

Question 5: Multi-Focus

4 points

General Scoring Notes

- Each part of the question (indicated by a letter) is initially scored by determining if it meets the criteria for essentially correct (E), partially correct (P), or incorrect (I). The response is then categorized based on the scores assigned to each letter part and awarded an integer score between 0 and 4 (see the table at the end of the question).
- The model solution represents an ideal response to each part of the question and the scoring criteria identify the specific components of the model solution that are used to determine the score.

Model Solution	Scoring
A i. Let random variable X represent the number of bedrooms in a randomly selected newly built house in the 2024 sample from Country B. The probability that a randomly selected house from the 2024 sample had fewer than 3 bedrooms is the probability that the house had either 1 or 2 bedrooms, which is $P(X < 3) = P(X = 1) + P(X = 2)$ $P(X < 3) = 0.12 + 0.22$ $P(X < 3) = 0.34.$ ii. The average number of bedrooms per house for the sample of newly built houses in 2024 is $E(X) = 1(0.12) + 2(0.22) + 3(0.28) + 4(0.22) + 5(0.14) + 6(0.02)$ $E(X) = 3.10 \text{ bedrooms.}$	Essentially correct (E) if the response satisfies at least three of the following four components: - In part A (i) the response correctly calculates the probability of 0.34. - In part A (i) the response provides supporting work or justification for component 1. - In part A (ii) the response correctly calculates the mean of 3.10. - In part A (ii) the response provides supporting work or justification for component 3. Partially correct (P) if the response satisfies only two of the components required for E. Incorrect (I) if the response does not meet the criteria for E or P.

Scoring Notes:

- An arithmetic or transcription error in a response can be ignored if correct work is shown.
- Supporting work for finding the expected value must include at least two of the terms in the equation to show the pattern, such as $1(0.12) + 2(0.22) + \dots$.
- A response to part A (ii) that indicates use of the appropriate formula for the mean of a discrete random variable in words (e.g., “the sum of x times p of x for all values of x ”) or describes an appropriate method (e.g., using the given distribution to create a hypothetical sample of 100 and then calculating the mean) may satisfy component 4.

Model Solution	Scoring
B i. Let μ = the population mean number of bedrooms in newly built houses in 2024 from Country B. The null hypothesis is $H_0: \mu = 2.9$ and the alternative hypothesis is $H_a: \mu \neq 2.9$. ii. A Type I error would be determining that the population mean number of bedrooms in newly built houses in 2024 from Country B is not equal to 2.9 when it is in fact 2.9.	Essentially correct (E) if the response satisfies the following four components: - In part B (i) the response states the correct equality for the null hypothesis with the value 2.9. - In part B (i) the response states the correct two-sided alternative hypothesis consistent with the null hypothesis. - In part B (i) the response provides sufficient context for the parameter by including a reference to the population mean AND the response variable (bedrooms) AND the sampling units (newly built houses). - In part B (ii) the response correctly defines a Type I error in context. Partially correct (P) if the response satisfies only three of the four components required for E. Incorrect (I) if the response does not meet the criteria for E or P.

Scoring Notes:

- Components 1 and 2 may be satisfied without regard to the symbol (or lack of symbol) used to represent the population parameter.
- Neither context nor the concept of the population is required to satisfy components 1 or 2.
- A response that states the hypotheses in words (e.g., “the null hypothesis is that the mean is 2.9, and the alternative hypothesis is that the mean is not equal to 2.9”) may satisfy components 1 and 2.
- A response that states the hypotheses in words AND refers to the population in context (e.g., “the null hypothesis is that the population mean number of bedrooms in newly built houses in 2024 from Country B is equal to 2.9, and the alternative hypothesis is that the population mean number of bedrooms in newly built houses in 2024 from Country B is not equal to 2.9”) may satisfy components 1, 2, and 3.
- If the response clearly refers to the **sample** mean instead of the **population** mean using words or a symbol (e.g., $\bar{x}$ or $\hat{\mu}$), then component 3 is not satisfied unless the symbol used is defined as the **population** mean.
- The phrase “mean number of bedrooms” or “mean number of bedrooms for houses” is not sufficient for the population aspect of component 3.
- A response may satisfy the population aspect of component 3 by the following:
 - Referring to the population by using words such as “population,” “all,” or “true” when defining the parameter.
 - Using notation such as μ when defining the hypothesis statements.
- A response that refers to “bedrooms” OR “rooms” may satisfy context for component 4.

Model Solution	Scoring
C Because the value 2.9 is not contained within the 97% confidence interval, the null hypothesis should be rejected. Therefore, there is convincing statistical evidence, at the $\alpha = 0.03$ level of significance, that the population mean number of bedrooms in newly built houses in 2024 from Country B is not equal to 2.9 (or is different than that in 2017).	Essentially correct (E) if the response satisfies the following two components: 1. States a conclusion consistent with and in terms of the alternative hypothesis using nondefinitive language 2. Provides a justification for the conclusion by indicating that 2.9 is not contained within the confidence interval Partially correct (P) if the response satisfies only one of the two components required for E. Incorrect (I) if the response does not meet the criteria for E or P.

Scoring Notes:

- A response that only provides an interpretation of the given confidence interval is scored I.
- A response may satisfy the conclusion aspect of component 1 by using words such as “there is evidence to support the alternative,” “there is statistical evidence that H_a is true,” “I am 97% confident there is a difference in means,” or “because 2.9 is not included in our interval, it is not a plausible value for the mean.”
- If an explicit decision is stated and the conclusion is inconsistent with the decision, component 1 is not satisfied. A response that incorrectly indicates that 2.9 is contained in the confidence interval and then provides an otherwise correct conclusion based on 2.9 being contained in the interval satisfies component 1 but not component 2.
- A response that provides a conclusion that is consistent with an incorrect alternative hypothesis identified in part B satisfies component 1.
- A definitive response that states that the average number of bedrooms is 3.10 (the value of the sample mean or any other number) does not satisfy component 1, even if the response makes additional correct statements about the alternative hypothesis.
- A response that draws a conclusion that is clearly about the sample mean (e.g., a statement about “average number of bedrooms for newly built houses in the study”) does not satisfy component 1.
- If the conclusion includes a definitive statement (e.g., “this proves that we have enough evidence that the mean number of bedrooms in newly built homes in 2024 is 2.9” or “Rodney is correct; the mean number of bedrooms in newly built homes in 2004 is 2.9”), then component 1 is not satisfied.
- A response that reverse engineers the given confidence interval to obtain a standard error using a reasonable critical value for a 97% confidence interval and then uses the standard error to compute a t -statistic and p -value to reach a correct conclusion satisfies component 1 but does not satisfy component 2.

