Question 1: Focus on Inference — 4 points

General Scoring Notes

- This question is scored in three sections. Each section is initially scored by determining if it meets the criteria for essentially correct (E), partially correct (P), or incorrect (I). The first section includes statements of the null and alternative hypotheses and identification of the appropriate hypothesis test. The second section includes verifying the conditions for the test identified in the first section and calculating the value of the test statistic and the corresponding *p*-value. The third section includes the conclusion for the test identified in the first section. The response is then categorized based on the scores assigned to each section and awarded an integer score between 0 and 4 (see the table at the end of the question).
- The model solution represents an ideal response to each section of the question, and the scoring criteria identify the specific components of the model solution that are used to determine the score.

	Model Solution	Scoring
Section 1	Let p_{younger} represent the proportion of all exercise center members from 18 to 55 years of age who would be interested in taking online fitness classes, and p_{older} represent the proportion of all exercise center members 56 years or older who would be interested in taking online fitness classes. The null hypothesis is $H_0: p_{\text{younger}} = p_{\text{older}}$ and the alternative hypothesis is $H_a: p_{\text{younger}} \neq p_{\text{older}}$. An appropriate inference procedure is a two-sample *z*-test for a difference of population proportions.	**Essentially correct (E)** if the response satisfies the following three components: 1. Identifies a two-sample *z*-test for a difference of population proportions by name (e.g., "two-proportion *z*-test" or "two-sample *z*-test") or by formula 2. States a correct null hypothesis of equal proportions *AND* a correct two-sided alternative hypothesis of unequal proportions 3. Provides sufficient context by referencing the two groups (18 to 55 years of age and 56 years or older) *AND* the populations (all exercise center members) **Partially correct (P)** if the response satisfies two of the three components required for E. **Incorrect (I)** if the response does not meet the criteria for E or P.

Additional Notes:

- If the response identifies the correct test by name but also states an incorrect formula, then component 1 is not satisfied.
- If the response identifies the test using the correct formula but equates it with a *t* instead of a *z*, then component 1 is not satisfied.
- Neither the names of the groups nor the concept of *population* is required to satisfy component 2.

- A response that states the hypotheses in words (e.g., the null hypothesis is that the proportions are equal, and the alternative hypothesis is that the proportions are not equal) may satisfy component 2. If the hypotheses also include the group names (younger and older members) and reference to the populations, then both components 2 and 3 may be satisfied.
- Group or population aspects of component 3 may be satisfied anywhere in the response, provided there are no incorrect references to groups or population elsewhere in the response (e.g., using $\hat{p}$ in hypotheses, referring to samples in conclusion, or describing groups as "those who are interested in online exercise classes" and "those who are not interested in online exercise classes").
- If the response clearly refers to *sample* proportions instead of *population* proportions using words or symbols (e.g., $\hat{p}_o$ and $\hat{p}_y$), then component 2 may be satisfied, but component 3 is not satisfied unless the symbols used are defined as *population* proportions.
- A response may satisfy the population aspect of component 3 by
  - Referring to the population by using words such as: "population," "all," or "true" when defining the parameter.
  - Using notation such as p_1, p_2, p_y, p_o, π_y, π_o, pi_y, pi_o, when defining the hypothesis statements.

    Note that subscripts y and o would be sufficient to also satisfy the group aspect of component 3, whereas subscripts of 1 and 2 are not sufficient to satisfy the group aspect of component 3 without further clarification.

Confidence Interval Approach:

- If a two-sample *z*-interval for a difference in population proportions is identified correctly by name (e.g., "two-proportion *z*-interval" or "two-sample *z*-interval") or by formula, then component 1 may be satisfied.
- If the response uses individual one-sample *z*-intervals for the two proportions, which is not a correct approach, then component 1 is not satisfied.
- For a response using a confidence interval approach, component 2 is satisfied if the response clearly indicates that the confidence interval is used to assess the correct two-sided alternative to the null hypothesis of equal proportions.
- If a response uses a two-sample *z*-interval for a population proportion, it must include references to group names and populations to satisfy component 3. For example, component 3 is satisfied if the response indicates that it is a confidence interval for the difference in population proportions of younger and older exercise center members.

Chi-Square Test for Homogeneity Approach:

- If the response uses a chi-square test approach, identifying the procedure name as a "chi-square test for homogeneity," component 1 may be satisfied.
- If the response identifies the procedure as a "chi-square test for independence" or just a "chi-square test," then component 1 is not satisfied.

