x}$), then component 3 is not satisfied unless the symbol used is defined as the population mean.
- For responses that are scored P, the number of components satisfied should be considered if holistic scoring is required.

Model Solution	Scoring
(b) (i) The distribution of the sample of whistle prices appears slightly skewed to the right, because the mean is slightly higher than the median. (ii) Based on the $1.5 \times \text{IQR}$ rule, there are not whistle prices in this sample that would be considered outliers. A whistle price is an outlier using this method if it is more than $1.5 \times \text{IQR}$ below the first quartile (Q_1) or more than $1.5 \times \text{IQR}$ above the third quartile (Q_3). Because $Q_1 - 1.5 \times \text{IQR} = 4.51 - 1.5(5.475 - 4.51) = 3.0625,$ and the minimum value (4.25) is greater than 3.0625, there are no outliers to the left. Because $Q_3 + 1.5 \times \text{IQR} = 5.475 + 1.5(5.475 - 4.51) = 6.9225,$ and the maximum value (6.58) is less than 6.9225, there are no outliers to the right.	Essentially correct (E) if the response satisfies the following four components: - In part (b-i) the response indicates the distribution is skewed to the right or approximately symmetric - In part (b-i) the response justifies the response to component 1 with appropriate reasoning based on the summary statistics - In part (b-ii) the response provides a justification for stating there are no outliers by correctly calculating the lower and upper outlier criteria with work shown - In part (b-ii) the response states whether there are outliers based on the calculated lower and upper outlier criteria Partially correct (P) if the response satisfies three of the four components required for E. <i>OR</i> satisfies components 1 and 2 <i>OR</i> components 3 and 4 of components 1–4 required for E. Incorrect (I) if the response does not meet the criteria for E or P.

Additional Notes:

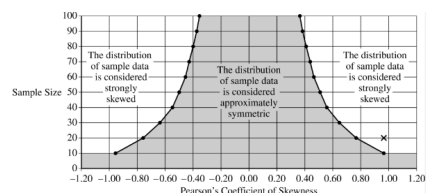
- If the response states skewed right, to satisfy component 2 the response should refer to the median being less than the mean. If the response states approximately symmetric, the response should refer to the median and mean being close together.
- A response that indicates skewed left does not satisfy components 1 or 2.
- If the response says the distribution is skewed to the right, examples of responses that satisfy component 2 include:
 - the mean is higher than the median,
 - the ratio between the mean and median is greater than 1,
 - the difference between the maximum and median is greater than the difference between the median and minimum, and
 - the difference between the maximum and third quartile is greater than the difference between the first quartile and minimum.
- For responses that score P, the number of components satisfied should be considered if holistic scoring is required.

Model Solution

Scoring

- (c) (i) Pearson's coefficient of skewness is
- $$\frac{3(5.12 - 4.885)}{0.743} \approx 0.949.$$

(ii)



Essentially correct (E) if the response satisfies the following two components:

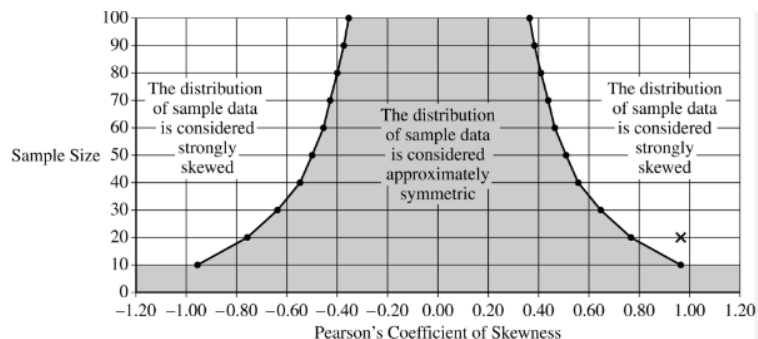
1. In part (c-i) the response calculates Pearson's coefficient of skewness with work shown
2. In part (c-ii) the response includes an x at approximately (0.949, 20) or an x consistent with the calculation in part (c-i)

Partially correct (P) if the response satisfies only one of the two components required for E.

Incorrect (I) if the response does not meet the criteria for E or P.

Additional Notes:

- An arithmetic or transcription error in a response can be ignored if correct work is shown.
- Responses must include an x that is located on the line for sample size of 20 and located between 0.90 and 1.00 on the horizontal axis or be consistent with the value given in part (c-i) to satisfy component 2.



Model Solution

Scoring

- (d) (i) Looking at the graph in part (c), for a sample size of 20, and a skewness coefficient of 0.949, this point falls in "the distribution of sample data is considered strongly skewed" region. Therefore, we would consider the shape of the distribution of the sample of whistle prices to be strongly skewed.
- (ii) No, based on the response to part (d-i), Julio's data would not satisfy the normality condition because neither of the criteria listed are met. Julio only has a sample size of 20, which is less than 30, and Pearson's coefficient of skewness indicates the distribution of sample data is strongly skewed.

Essentially correct (E) if the response satisfies the following five components:

1. In part (d-i) the response indicates the sample is strongly skewed or makes a statement consistent with the graph in part (c)
2. In part (d-i) the response uses or refers to the graph or the Pearson coefficient of skewness to make the correct determination of skewness
3. In part (d-ii) the response states that the condition of normality is not met or makes a statement consistent with the explanation in part (d-i)
4. In part (d-ii) the response explains the sample size of 20 is less than 30
5. In part (d-ii) the response explains that the distribution of sample data is strongly skewed or makes a statement consistent with the explanation in part (d-i)

Partially correct (P) if the response does not meet the criteria for E but satisfies three or four of components 1–5 required for E.

Incorrect (I) if the response does not meet the criteria for E or P.

Additional Notes:

- A response that determines the sample is strongly skewed, but says the normality condition is satisfied, should be scored I.
- In part (d-i) a response that indicates the distribution is strongly skewed to the left does not satisfy component 1.
- If a response in part (c-ii) has an x on the curve, the response in part (d-i) must indicate the sample is skewed (not strongly skewed or approximately symmetric), or indicate it is not possible to choose whether it is strongly skewed or approximately symmetric to satisfy component 1.
- In part (d-i) a response that indicates the distribution is skewed, but not strongly skewed, because the Pearson coefficient of skewness is not that far from the curve on the graph satisfies component 1 and component 2.
- In part (d-i) a response that indicates the distribution is skewed, but not strongly skewed, may satisfy components 3–5 if in part (d-ii) the response indicates the conditions are met because the distribution is not very skewed and there are no outliers. The response does not need to indicate the sample size is less than 30.
- In part (d-ii) a response that indicates the sample size is small without explicitly referring to the values 20 and 30 may satisfy component 4.
- The quality of communication, or the number of components satisfied for responses with score P, should be considered if holistic scoring is required.

Scoring for Question 6

Each essentially correct (E) part counts as 1 point, and each partially correct (P) part counts as $\frac{1}{2}$ point.

Score**Complete Response****4****Substantial Response****3****Developing Response****2****Minimal Response****1**

If a response is between two scores (for example, 2 $\frac{1}{2}$ points), use a holistic approach to decide whether to score up or down, depending on the strength of the response and quality of the communication.