Scoring for Question 5	Score
Complete Response Three parts essentially correct	4
Substantial Response Two parts essentially correct and one part partially correct	3
Developing Response Two parts essentially correct and no part partially correct <i>OR</i> One part essentially correct and one or two parts partially correct <i>OR</i> Three parts partially correct	2
Minimal Response One part essentially correct and no parts partially correct <i>OR</i> No part essentially correct and two parts partially correct	1

Question 4: Focus on Inference

4 points

General Scoring Notes

- This question is scored in four sections. Each section is initially scored by determining if it meets the criteria for essentially correct (E), partially correct (P), or incorrect (I). The first section includes statements of the null and alternative hypotheses and identification of the appropriate hypothesis test in part (a). The second section includes verifying the conditions for the test identified in part (a) and calculating the value of the test statistic and the corresponding p -value. The third section includes the conclusion for the test identified in part (a). The fourth section includes the response to part (b). The response is then categorized based on the scores assigned to each section and awarded an integer score between 0 and 4 (see the table at the end of the question).
- The model solution represents an ideal response to each section of the question, and the scoring criteria identify the specific components of the model solution that are used to determine the score.

	Model Solution	Scoring
(a) Section 1	Let p represent the proportion of all customers of the pet supply company who would place an order within 30 days after receiving an e-mail with a coupon for \$10 off the next purchase. The null hypothesis is $H_0: p = 0.40$, and the alternative hypothesis is $H_a: p > 0.40$. An appropriate test is a one-sample z-test for a population proportion.	Essentially correct (E) if the response satisfies the following three components: - States the correct equality for the null hypothesis for a proportion (e.g., $p = 0.40$) AND the correct direction of the one-sided alternative hypothesis for a proportion (e.g., $p > 0.40$) - Provides sufficient context for the parameter by including reference to the <i>population</i> proportion AND the sampling units (customers) AND the response variable (placing an order after receiving a coupon) - Identifies a one-sample z-test for a population proportion by name (e.g., “one-proportion z-test”) but <u>not</u> merely “one-sample z-test”) or by formula Partially correct (P) if the response does not meet the criteria for E but satisfies either component 1 and/or component 3. Incorrect (I) if the response does not meet the criteria for E or P.

Additional Notes:

- The elements of component 2 do not have to be satisfied with the statement of the hypotheses. They may be satisfied by work presented anywhere in the response, most likely by the statement of the conclusion.
- If the statement of the hypotheses refers to population proportion and the conclusion refers to sample proportion (or vice versa), then the population aspect of component 2 is not satisfied.

- A response that states the null hypothesis as $H_0: p \leq 0.40$ may satisfy component 1.
- To satisfy component 1, the hypotheses must be stated in terms of a proportion. If a symbol other than p or π is used to denote the proportion, it must be clearly defined as a proportion (but does not need to reflect the context of customers who would place an order within 30 days after receiving a coupon) in order for the response to satisfy component 1. It is acceptable to use “ p_0 ” to denote the proportion.
- A response that states the hypotheses in words (e.g., “the null hypothesis is that the proportion is 0.40, and the alternative hypothesis is that the proportion is greater than 0.40”) may satisfy component 1. Neither context nor the concept of the *population* is required to satisfy component 1.
- A response that states the hypotheses in words (e.g., “the null hypothesis is that the proportion of all customers who would place an order within 30 days after receiving a coupon is equal to 0.40, and the alternative hypothesis is that the proportion is greater than 0.40”) may satisfy component 1 and component 2.
- If the response clearly refers to the *sample* proportion instead of the *population* proportion using words or a symbol (e.g., $\hat{p}$), then component 2 is not satisfied unless the symbol used is defined as the *population* proportion.
- A response may satisfy the population aspect of component 2 by doing the following:
 - referring to population in the statement of the conclusion of the inferential procedure
 - using notation such as p , p_0 , or π when defining the hypothesis statements.
- A response may satisfy the sampling units aspect of component 2 by referring to “people who place an order” or similar statement.
- If the response identifies the correct test by name, but also states an incorrect formula, then component 3 is not satisfied.
- If the response identifies the test by formula using a t -percentile instead of a z -percentile, then component 3 is not satisfied.

Confidence Interval Approach:

- If a one-sample z -interval for a population proportion is identified correctly by name (e.g., “one-proportion z -interval”) but not merely “one-sample z -interval”) or by formula, then component 3 is satisfied.
- If a response uses a one-sample z -interval for a population proportion, then component 2 is satisfied if the response indicates that it is a confidence interval for the proportion of all customers who would place an order within 30 days after receiving a coupon, even if the hypotheses are not stated.

	Model Solution	Scoring
(a) Section 2	The independent observations condition for performing the one-sample z-test for a population proportion is satisfied because the data were obtained from a random sample of 90 customers who placed an order in the past year and, because sampling of customers is done without replacement, it is assumed that this large online company has more than $10(90) = 900$ customers. The sample size is large enough to support an assumption that the sampling distribution of $\hat{p}$ is approximately normal because $(90)(0.4) = 36$ and $(90)(1 - 0.4) = 54$ are both at least 10. The value of the sample proportion is $\hat{p} = \frac{38}{90} \approx 0.422$ and the value of the test statistic is $z = \frac{\frac{38}{90} - 0.40}{\sqrt{\frac{(0.40)(0.60)}{90}}} \approx 0.430$. The corresponding p-value is $P(z > 0.430) \approx 0.333$.	Essentially correct (E) if the response satisfies the following four components: 1. Checks the independence condition by referring to the random selection of 90 customers AND indicating that the company is assumed to have at least 900 customers (i.e., $90 \leq 0.10N$) 2. Checks that the sample size is large enough to support the assumption that the sampling distribution of $\hat{p}$ is approximately normal by verifying that $(90)(0.4)$ and $(90)(1 - 0.4)$ are both at least 10 (or 5) 3. Correctly reports the value of the z-statistic 4. Correctly reports the p-value, consistent with the reported test statistic and stated alternative hypothesis Partially correct (P) if the response satisfies only two or three of the four components. Incorrect (I) if the response does not meet the criteria for E or P.

Additional Notes:

- In order to satisfy the reference to the random selection of 90 customers in component 1 it is minimally acceptable to state “random sample – check” or “SRS – check.” However, component 1 is not satisfied if the response implies that random *assignment* was used or only states “random - check.”
- In order to satisfy component 2, the response must include actual values of the observed successes and failures, or values for the expected successes and failures, or formulas for the expected number of successes and failures with values inserted AND the response must make a comparison of the two values with some standard criterion, such as 5 or 10. If expressions such as $(90)(0.4)$ and $(90)(1 - 0.4)$ are used, simplification is not required.
 - Examples of acceptable quantities (comparisons must still be made):
 - 38 and 52 (observed counts)
 - 36 and 54 (expected counts under the null hypothesis)
 - $(90)(0.4)$ and $(90)(1 - 0.4)$
 - $(90)(0.4222)$ and $(90)(1 - 0.422)$
 - Unless values of all parameters are explicitly defined in the response, the following quantities are unacceptable:
 - $90p$, $90(1 - p)$, np , $n(1 - p)$
 - $90\hat{p}$, $90(1 - \hat{p})$, $n\hat{p}$, $n(1 - \hat{p})$
 - $n(0.4)$, $n(0.6)$, $n(0.4222)$, $n(1 - 0.4222)$

- When computing the test statistic, using a $\hat{p}$ of 0.42 in the numerator results in a test statistic equal to 0.39 with a p -value of 0.36. These values satisfy components 3 and 4.
- If the response uses $\hat{p}$ in the null standard error formula to calculate the z -statistic, component 3 may be satisfied.
- A response that reports the correct value for the z -statistic but contains errors in supporting work may still satisfy component 3.
- If the response satisfies component 4, any supporting work for the p -value may be treated as extraneous.
- If the response compares the value of the test statistic to a critical value instead of reporting a p -value, then the critical value (1.645), or a critical value consistent with the stated alternative hypothesis, satisfies component 4.
- If a two-tailed alternative hypothesis is stated or the direction of the stated one-tailed alternative hypothesis is incorrect, then the p -value must be consistent with the stated alternative hypothesis to satisfy component 4.
- If the response omits identifying the hypotheses, the correct one-sided alternative hypothesis is assumed when scoring component 4.
- If an incorrect alternative hypothesis is stated, then the p -value must be consistent with the stated alternative hypothesis to satisfy component 4.