	Model Solution	Scoring
Section 2	The independent observations condition for performing the two-sample z-test for a difference in population proportions is satisfied because the data were obtained from a random sample of 170 exercise center members ages 18 to 55 years and a second random sample of 230 exercise center members ages 56 years and older. The 10% condition must be met by both samples because sampling of exercise center members is done without replacement. There are more than $10(170) = 1{,}700$ adults from 18 to 55 years of age who are members of the exercise center and more than $10(230) = 2{,}300$ adults ages 56 years and older who are members of the exercise center. The value of the sample proportions are $\hat{p}_{\text{younger}} = \frac{51}{170} = 0.3$ and $\hat{p}_{older} = \frac{79}{230} \approx 0.3435.$ The combined proportion is $\hat{p}_{\text{c}} = \frac{170(0.3) + 230(0.3435)}{170 + 230} \approx 0.325.$ The sample size is large enough to support an assumption that the sampling distribution of $\hat{p}_{\text{younger}} - \hat{p}_{\text{older}}$ is approximately normal because $170(0.325) = 55.25$, $(170)(1 - 0.325) = 114.75$, $230(0.325) = 74.75$, and $(230)(1 - 0.325) = 155.25$ are all at least 10. The value of the test statistic is $z = \frac{0.3 - 0.3435}{\sqrt{0.3250(1 - 0.3250)}\sqrt{\frac{1}{170} + \frac{1}{230}}}$ $z \approx -0.918.$ The corresponding p-value is $2 * P(z < -0.918) \approx 0.359.$	**Essentially correct (E)** if the response satisfies the following four components: 1. Checks the independence condition by referring to the two random samples *AND* indicates that there are more than $10(170) = 1{,}700$ exercise center members from 18 to 55 years of age and more than $10(230) = 2{,}300$ exercise center members ages 56 years and older 2. Checks that the sample sizes are large enough by verifying that $170(0.325) = 55.25$, $(170)(1 - 0.325) = 114.75$, $230(0.325) = 74.75$, and $(230)(1 - 0.325) = 155.25$ are all at least 5 (or 10) 3. Correctly reports the value of the z-statistic 4. Correctly reports the p-value, consistent with the reported test statistic and stated alternative hypothesis **Partially correct (P)** if the response satisfies only two or three of the four components required for E. **Incorrect (I)** if the response does not meet the criteria for E or P.

Additional Notes:

- In order to satisfy the reference to the random selection of exercise center members in component 1, it is minimally acceptable to state "random samples—check" or "SRSs—check." However, component 1 is not satisfied if the response implies that random *assignment* was used or only states "random—check."

- If the response states that the random sample of members in each age group is less than 10 percent of total membership in that group (with sample sizes provided), component 1 may be satisfied. For example, "$170 < 10\%$ of all members ages 18 to 55, and $230 < 10\%$ of all members ages 56 and older."
- In order to satisfy component 2, the response must include values of the observed successes and failures, or values for the expected successes and failures, or formulas for the expected number of successes and failures with values inserted *AND* the response must make a comparison of the four values with some standard criterion, such as 5 or 10. If expressions such as $170(0.325)$ and $(170)(1 - 0.325)$ are used, simplification is not required.
- Examples of acceptable quantities (comparisons must still be made):
  - $55.25,\ 114.75,\ 74.75,$ and 155.25
  - $(170)(0.325),\ (170)(1 - 0.325),\ 230(0.325),$ and $(230)(1 - 0.325)$
  - 51, 119, 79, and 151 (observed counts)
  - $(170)(0.3),\ (170)(0.7),\ (230)(0.3435),$ and $(230)(0.6565)$
- Unless values of relevant parameters are explicitly identified in the response, the following quantities are unacceptable:
  - $170p_1,\ 170(1 - p_1),\ n_1p_1,\ n_1(1 - p_1),\ 230p_2,\ 230(1 - p_2),\ n_2p_2,\ n_2(1 - p_2)$
  - $170\hat{p}_1,\ 170(1 - \hat{p}_1),\ n_1\hat{p}_1,\ n_1(1 - \hat{p}_1),\ 230\hat{p}_2,\ 230(1 - \hat{p}_2),\ n_2\hat{p}_2,\ n_2(1 - \hat{p}_2)$
- A response that reports the correct value for the *z*-statistic but contains errors in supporting work may still satisfy component 3.
- A response that reports a value for the *z*-statistic using the incorrect estimate of the standard error of the difference $\hat{p}_{younger} - \hat{p}_{older}$ based on individual estimates for each group may still satisfy components 3 and 4, in which case $z = -0.9237$ and p-value $= 0.3557$.
- A response that uses Table A to determine the *p*-value from the rounded $z = -0.92$ should report p-value $= 0.3576$.
- A response that inputs correct values into the *z*-statistic formula but computes an incorrect value for the *z*-statistic, may satisfy component 3.
- If the response compares the value of the test statistic to a critical value instead of computing a *p*-value, then a comparison consistent with the stated alternative hypothesis satisfies component 4.
- If the response omits the hypotheses, the correct two-sided alternative hypothesis is assumed when scoring component 4.
- If an incorrect alternative hypothesis is stated, then the *p*-value must be consistent with the stated alternative hypothesis to satisfy component 4.
- If the response satisfies component 4, any supporting work for the *p*-value may be treated as extraneous.