Confidence Interval Approach:

- If either a one-sided 95 percent confidence interval is correctly calculated as $(0.337, 1)$, or a two-sided 90 percent confidence interval is correctly calculated as $(0.337, 0.508)$, then component 3 is satisfied.
- If only the lower end of a confidence interval is used to reach a conclusion, then component 4 is satisfied.
- Application of a confidence interval approach must be consistent with the stated alternative to satisfy component 4. A two-sided 95 percent confidence interval is $(0.320, 0.524)$, and a lower one-sided 95 percent confidence interval is $(0, 0.508)$.

	Model Solution	Scoring
(a) Section 3	Because the p -value is greater than $\alpha = 0.05$, the null hypothesis should not be rejected. The results from this study do not provide convincing statistical evidence that the manager's belief is correct. That is, there is not convincing statistical evidence that more than 40 percent of all customers of the pet supply company would place an order within 30 days after receiving an e-mail with a coupon for \$10 off the next purchase.	Essentially correct (E) if the response satisfies the following two components: 1. Provides a correct justification of the conclusion based on whether the p-value is less than $\alpha = 0.05$ (or a comparison of the value of the test statistic to an appropriate critical value, e.g., $z < -1.645$) 2. States a correct conclusion consistent with the stated alternative hypothesis OR states a conclusion that answers the inference question (e.g., states the conclusion in terms of the manager's belief) Partially correct (P) if the response satisfies only one of the two components. Incorrect (I) if the response does not meet the criteria for E or P.

Additional Notes:

- Although including proper context (the concept of population proportion and referencing the response variable) is important in stating the conclusion, context displayed in stating the conclusion is considered in scoring component 2 of Section 1.
- The response need not make an explicit decision about the null hypothesis (reject H_0 or fail to reject H_0) in order to satisfy component 1. However, if an incorrect decision is stated, then component 1 is not satisfied.
- If the conclusion and justification are consistent with an incorrect p -value (or an incorrect value of the test statistic, or an incorrect confidence interval), the response may satisfy component 1 and component 2.
- If the response omits hypotheses, assume the correct alternative hypothesis, $H_a: p > 0.40$, was provided when scoring component 1 and component 2.
- If the conclusion includes a definitive statement (e.g., "this proves that we do not have enough evidence to claim..." or "there is no evidence..."), then component 2 is not satisfied.
- If the response includes a statement that is equivalent to accepting the null hypothesis (e.g., "we conclude that the proportion of customers who will place an order is 0.40"), then component 2 is not satisfied.
- If the response includes an incorrect interpretation of the p -value, then the score for Section 3 is lowered one level (that is, from E to P or from P to I).
- The clarity and quality of the statement of the conclusion and the statement of the justification may be used in a holistic approach to decide whether to score up or down (e.g., raising a score of 2.5 to 3 or reducing a score of 2.5 to 2).

Confidence Interval Approach:

- If the alternative hypothesis is specified correctly as $H_a: p > 0.40$, then component 1 is satisfied if the justification is based on whether 0.40 is below the lower end of the confidence interval. If the alternative hypothesis is stated in the wrong direction, then component 1 is satisfied if the justification is based on

- If no alternative hypothesis is specified in the response, then assume the correct alternative hypothesis is provided when scoring component 2.
- If an incorrect two-sided alternative hypothesis is specified, then component 2 is satisfied if the justification is based on whether 0.40 is included in the confidence interval.
- If the response includes an incorrect interpretation of the confidence interval, then the score for Section 3 is lowered one level (that is, from E to P or from P to I).

	Model Solution	Scoring
(b) Section 4	Because the null hypothesis was not rejected in part (a), a Type II error could have been made. A Type II error occurs when the null hypothesis is false and is not rejected. In this case, a Type II error is made by failing to reject the null hypothesis that 40 percent (or less) of all customers of the pet supply company would place an order within 30 days after receiving an e-mail with a coupon for \$10 off the next purchase, when in fact, more than 40 percent would do so. Consequently, the manager may decide not to use the coupon promotion when it actually would result in more than 40 percent of their customers making a purchase within 30 days.	Essentially correct (E) if the response satisfies the following two components: 1. States that a Type II error could have been made 2. Provides a reasonable interpretation of the consequence of the stated error AND uses sufficient context, by including at minimum "those who would place an order" or "coupon" or "sales" (e.g., indicating the manager may not use the coupon promotion when it would actually lead to more than 40% of customers placing an order) Partially correct (P) if the response satisfies only one of the two components. Incorrect (I) if the response does not meet the criteria for E or P.

Additional Notes:

- If the response to part (a) rejects the null hypothesis, then
 - Component 1 is satisfied if the response states that a Type I error could have been made.
 - Component 2 is satisfied if the response provides a reasonable interpretation of the consequence of a Type I error AND uses sufficient context by including at minimum "those who would place an order" or "coupon" or "sales."
- If the response states that a Type II error could have been made, followed by an incorrect description of a Type II error (e.g., "did not find convincing evidence that more than 40% of customers will place an order when there actually was evidence of more than 40%"), component 1 is not satisfied.
- The clarity and quality of the statement of the consequence may be used in a holistic approach to decide whether to score up or down (e.g., raising a score of 2.5 to 3 or reducing a score of 2.5 to 2).

Scoring for Question 4

Each essentially correct (E) part counts as 1 point, and each partially correct (P) part counts as $\frac{1}{2}$ point.

Score**Complete Response****4****Substantial Response****3****Developing Response****2****Minimal Response****1**

If a response is between two scores (for example, 2 $\frac{1}{2}$ points), use a holistic approach to decide whether to score up or down, depending on the strength of the response and quality of the communication.

2018**AP** CollegeBoard

AP Statistics

Scoring Guidelines

AP Statistics

2018 SCORING GUIDELINES

Question 1**Intent of Question**

The primary goals of this question were to assess a student's ability to (1) identify various values in regression computer output; (2) interpret the intercept of a regression line in context; (3) interpret the coefficient of determination (r^2) in context; and (4) identify an outlier from a scatterplot.

Solution**Part (a):**

The estimate of the intercept is 72.95. It is estimated that the average time to finish checkout if there are no other customers in line is 72.95 seconds.

Part (b):

The coefficient of determination is $r^2 = 73.33\%$. This value indicates that 73.33% of the variability in the times it takes customers to finish checkout, including time waiting in line, can be explained by knowing how many customers are in line in front of the selected customer.

Part (c):

The outlier is the point with $x = 3$ and y close to 0. This point is considered an outlier because the combination of x and y values differs from the pattern of the rest of the data. Specifically, the value of y (time to finish checkout) is much lower than would be expected when there are $x = 3$ customers in line in front of the selected customer, given the remaining data.

Scoring

Parts (a), (b), and (c) are scored as essentially correct (E), partially correct (P), or incorrect (I).

Part (a) is scored as follows:

Essentially correct (E) if the response satisfies the following three components:

1. Correctly identifies 72.95 as the intercept.
2. Communicates the concept of a y -intercept in a context that includes both time and zero customers.
3. Indicates that the value of the intercept is a prediction by using language such as "predicted," "estimated," or "average" value of y .

Partially correct (P) if the response includes only two of the three components.

Incorrect (I) if the response includes at most one of the three components.

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Question 1 (continued)

Notes:

- Regression equations (such as $\hat{y} = 72.95 + 174.40x$) cannot be used to satisfy identification of the intercept in component 1, unless the intercept is explicitly labeled as such.
- A regression equation cannot be used to satisfy component 3.
- Incorrect regression equations are treated as extraneous and do not affect the scoring of any component.
- A response that interprets 72.95 as a slope does not satisfy components 1 or 2.

Part (b) is scored as follows:

Essentially correct (E) if the response satisfies the following three components:

1. Correctly identifies 73.33% as the coefficient of determination.
2. Provides a correct (possibly generic) interpretation of r^2 .
3. Interpretation includes context.

Partially correct (P) if the response satisfies only two of the three components;

OR

if the response satisfies the three components, but reverses the roles of number of customers in line and time to finish checkout in the interpretation.

Incorrect (I) if the response satisfies at most one of the three components.

Notes:

- In component 2 the correct interpretation of the coefficient of determination can take any of several equivalent forms, such as:
 - The percent variability in y that is attributed to the linear relationship between y and x or between x and y .
 - The proportion of the total variability in the dependent variable y that is explained by the independent variable x .
 - The proportion of variation in y that is accounted for by the linear model.
 - The proportionate reduction of total variation of the y values that is associated with the use of the independent variable x .
 - The proportionate reduction in the sum of the squares of vertical deviations obtained by using the least-squares line instead of the naïve prediction of $\bar{y}$.
- In component 2 common *incorrect* interpretations of the coefficient of determination include:
 - The percent variability in the *predicted* y values that is explained by the linear relationship between y and x .
 - The percent variability in the *data* that is explained by the linear relationship between y and x .
 - The percent variability that is explained by the linear relationship between y and x .
 - The percent variability in y that is *on average* explained by the linear relationship between y and x .
- For component 3 context must include mention of time or customers.

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Question 1 (continued)

Part (c) is scored as follows:

Essentially correct (E) if the response satisfies the following two components:

1. Correctly identifies the outlier.
2. Describes an unusual feature of the identified scatter plot point, relative to the remaining data points, that is sufficient to identify it as the outlier. Examples include:
 - The combination of x and y values is unusual compared to the other points.
 - The value of y is much lower than would be expected (or predicted), given the remaining data.
 - The residual for the point is unusually large relative to the other residuals.

Partially correct (P) if the response satisfies component 1 but does not satisfy component 2.

Incorrect (I) if the response does not meet the criteria for E or P.

Notes:

- In the absence of any point being circled on the graph, component 1 can still be satisfied by explicitly referring to the coordinates of the outlier. Valid coordinates for outlier identification must specify an x value of 3 and a y value that is strictly between 0 and 250.
- A response that does not make a comparison to the remaining data points, such as stating the outlier has a large residual or is nowhere near the regression line, does not satisfy component 2.
- A response that makes a comparison to the remaining data points based upon an unusual feature that is *insufficient* for outlier identification, such as stating the point is the only point with that particular y value, does not satisfy component 2.
- In the absence of explicit numerical calculation, a response that appeals to the influence that the outlier has on the regression coefficient estimates or on the sample correlation coefficient does not satisfy component 2.

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Question 1 (continued)

4 Complete Response

Three parts essentially correct

3 Substantial Response

Two parts essentially correct and one part partially correct

2 Developing Response

Two parts essentially correct and no parts partially correct

OR

One part essentially correct and one or two parts partially correct

OR

Three parts partially correct

1 Minimal Response

One part essentially correct

OR

No parts essentially correct and two parts partially correct

2018 SCORING GUIDELINES

Question 2

Intent of Question

The primary goals of this question were to assess a student's ability to (1) calculate the sample size when given the endpoints of a confidence interval for a proportion; (2) explain how bias could be present in a particular survey method; and (3) estimate a proportion from sample data collected using a method designed to decrease bias.

Solution

Part (a):

Using the standard formula for a confidence interval for one proportion, the interval (0.584 to 0.816) is found as follows. $\hat{p} \pm z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$ where $\hat{p} = \frac{0.584 + 0.816}{2} = 0.7$, the margin of error is $0.816 - 0.7 = 0.116$, and $z^* = 1.96$.

Solving $1.96 \sqrt{\frac{0.7(1-0.7)}{n}} = 0.116$ yields $n = \frac{(1.96)^2(0.7)(1-0.7)}{(0.116)^2} \approx 59.95$. The sample size was 60.

Part (b):

Bias might have been introduced because students responded directly to the environmental science teacher. Because the students would know that an environmental science teacher cares about the environment, they might say yes when they actually don't recycle. This would result in a point estimate that is greater than the proportion of all students who would respond yes to the question.

Part (c):

(i) The expected number is $(300)\left(\frac{1}{2}\right) = 150$.

(ii) The point estimate is based on expecting 150 students to be required to say no and 150 students to truthfully answer the question. Of the 213 answers of no, we expect that $213 - 150 = 63$ were from students who truthfully answered the question. That means we expect that the remaining $150 - 63 = 87$ students truthfully answered the question and responded yes. So the point estimate for the proportion of all students at the high school who would respond yes to the question is $\frac{87}{150} = 0.58$.

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Question 2 (continued)

Scoring

Parts (a), (b), and (c) are scored as essentially correct (E), partially correct (P), or incorrect (I).

Part (a) is scored as follows:

Essentially correct (E) if the response satisfies the following five components:

1. Uses a standard error in the form $\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$ where $\hat{p}$ is between 0 and 1.
2. Shows evidence that $\hat{p} = 0.7$ was correctly used in the standard error.
3. Shows evidence that 0.116 was correctly used as the margin of error in the calculation.
4. Shows evidence that $z^* = 1.96$ was correctly used as the critical value in the calculation.
5. Includes a single, positive whole-number answer.

Partially correct (P) if the response satisfies only three or four of the five components.

Incorrect (I) if the response satisfies at most two of the five components.