Confidence Interval Approach:

- If the stated alternative hypothesis is correct or no alternative hypothesis is provided:
  - If a two-sided 95 percent confidence interval for $p_{\text{younger}} - p_{\text{older}}$ is correctly calculated as $(-0.1357,\ 0.0488)$, then component 3 is satisfied.
  - If the two-sided confidence interval is correctly interpreted based on whether zero is in the interval, then component 4 is satisfied.

- If the stated alternative hypothesis is incorrect (one-sided), the confidence interval approach must be consistent with the stated alternative to satisfy components 3 and 4:
  - An interval consistent with the stated alternative will satisfy component 3. A lower one-sided 95 percent confidence interval for $p_{\text{younger}} - p_{\text{older}}$ is $(-1, 0.3395)$ or an upper one-sided 95 percent confidence interval for $p_{\text{younger}} - p_{\text{older}}$ is $(-0.1209, 1)$.
  - If only the upper end of the lower one-sided confidence interval is used to reach a conclusion or only the lower end of the upper one-sided confidence interval is used to reach a conclusion, then component 4 is satisfied.
- A response that provides an interval for $p_{\text{younger}} - p_{\text{older}}$ should be scored the same way as the interval for $p_{\text{older}} - p_{\text{younger}}$.

Chi-Square Test for Homogeneity Approach:

- Component 2 is satisfied if the response verifies that all four expected counts are at least 5 (or 10), so that the test statistic has an approximate chi-square distribution.
- Component 3 is satisfied if the response correctly reports the chi-square statistic (no work is required).

	Model Solution	Scoring
Section 3	Because the *p*-value of approximately 0.359 is greater than $\alpha = 0.05$, the null hypothesis should not be rejected. The results from this study do not provide convincing statistical evidence that the population proportion of exercise center members from 18 to 55 years of age who would be interested in taking online fitness classes is different from the population proportion of adults ages 56 years and older who would be interested in taking online fitness classes.	**Essentially correct (E)** if the response satisfies the following two components: 1. Provides correct comparison of the *p*-value to alpha (*p*-value is greater than alpha) *AND* provides a correct decision about the null and/or alternative hypothesis 2. States a conclusion in context, consistent with, and in terms of the alternative hypothesis using non-definitive language **Partially correct (P)** if the response satisfies only one of the two components required for E. **Incorrect (I)** if the response does not meet the criteria for E or P.

Additional Notes:

- In order to satisfy component 1, the response must clearly identify the number that is compared to alpha as a *p*-value (which can be identified anywhere in the response).
- If the response provides an unreasonable *p*-value (that is clearly identified as the *p*-value) and correctly compares it to alpha, component 1 may be satisfied.
- To satisfy the *p*-value comparison in component 1, the response can compare the value of the test statistic to an appropriate critical value, e.g., $|z| > 1.96$.
- An explicit decision about the null hypothesis is not required to satisfy component 1.
- If an explicit decision is stated and the conclusion is inconsistent with the decision, component 1 is not satisfied.
- The decision part of component 1 may be satisfied by implying the decision within the conclusion statement (sufficient evidence/insufficient evidence for the alternative hypothesis).
- If the comparison and decision are consistent with an incorrect *p*-value (or an incorrect value of the test statistic, or an incorrect confidence interval), the response may satisfy component 1.
- To satisfy the context in component 2, the response must include references to proportions, the groups (ages 18 to 55, 56 or older), the sampling units (members), and the variable of interest (interest in online fitness classes).
- A response that clearly refers to samples in the conclusion cannot satisfy component 3 in section 1 but may satisfy components 1 and 2 in section 3.
- If the response omits hypotheses, assume the correct alternative hypothesis is provided when scoring component 2.
- If the response states incorrect hypotheses, component 2 may be satisfied by either stating a conclusion in terms of the stated alternative hypothesis or by answering the inference question.
- Examples of non-definitive language in component 2 include “evidence to accept the alternative,” “there is evidence for the alternative,” and “there is not sufficient evidence for the alternative.”