Notes:

- Using an equation in the form $n = \frac{z^2 \hat{p}(1-\hat{p})}{\text{MOE}^2}$ satisfies component 1.
- A value of 0.21 in the numerator of the standard error implies that $\hat{p} = 0.7$ was correctly used in the standard error and satisfies component 2.
- An equation such as $0.816 = 0.7 + \text{MOE}$ implies that 0.116 was correctly used for the margin of error and satisfies component 3.
- Statements that suggest a whole-number answer is approximate (such as, “about 60” or “ ≈ 60 ”) satisfy component 5.
- Algebraic work between the set-up and final answer does not need to be shown to satisfy component 5.
- When calculating the values 0.7, 0.116, or 1.96, ignore minor arithmetic errors or transcription errors if they can be identified by the work shown.

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Question 2 (continued)

Part (b) is scored as follows:

Essentially correct (E) if the response satisfies the following three components:

1. Explains why the responses to the survey might differ from the truth about student recycling in this context (for example, the survey was not anonymous, the question was asked by an authority figure).
2. Explains how the responses to the survey might differ from the truth about student recycling (for example, “students might say yes when they actually don’t recycle,” “students lie and say yes,” “students don’t recycle but lie to the teacher”).
3. Describes the effect of the bias on the point estimate (or the proportion, percentage, number of yes responses in the sample) and doesn’t contradict the bias described.

Partially correct (P) if the response satisfies only two of the three components.

Incorrect (I) if the response satisfies at most one of the three components.

Notes:

- To satisfy component 1 the response must provide a reason that is based on a bias created by the teacher asking students in person. For example, a response that addresses the wording of the question, voluntary response, or sampling variability does not satisfy component 1.
- To satisfy component 2 the response needs to explicitly contrast what the students say with what they do.
- Evidence used to address component 3 cannot also be used to address component 2. For example, a response that says “Students might lie, producing an estimate that is too high” addresses the effect of the bias on the point estimate but should not be combined with the statement about students lying to infer that students do not actually recycle. However, a response that says “Students may lie and say yes, producing an estimate that is too high” satisfies both components 2 and 3.
- If the response is clearly about the population proportion and not about the point estimate, component 3 cannot be satisfied.
- Statements such as “the interval will be too high” do not satisfy component 3 because they don’t specifically address the point estimate.

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Question 2 (continued)

Part (c) is scored as follows:

Essentially correct (E) if the response gives an answer of 150 in (c-i) and gives an answer of 0.58 (or equivalent) in (c-ii).

Partially correct (P) if the response gives an answer of 150 in (c-i) and gives an answer of 0.42 (or equivalent) in (c-ii);

OR

if the response does not give an answer of 150 in (c-i) but gives an answer of 0.58 (or equivalent) with supporting work in (c-ii).

Incorrect (I) if the response does not meet the criteria for E or P.

Notes:

- In part (c-i) the answer must be a single number. Responses such as “at least 150” or “147–153” are incorrect. However, responses such as “about 150” or “ ≈ 150 ” are acceptable.
- In part (c-ii) the proportion can be described verbally (e.g., “87 out of 150”).
- In part (c-ii) if the response clearly indicates that 0.58 (or 0.42) is the population proportion, lower the overall score in part (c) by one level (that is, from E to P, or from P to I). Using probability notation such as $P(\text{yes})$ does not clearly indicate a population proportion.
- In part (c-ii) if the response includes a point estimate of 0.58 or 0.42 but uses a confidence interval as the final answer, lower the overall score in part (c) by one level (that is, from E to P, or from P to I).
- If the answer is incorrect in part (c-i) and the answer in part (c-ii) uses numerator = 87 and denominator = 300 – answer to (c-i), the response should be scored P.

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Question 2 (continued)

4 Complete Response

Three parts essentially correct

3 Substantial Response

Two parts essentially correct and one part partially correct

2 Developing Response

Two parts essentially correct and no parts partially correct

OR

One part essentially correct and one or two parts partially correct

OR

Three parts partially correct

1 Minimal Response

One part essentially correct

OR

No parts essentially correct and one or two parts partially correct

2018 SCORING GUIDELINES

Question 3

Intent of Question

The primary goals of this question were to assess a student's ability to (1) compute a probability based on a weighted mixture of two populations; (2) compute a conditional probability; and (3) recognize a binomial random variable and compute the probability associated with it.

Solution

Part (a):

Let L denote left-handed, M denote multiple birth, and S denote single birth.
The probability that a randomly selected child born in the region is left-handed is:

$$P(L) = P(M)P(L | M) + P(S)P(L | S) = (0.035)(0.22) + (0.965)(0.11) = 0.0077 + 0.10615 = 0.11385.$$

Part (b):

From part (a), $P(L) = 0.11385$. Therefore,

$$P(M | L) = \frac{P(L \text{ and } M)}{P(L)} = \frac{(0.035)(0.22)}{0.11385} = \frac{0.0077}{0.11385} \approx 0.0676.$$

Part (c):

Let X represent the number of children who are left-handed in a random sample of 20 children from the region. X has a binomial distribution with $n = 20$ and $p = 0.11385$ (found in part (a)). Using the binomial distribution,

$$\begin{aligned} P(X \geq 3) &= 1 - P(X \leq 2) \\ &= 1 - \binom{20}{0}(0.11385)^0(0.88615)^{20} - \binom{20}{1}(0.11385)^1(0.88615)^{19} - \binom{20}{2}(0.11385)^2(0.88615)^{18} \\ &\approx 1 - 0.598 \approx 0.402. \end{aligned}$$

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Question 3 (continued)

Scoring

Parts (a), (b), and (c) each scored as essentially correct (E), partially correct (P), or incorrect (I).

Part (a) is scored as follows:

Essentially correct (E) if the probability is computed correctly, *AND* work is shown that includes correct numerical values using a formula, end results from a tree diagram, or some other appropriate strategy.

Partially correct (P) if the response provides a reasonable strategy for finding the probability, such as a formula or tree diagram, but uses one or more inappropriate values;

OR
if the response gives the correct probability but not enough work is shown to determine how that probability was found.

Incorrect (I) if the response does not meet the criteria for E or P.

Note: A reasonable strategy needs to include summing the results of two multiplications.

Part (b) is scored as follows:

Essentially correct (E) if the probability is computed correctly, with work shown that includes appropriate numerical values for both the numerator and denominator.

Partially correct (P) if the response includes a numerator and denominator in calculating the conditional probability, with one appropriate term (numerator or denominator) and the other inappropriate.

Incorrect (I) if the response does not meet the criteria for E or P.

Note: Appropriate values include incorrectly calculated values from part (a).

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Question 3 (continued)

Part (c) is scored as follows:

Essentially correct (E) if the response satisfies the following five components:

1. Uses a calculation based on the binomial distribution to find the probability of the number of children in the sample who are left-handed.
2. Specifies appropriate values for n and p .
3. Uses correct endpoint value for the probability.
4. Uses correct direction to calculate the probability of at least three left-handed children.
5. Correctly calculates a binomial probability consistent with the previous work.