- Examples of definitive language in component 2 include "accepts the null," "proves the null," "proves the alternative," "accepts the alternative," "there is not evidence for the alternative," and "no evidence for the alternative."
- If components 1 and/or 2 are satisfied and the response provides an incorrect interpretation of the *p*-value, the score is lowered from E to P or P to I.

Confidence Interval Approach:

- Component 1 is satisfied if the justification is based on whether zero is included in the confidence interval.
  - If only the upper end of the lower one-sided confidence interval is used to reach a conclusion or only the lower end of the upper one-sided confidence interval is used to reach a conclusion, then component 1 is satisfied. A lower one-sided 95 percent confidence interval for $p_{\text{younger}} - p_{\text{older}}$ is $(-1, 0.3395)$ or an upper one-sided 95 percent confidence interval for $p_{\text{younger}} - p_{\text{older}}$ is $(-0.1209, 1)$.
- If no alternative hypothesis is specified in the response, then assume the correct alternative hypothesis is provided when scoring component 2.
- If the response includes an incorrect interpretation of the confidence interval, then the score for section 3 is lowered from E to P or from P to I.

Scoring for Question 1	Score
Complete Response Three sections essentially correct	**4**
Substantial Response Two sections essentially correct and one section partially correct	**3**
Developing Response Two sections essentially correct and no section partially correct *OR* One section essentially correct and one or two sections partially correct *OR* Three sections partially correct	**2**
Minimal Response One section essentially correct and no section partially correct *OR* No section essentially correct and two sections partially correct	**1**

2019

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Scoring Guidelines

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Question 1

Intent of Question

The primary goals of this question were to assess a student's ability to (1) describe features of a distribution of sample data using information provided by a histogram; (2) identify potential outliers; (3) sketch a boxplot; and (4) comment on an advantage of displaying data as a histogram rather than as a boxplot.

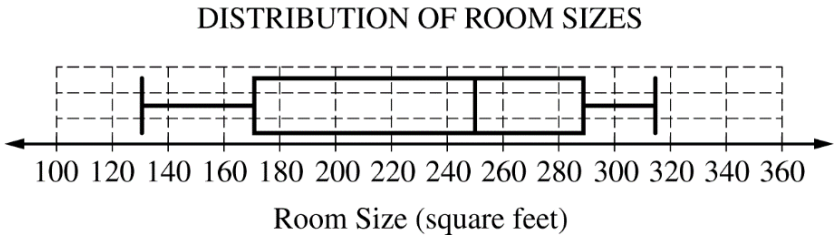
Solution

Part (a):

The distribution of the sample of room sizes is bimodal and roughly symmetric with most room sizes falling into two clusters: 100 to 200 square feet and 250 to 350 square feet. The center of the distribution is between 200 and 300 square feet. The range of the distribution is between 150 and 250 square feet. There are no apparent outliers.

Part (b):

The interquartile range is $\text{IQR} = 292 - 174 = 118$ square feet. There are no potential outliers because the minimum room size of 134 square feet does not fall below $Q_1 - 1.5(\text{IQR}) = -3$ square feet, and the maximum room size of 315 square feet does not exceed $Q_3 + 1.5(\text{IQR}) = 469$ square feet.



Part (c):

The histogram clearly shows the bimodal nature of the distribution of room sizes, but this is not apparent in the boxplot.

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Question 1 (continued)

Scoring

This question is scored in three sections. Section 1 consists of part (a); Section 2 consists of the outlier determination in part (b); Section 3 consists of the boxplot sketch in part (b) and part (c). Each section is scored as essentially correct (E), partially correct (P), or incorrect (I).

Section 1 is scored as follows:

Essentially correct (E) if the description of the distribution of room sizes satisfies the following four components:

1. The shape is bimodal OR there are two peaks OR there are two clusters.
2. The center is between 200 and 300 square feet.
3. The spread is addressed by stating the range is a value between 150 and 250 square feet OR the interquartile range is a value between 50 and 150 square feet OR all room sizes are between 100 and 350 square feet.
4. The response includes context.

Partially correct (P) if the response satisfies two or three of the four components.

Incorrect (I) if the response does not satisfy the criteria for E or P.

Notes:

- Shape: Component 1 cannot be satisfied if a response describes the histogram as unimodal or describes the entire histogram as normal or approximately normal.
- Shape: A response that addresses symmetry, while appropriate, does not impact the scoring of section 1.
- Center: A response that states one cluster of the distribution is centered between 150 and 200 square feet and the other cluster is centered between 250 and 300 square feet satisfies both components 1 and 2.
- Center:
    - Responses that address center using interval language such as "the mean of the distribution is *between* 200 and 300" must, for any single measure of center, provide an interval with lower endpoint not below 200 square feet, and with upper endpoint not above 300 square feet to satisfy component 2.
    - Responses that address center using approximate language such as "the median of the distribution is *approximately* 225" must, for any single measure of center, specify a numeric value that is not less than 200 square feet, and that is not greater than 300 square feet to satisfy component 2.
    - Responses that use definitive language such as "the mean of the distribution *is* 231.4" must identify the corresponding numeric value correctly to satisfy component 2. Specifically, the median of the distribution can be correctly identified as any value between 250 and 253.5 square feet, inclusive; the mean of the distribution is 231.4 square feet; and the center (or average) of the distribution can be any value that is a correct median or mean.

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Question 1 (continued)

- Spread: A response recognizing all values in the sample fall between 100 and 350 square feet (or between 134 and 315 square feet) satisfies component 3 *only* for these exact endpoints and need not appeal to a specific measure of spread such as range or interquartile range (IQR).
- Spread:
    - Responses that appeal to a specific measure of spread using interval language, such as "the IQR is *between* 50 and 150," must provide bounds appropriate to the corresponding measure of spread. For range, the lower endpoint must not be below 150 square feet and the upper endpoint cannot exceed 250 square feet; for IQR, the lower endpoint must not be below 50 square feet, with upper endpoint not to exceed 150 square feet; for standard deviation, the lower endpoint must not be below 25 square feet, with upper endpoint not to exceed 100 square feet.
    - Responses that appeal to a specific measure of spread using approximate language, such as "the range is *approximately* 250," must specify a numeric value within the bounds appropriate to that measure of spread. For range, the value must be between 150 and 250 square feet(inclusive); for IQR, the value must be between 50 and 150 square feet (inclusive); for standard deviation, the value must be between 25 and 100 square feet (inclusive). Responses that appeal to a specific measure of spread using definitive language, such as "the range of the distribution *is* 181," must identify the corresponding numeric value correctly to satisfy component 3. Specifically, the range of the distribution is 181 square feet; the IQR of the distribution is 118 square feet; and the standard deviation of the distribution is 68.12 square feet.

Section 2 is scored as follows:

Essentially correct (E) if the response satisfies the following three components:

1. Computation of both upper and lower outlier boundary fences that also shows the fences formulas either in words, symbols $Q_1 - 1.5(\text{IQR})$ and $Q_3 + 1.5(\text{IQR})$, or with values substituted from the table $174 - 1.5(118)$ and $292 + 1.5(118)$, or $(174 - 177)$ and $(292 + 177)$.
2. A correct decision regarding the presence of outliers.
3. Correct justification that compares the data with the fences.

Partially correct (P) if the response satisfies only two of the three components OR if the response omits exactly one of the fences but otherwise satisfies all three components.

Incorrect (I) if the response does not satisfy the requirements for E or P.

Notes:

- A response that identifies both fence formulas using symbols, but does not substitute values for all symbols, must also include the correct fence values of -3 and 469 to satisfy component 1.
- In place of an appeal to fences, a response may compute outlier bounds representing k standard deviations from the sample mean, where k is a number from 2 to 3 (inclusive), and must include formulas for both endpoints either in words, symbols $\bar{x} \pm k(\text{standard deviation})$, or with values substituted from the table. When $k = 2$ the outlier bounds are $(95.16, 367.64)$; when $k = 3$ the bounds are $(27.04, 435.76)$.