Partially correct (P) if the response satisfies component 1 and only two or three of the other four components;
OR

if components 2, 3, 4, and 5 are met, and the response does not explicitly indicate the binomial distribution is used by name or formula.

Incorrect (I) if the response does not meet the criteria for E or P.

Notes:

- “Appropriate” values include incorrectly calculated values from part (a) or a recalculated probability from part (b).
- An unlabeled numerical value in a calculator statement cannot be used to satisfy a component.
- A response which calculates $P(X \leq 3)$ satisfies component 3 but does not satisfy component 4.
- A normal approximation to the binomial is not appropriate because $np = 20 \times 0.11385 = 2.277 < 5$.
A response using the normal approximation can score at most P. To earn a score of P, the response must include all of the following:
 - a correct mean and standard deviation based on the binomial parameters
 - clear indication of boundary and direction with a z-score or diagram
 - the probability computed correctly

Notes for all parts:

- If the resulting probability or part of the calculation of the probability uses a value that is not between 0 and 1, inclusive, the score for that part is lowered by one level (that is, from E to P, or from P to I).
- An arithmetic or transcription error in a response can be ignored if correct work is shown. For example, $0.0077 + 0.10615 = 0.1385$.

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Question 3 (continued)

4 Complete Response

Three parts essentially correct

3 Substantial Response

Two parts essentially correct and one part partially correct

2 Developing Response

Two parts essentially correct and no parts partially correct

OR

One part essentially correct and one or two parts partially correct

OR

Three parts partially correct

1 Minimal Response

One part essentially correct

OR

No parts essentially correct and two parts partially correct

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Question 4

Intent of Question

The primary goals of this question were to assess a student's ability to (1) determine whether a cause-and-effect conclusion can be made based on how a study was conducted and (2) set up, perform, and interpret the results of a hypothesis test, in the context of the problem.

Solution

Part (a):

Yes, it would be reasonable to conclude that the new procedure causes a reduction in recovery time, for patients similar to those in the study. The patients in the study were randomly assigned to the two procedures, which reduces the chance that confounding variables will affect the results. Therefore the statistically significant reduction in mean recovery time can be attributed to the new procedure being superior to the standard procedure.

Part (b):

Step 1: State a correct pair of hypotheses.

Let μ_S represent the mean recovery time among all patients similar to those in the study if they were to receive the standard treatment.

Let μ_N represent the mean recovery time among all patients similar to those in the study if they were to receive the new treatment.

The hypotheses to be tested are $H_0 : \mu_S = \mu_N$ versus $H_a : \mu_S > \mu_N$.

Step 2: Identify a correct test procedure (by name or by formula) and check appropriate conditions.

The appropriate procedure is a two-sample t -test for a difference between means.

Because this is an experiment, the first condition is that subjects were randomly assigned to one treatment group or the other. In this case the condition is satisfied because we were told that the subjects were randomly assigned to either the standard or new procedure.

The second condition is that the recovery times of the two populations are normally distributed or the sample sizes are sufficiently large to presume that the distribution of the difference in the sample means is approximately normal. In this case the condition is met because the sample sizes of 110 and 100 are both sufficiently large.

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Question 4 (continued)

Step 3: Correct mechanics, including the value of the test statistic, degrees of freedom, and p -value (or rejection region).

$$\text{The test statistic is } t = \frac{\bar{x}_S - \bar{x}_N}{\sqrt{\frac{s_S^2}{n_S} + \frac{s_N^2}{n_N}}} = \frac{217 - 186}{\sqrt{\frac{34^2}{110} + \frac{29^2}{100}}} \approx 7.13.$$

The p -value is the area greater than 7.13 for a t -distribution with $df = 207.18$, which is essentially 0 (8.36×10^{-12}).

Step 4: State a correct conclusion in the context of the problem, using the result of the statistical test.

Because the p -value is very small, we have sufficient evidence to conclude that for patients similar to the ones in the study, those receiving the new procedure would have less recovery time, on average, than those receiving the standard procedure.

Scoring

This question is scored in three sections. Section 1 consists of part (a); section 2 consists of step 1, step 2, and the test statistic in step 3 in part (b); and section 3 consists of the p -value in step 3 and step 4 in part (b). Sections 1, 2, and 3 are each scored essentially correct (E), partially correct (P), or incorrect (I).

Section 1 is scored as follows:

Essentially correct (E) if the response satisfies the following three components:

1. Correctly states that it is reasonable to make a causal conclusion.
2. Justifies the causal conclusion based on random assignment of patients to procedures (or procedures to patients);
OR
justifies the causal conclusion by stating that a randomized experiment was conducted.
3. Includes the context of the situation.

Partially correct (P) if the response satisfies component 1 AND provides WEAK justification of the causal conclusion by stating that there was random assignment or a randomized experiment was conducted, but with no context;

OR
by stating that an experiment was conducted or there was assignment (without the word "randomized") AND the response includes context of the situation;

OR
by stating that the study design reduces the chance of confounding variables or balances the effects of uncontrolled variables across both groups in context without explicitly referring to the random assignment.

Incorrect (I) if the response does not meet the criteria for E or P.

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Question 4 (continued)

Notes:

- If the response states that it is *not* reasonable to make a causal conclusion because the result could have been due to random chance *AND* explains that there is evidence for a causal conclusion based on random assignment of patients to procedures or by stating that a randomized experiment was conducted, then the response is scored E.
- If the response discusses aspects of an experiment other than random assignment (such as, control, replication, or large samples), then those aspects are considered extraneous and the response can be scored E unless those aspects are incorrect for this study (such as, blocking is a requirement, or the study used blocking, or the study used a placebo) in which case the score should be lowered one level (that is, from E to P, or from P to I).
- If the response correctly states in context that it is reasonable to make a causal conclusion but includes incorrect or contradictory justification (such as, random selection of patients), then the response is scored I.

Section 2 is scored as follows:

Essentially correct (E) if the response satisfies the following four components:

1. Parameters are defined correctly.
2. Hypotheses imply equality in the null and correct direction in the alternative.
3. Correct test is identified by name or formula.
4. Correct test statistic for a difference in means is calculated.

Partially correct (P) if the response satisfies only two or three of the four components.

Incorrect (I) if the response satisfies at most one of the four components.

Notes:

- If standard symbols are used for the parameters with appropriate group labels (such as, μ_S, μ_N), component 1 is satisfied.
- If the correct test is identified, but the response states an incorrect formula or uses incorrect notation in the formula, component 3 is not satisfied.
- A pooled two-sample t -test is acceptable for component 3, but the student must also state and comment on the plausibility of the equal population variances assumption.
- If the response identifies a z -test for equal means as the correct test identification, component 3 is not satisfied but component 4 could be satisfied.

Confidence Interval approach:

- If a single two-sample t -interval for the difference in means is used, components 3 and 4 can be satisfied. Component 3 is satisfied if the t -interval is correctly identified by name or formula. Component 4 is satisfied if the correct interval is calculated. If an alpha level is stated, then an appropriate adjustment to the confidence level must be made because the appropriate test is one-sided.
- If two one-sample t -intervals are used, while not a recommended approach, component 3 is not satisfied but component 4 could be satisfied. Component 4 is satisfied if both intervals are calculated correctly.