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Question 1 (continued)

- A response that identifies the standard deviation bounds using symbols, but that does not substitute values for all symbols, does not satisfy component 1 unless the correct numeric bounds are provided.
- Component 3 is satisfied if the response states the outlier decision criterion: any data values falling outside of the interval from -3 to 469 are potential outliers.

Section 3 is scored as follows:

Essentially correct (E) if the response satisfies the following two components:
1. A correct sketch of the boxplot.
2. A response for part (c) that indicates the bimodal shape of the room size distribution is apparent in the histogram but not in the boxplot.

Partially correct (P) if the response satisfies only one of the two components.

Incorrect if the response does not meet the criteria for E or P.

Notes:
- The boxplot must be completely correct to satisfy component 1. Specifically:
  - The minimum is positioned between grid lines at 120 and 140 square feet.
  - Q_1 is positioned between grid lines at 160 and 180 square feet.
  - The median is positioned between grid lines at 240 and 260 square feet.
  - Q_3 is positioned between grid lines at 280 and 300 square feet.
  - The maximum is positioned between grid lines at 300 and 320 square feet.
- If a *mean* is included as a part of the boxplot, component 1 cannot be satisfied.
- A response based on skewness or symmetry does not satisfy component 2.
- A response stating the unimodal OR normal shape of the histogram of room sizes is apparent in the histogram but not in the boxplot will satisfy component 2 *only* if the shape description in section 1 component 1 was also unimodal OR normal, respectively.

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Question 1 (continued)

4 Complete Response

Three sections essentially correct

3 Substantial Response

Two sections essentially correct and one section partially correct

2 Developing Response

Two sections essentially correct and no sections partially correct

OR

One section essentially correct and one or two sections partially correct

OR

Three sections partially correct

1 Minimal Response

One section essentially correct

OR

No sections essentially correct and two sections partially correct

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Question 2

Intent of Question

The primary goals of this question were to assess a student's ability to (1) identify components of an experiment; (2) determine if an experiment has a control group; and (3) describe how experimental units can be randomly assigned to treatments.

Solution

Part (a):

Treatments: Sprays with four different concentrations of the fungus (0 ml/L, 1.25 ml/L, 2.5 ml/L, and 3.75 ml/L)

Experimental units: 20 containers, each containing the same number of insects

Response variable: Number of insects that are still alive in each container one week after spraying

Part (b):

Yes. Because the 0 ml/L concentration contains no fungus, the containers that are sprayed with the 0 ml/L concentration form the control group.

Part (c):

Label each container with a unique integer from 1 to 20. Then use a random number generator to choose 15 integers from 1 to 20 without replacement. Use the first five of these numbers to identify the five containers that will receive the 0 ml/L treatment. Use the second five of these numbers to identify the five containers that will receive the 1.25 ml/L treatment. Use the third five of these numbers to identify the five containers that will receive the 2.5 ml/L treatment. The remaining five containers will receive the 3.75 ml/L treatment.

(Alternative solution) Using 20 equally sized slips of paper, label five slips with 0 ml/L, five slips with 1.25 ml/L, five slips with 2.5 ml/L, and five slips with 3.75 ml/L. Mix the slips of paper in a hat. For each container, select a slip of paper from the hat (without replacement) and spray that container with the treatment selected.

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Question 2 (continued)

Scoring

Parts (a), (b), and (c) are each scored as essentially correct (E), partially correct (P), or incorrect (I).

Part (a) is scored as follows:

Essentially correct (E) if the response satisfies the following three components:
1. Identifies the 4 concentrations (or mixtures or sprays) as the treatments
2. Identifies the 20 containers as the experimental units
3. Identifies the number of insects that are still alive in each container as the response variable

Partially correct (P) if response satisfies only two of the three components.

Incorrect (I) if the response does not meet the criteria for E or P.