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Question 4 (continued)

Section 3 is scored as follows:

Essentially correct (E) if the response satisfies the following three components:

1. Makes reference to an approximately correct p -value that is consistent with the test statistic and alternative hypothesis for a difference in means.
2. Correctly justifies the conclusion based on the size of the p -value or the test statistic.
3. Correctly states the conclusion in context.

Partially correct (P) if the response satisfies only two of the three components.

Incorrect (I) if the response does not meet the criteria for E or P or includes a justification not based on the inferential results.

Notes:

Component 1:

- Is satisfied if the response makes reference to a large test statistic without referring to a p -value.

Component 2:

- No alpha level is needed to provide justification of the conclusion based on the size of the p -value.
- Is satisfied if the response states the p -value without reference to size, but it is contiguous to the conclusion and clearly indicates a continuous train of thought.
- A correct interpretation of the p -value with a complete explanation that obtaining a test statistic at least this extreme is unlikely due to chance alone is considered justification based on the size of the p -value.
- If an incorrect interpretation of the p -value is given, the score is lowered one level (that is, from E to P, or from P to I).
- A decision about the null hypothesis (reject H_0 or fail to reject H_0) is not required, but if an incorrect decision is stated based on the given p -value then component 2 is not satisfied.
- If a rejection region approach is used, a reasonable critical value replaces the p -value.

Component 3:

- A correct conclusion must be related to the alternative hypothesis in order to satisfy component 3.
- The following responses do not satisfy component 3:
 - States or implies that the null hypothesis is *accepted*
 - States or implies that the alternative hypothesis has been *proven*
 - States the conclusion in past tense (unless the response did not satisfy a component of section 2 for the use of past tense)

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Question 4 (continued)

Confidence Interval approach:

- If a single two-sample t -interval for the difference in means is used:
 - Component 1 is satisfied if the response indicates that zero is either included or not included in the calculated interval.
 - Component 2 is satisfied if the response indicates that the bounds are either both above or both below zero (consistent with alternative hypothesis) and uses that as justification for the conclusion.
 - Component 3 is satisfied if the conclusion is stated in context.
- If two one-sample t -intervals are used (which is not recommended) the response is scored at most P if all three components are satisfied, otherwise scored I:
 - Component 1 is satisfied if the response states that the intervals do not overlap.
 - Component 2 is satisfied if the conclusion indicates that the confidence interval for the new procedure lies below the confidence interval for the standard procedure.
 - Component 3 is satisfied if the conclusion is stated in context.

Note: If the three sections of the response are scored as E, to earn a score of 4 as a complete response, both conditions in step 2 must be correctly stated and justified. Additional condition(s) inappropriate for a two-sample t -test must not be stated. Otherwise, the response earns a score of 3 a substantial response.

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Question 4 (continued)

4 Complete Response

Three sections essentially correct with conditions for inference

3 Substantial Response

Three sections essentially correct without conditions for inference

OR

Two sections essentially correct and one section partially correct

2 Developing Response

Two sections essentially correct and no sections partially correct

OR

One section essentially correct and one or two sections partially correct

OR

Three sections partially correct

1 Minimal Response

One section essentially correct

OR

No sections essentially correct and one or two sections partially correct

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Question 5

Intent of Question

The primary goals of this question were to assess a student's ability to (1) determine which of two histograms represents data with a larger median; (2) calculate the mean of a combined data set when the separate means and sample sizes are known; and (3) calculate the probability that an individual randomly chosen from a finite population will have a value within one standard deviation of the mean, when provided with values for the mean, standard deviation, and all members of the population.

Solution

Part (a):

The median teaching year for High School A is any value with 100 data values at or below it and 100 data values at or above it. The median teaching year for High School B is the 111th value in the ordered list of values. For High School A the median is in the interval that starts at 7 and ends just before 10, because there are only 94 data values below 7 and 106 data values of at least 7. Therefore the median cannot be less than 7. For High School B the median is in the interval that starts at 4 and ends just before 7 because there are more than half (113) of the data values less than 7. Therefore the median must be less than 7. So High School A must be the one with a median of 7, and High School B must be the one with a median of 6.

Another way to determine which school has the median of 7 is to notice that the distribution for High School B is highly skewed to the right, whereas the distribution for High School A is bimodal with a few possible outliers on the right. A distribution that is highly right-skewed is likely to have a substantially larger mean than median. The mean of both distributions is given as 8.2 years, so it makes sense that the highly right-skewed distribution (High School B) is the one with the bigger gap between the mean and median and, therefore, the one with the lower median of 6.

Part (b):

The mean for the original 200 teachers was given as 8.2 years, and the mean for the additional 18 teachers is 2.5 years. Therefore the mean for the combined data set is:

$$\frac{(200)(8.2) + (18)(2.5)}{200 + 18} = \frac{1,640 + 45}{218} \approx 7.73 \text{ years.}$$

Part (c):

The interval mean plus or minus 1 standard deviation on either side of the mean is 8.2 ± 7.2 , or from 1.0 year to 15.4 years. Because teaching year is recorded as an integer, the interval includes teaching years 1 to 15. The number of teachers in that interval can be found by adding the heights of the five bars in the histogram for the intervals from 1 to 16, which includes $79 + 34 + 28 + 29 + 19 = 189$. Therefore the probability is $\frac{189}{221} \approx 0.8552$.

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Question 5 (continued)

Scoring

Parts (a), (b), and (c) are scored as essentially correct (E), partially correct (P), or incorrect (I).

Part (a) is scored as follows:

Essentially correct (E) if the response satisfies the following three components:

1. States that the median is 6 for High School B and the median is 7 for High School A.
2. Provides a reasonable explanation of how the decision was made.
3. Provides the definition of the median or explicitly applies the definition of a median as a criterion in reaching their decision.

OR

Essentially correct (E) if the response satisfies the following three components:

1. States that the median is 6 for High School B and the median is 7 for High School A.
2. States that High School B shows a skewed distribution (or High School A shows a less skewed distribution).
3. Provides a reasonable explanation of how the more skewed distribution (High School B) would be the one with a larger separation between the mean and median.

Partially correct (P) if the response satisfies the first component and only one of the other two components required for E.

Incorrect (I) if the response does not meet the criteria for E or P.

Note: An incorrect statistical statement in the response will result in E being lowered to P, but not P being lowered to I. For example,

- If either distribution is described as left skewed, normal, or approximately normal;
- If the discussion would indicate a median different than 7 for High School A or a median different than 6 for High School B.

Part (b) is scored as follows:

Essentially correct (E) if the response satisfies the following two components:

1. The correct answer that the mean is 7.73.
2. Enough work to show that the answer was obtained as a weighted average of the two individual means.

Partially correct (P) if the response satisfies only one of the two components.

Incorrect (I) if the response does not satisfy the requirements for E or P.

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Question 5 (continued)

Part (c) is scored as follows:

Essentially correct (E) if the response satisfies the following three components:

1. Calculates that the appropriate interval is 1 to 15.4 or 1 to 15 teaching years.
2. Correctly sums the counts of data values in the numerator based on the intervals provided.
3. Computes the probability using 221 as the denominator.

Partially correct (P) if the response satisfies only two of the three components;

OR

if the response reports the correct probability (0.8552) without supporting work.

Incorrect (I) if the response satisfies at most one of the three components.

Notes:

- If the response attempts to use the Empirical Rule or normal distribution to provide the desired probability, the response is scored I.
- If an incorrect count is shown in component 2, for instance by including the interval from 16 to 19, then component 3 is satisfied if that incorrect count is divided by 221 to find the reported probability.
- It is acceptable if the count is slightly off because of difficulty reading the exact heights of the bars in the histogram.
- If only one of component 2 or component 3 is missing, but the correct probability (0.8552) is reported, the response can be scored E.
- If the response recognizes that all values in the histogram bins up to 16 fall within one standard deviation of the mean and reports the interval as 1 to 16, component 1 is satisfied.

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Question 5 (continued)

4 Complete Response

Three parts essentially correct

3 Substantial Response

Two parts essentially correct and one part partially correct

OR

Part (a) essentially correct and two parts partially correct

2 Developing Response

Two parts essentially correct and no parts partially correct

OR

Part (b) or part (c) essentially correct and one or two parts partially correct

OR

Three parts partially correct

1 Minimal Response

One part essentially correct

OR

No parts essentially correct and one or two parts partially correct

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Question 6

Intent of Question

The primary goals of this question were to assess a student's ability to (1) describe what constitutes a Type II error for a specific hypothesis test; (2) specify a rejection region in terms of values of the sample mean; (3) compute the power of a test for a specific value in the alternative hypothesis; (4) recognize the definition of power; and (5) understand the impact of increasing the sample size on the power of a test.

Solution

Part (a):

A Type II error occurs when the alternative hypothesis is true, but the null hypothesis is not rejected. In this situation a Type II error would happen if the mean systolic blood pressure of the population of employees is greater than 122 mmHg, but the null hypothesis that it is 122 mmHg is not rejected. In other words a Type II error would happen if the mean blood pressure for the population of employees is higher than the national average, but the test does not conclude that it is higher.

Part (b):

The test is one-sided and the standard deviation is known, so the null hypothesis will be rejected if the test

statistic $z = \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}} > 1.645$. With $\mu = 122$, $\sigma = 15$, and $n = 100$, we get $\frac{\bar{x} - 122}{1.5} > 1.645$. Therefore,

$$\bar{x} > 124.4675.$$

Part (c):

If the actual population mean is 125, with $\sigma = 15$ and $n = 100$, then the sampling distribution of $\bar{x}$ is

approximately normal with mean of 125 and standard deviation $\frac{15}{\sqrt{100}} = 1.5$. Therefore,

$$P(\bar{x} > 124.4675) = P\left(\frac{\bar{x} - \mu_{\bar{x}}}{\sigma_{\bar{x}}} > \frac{124.4675 - 125}{1.5}\right) = P(z > -0.355) = 0.64.$$

Part (d):

The probability found in part (c) is called the power of the test.

Part (e):

If the sample size is increased from 100 to something larger, the probability of rejecting the null hypothesis when the population mean is 125 will be higher than it is for a sample of size 100. Intuitively, more data provide a higher probability of a correct conclusion. The technical explanation is that the rejection region will still be $z > 1.645$, but the sampling distributions of the sample mean will have a smaller standard deviation; therefore, the minimum value of $\bar{x}$ for which we would reject the null hypothesis would be lower and, in return, the probability the null hypothesis is rejected will increase.

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Question 6 (continued)

Scoring

This question is scored in three sections. Section 1 consists of part (a) and part (b), section 2 consists of part (c), and section 3 consists of part (d) and part (e). Sections 1, 2, and 3 are scored as essentially correct (E), partially correct (P), or incorrect (I).

Section 1 is scored as follows:

Essentially correct (E) if the response satisfies the following four components:

1. Part (a) includes in the description of a Type II error the fact that the alternative hypothesis is true, either generically or in context of the situation.
2. Part (a) includes in the description of a Type II error the fact that the null hypothesis is not rejected, either generically or in context of the situation.
3. Part (b) includes a correct z-score for the upper 5 percent tail and indicates the correct direction for the rejection region.
4. Part (b) includes $\mu_{\bar{x}} = 122$, $\sigma_{\bar{x}} = 1.5$, and the resulting $\bar{x}$ value.

Partially correct (P) if the response satisfies only two or three of the four components.

Incorrect (I) if the response does not meet the criteria for E or P.

Notes:

- If the response in part (a) does not include context, the number of components satisfied is reduced by one (that is, from four to three, or from three to two, and so on). Context includes a reference to units, to blood pressure, to employees, etc.
- If a response in part (a) is clearly referring to an individual's blood pressure as opposed to the mean blood pressure of all employees, neither components 1 nor 2 are satisfied.

Section 2 is scored as follows:

Essentially correct (E) if the response satisfies the following three components:

1. Recognizes that the null hypothesis will be rejected when $\bar{x} \geq 124.4675$, as found in part (b).
2. Provides the correct sampling distribution for the sample mean when the true mean is 125, including correct values for the mean and standard deviation, either explicitly or by plugging them into the test statistic formula.
3. Provides evidence of using the normal curve and finds the correct probability value.

Partially correct (P) if the response satisfies only two of the three components.

Incorrect (I) if the response does not meet the criteria for E or P.

Notes:

- Components 1 and 3 can still be satisfied if errors made in finding the rejection region in part (b) are carried into part (c).
- A calculator statement that does not include labels for input values does not satisfy component 2 but may still satisfy components 1 and 3.

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Question 6 (continued)

Section 3 is scored as follows:

Essentially correct (E) if the response satisfies the following four components:

1. Part (d) specifies power as the name of the probability.
2. Part (e) correctly states that the probability would be greater.
3. Part (e) correctly implies that the standard deviation of the sampling distribution decreases, either explicitly or by substituting values into a formula.
4. Part (e) indicates the minimum value of $\bar{x}$ for which the null hypothesis is rejected decreases, either explicitly or by substituting values into a formula.

Note: Component 4 can still be satisfied if a response indicates that a maximum value of $\bar{x}$ for which the null hypothesis is rejected increases if this direction is consistent with answers in parts (b) and (c).

Partially correct (P) if the response satisfies only two or three of the four components.

Incorrect (I) if the response does not meet the criteria for E or P.

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Question 6 (continued)

4 Complete Response

Three sections essentially correct

3 Substantial Response

Two sections essentially correct and one section partially correct

2 Developing Response

Two sections essentially correct and no sections partially correct

OR

One section essentially correct and one or two sections partially correct

OR

Three sections partially correct

1 Minimal Response

One section essentially correct

OR

No sections essentially correct and two sections partially correct

OR

Sections 1 and 2 incorrect, and section 3 partially correct with exactly three of the four components satisfied

OR

Section 1 partially correct with exactly three of the four components satisfied, and sections 2 and 3 incorrect