Notes:
- Listing the four treatments satisfies component 1 (including ml/L is not required). However, if the list does not include all four treatments, component 1 is not satisfied.
- To satisfy component 1, the response must refer to plural concentrations/mixtures/sprays (e.g., the mixtures, the levels of the concentration). Referring only to the explanatory variable (concentration) does not satisfy component 1.
- The following responses satisfy component 2: "the 20 containers"; "the containers"; "the 20 groups of insects"; or "the groups of insects in each container." References to only "groups of insects" do not satisfy component 2 because it is unclear if these groups are formed by treatment or by container.
- To satisfy component 3, it must be clear that the response variable is being measured separately for each experimental unit. A response that says only "number of insects alive" does not satisfy component 3 because it could be referring to the total number of insects alive.
- To satisfy component 3, the response must be stated as a variable by using "number of" or equivalent. For example, "insects alive in each container" is not a variable and would not satisfy component 3.
- If the response states that the insects are the experimental units, then component 3 can still be satisfied by providing a binary response variable for each insect (e.g., whether the insect lived or died, survival status).

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Question 2 (continued)

Part (b) is scored as follows:

Essentially correct (E) if the response indicates that there is a control group and justifies this claim by identifying the control group or by explaining that there is a treatment which contains no fungus.

Partially correct (P) if the response indicates that there is no control group because every container is sprayed with some mixture

OR

if the response states that there is a control group but implies that 0 ml/L is not a treatment (e.g., "the containers with 0 ml/L form a control group because they don't receive a treatment"; "yes, there is a group that got no treatment").

Incorrect (I) if the response does not meet the criteria for E or P.

Notes:

- The response does not need to explain the purpose of a control group.
- The response does not need to explicitly say "yes"—it can be implied by stating that there is a control group or saying "the control group is …."

Part (c) is scored as follows:

Essentially correct (E) if the response satisfies the following three components:

1. Creates appropriate labels for the units/treatments (e.g., label the containers from 1 through 20, label 20 slips of paper with five for each treatment)
2. Describes how to correctly implement the random assignment process
3. The random assignment process results in an equal number of experimental units assigned to each treatment

Partially correct (P) if response satisfies only two of the three components.

Incorrect (I) if the response does not meet the criteria for E or P.

Notes:

- If the response states that insects are the experimental units in part (a), the response in part (c) can be in terms of insects or containers. In either case, the same three components are used to determine the score.
- If the response states that the containers are the experimental units in part (a), but only describes how to assign insects to treatments in part (c), component 1 is not satisfied.
- For responses that use slips of paper:
    - If the number of slips of paper is not equal to the number of experimental units, then component 1 is not satisfied. The slips of paper do not need to be specifically identified as equally-sized.
    - If the slips of paper are not mixed/shuffled or the slips are not "selected at random," component 2 is not satisfied. Sampling without replacement is implied when using slips of paper, unless the response specifies sampling with replacement.

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Question 2 (continued)

- For responses that use random number generators (or a 20-sided die):
  - If the initial assignment of numbers to units does not give each unit the same probability of being assigned to each treatment (e.g., units are represented by different numbers of integers), then component 1 is not satisfied.
  - If the response does not indicate that the numbers are selected without replacement or that different numbers must be used, the response does not satisfy component 2. The response does not need to specify the interval of numbers from which they are selecting (e.g., randomly generate a number from 1 to 20).
- For responses that use a table of random digits:
  - If the initial assignment of numbers to units does not give each unit the same probability of being assigned to each treatment, component 1 is not satisfied. For example, responses that use the labels 1 to 20 (not 01 to 20) do not satisfy component 1 because label 1 has a $\frac{1}{10}$ probability of being selected but label 20 has a $\frac{1}{100}$ probability of being selected.
  - If the response does not indicate that the numbers are selected without replacement or that different numbers must be used, the response does not satisfy component 2. The response does not need to specify the interval of numbers from which they are selecting or state that the numbers corresponding to unused labels will be skipped (e.g., skip numbers 00 and 21 to 99).
- For responses that use a 4-sided die (or random integers from 1 to 4):
  - If the die is rolled for each experimental unit, then component 3 is not satisfied because an equal number of units per treatment is not guaranteed.
  - If the die is rolled for each experimental unit until treatments are "full," then component 1 is not satisfied because this setup doesn't allow for all possible random assignments to be equally likely (unless the order of the units is randomized initially).
- If a response groups the experimental units before any random assignment (e.g., forms five groups of four containers or four groups of five containers), and then randomly assigns treatments to the groups or randomly assigns treatments within each group, component 1 is not satisfied. However, if a response forms groups in the context of a randomized block design with a reasonable blocking variable, component 1 can be satisfied.
- If a response describes two different random assignment processes in detail (e.g., how to randomly assign insects to containers and how to assign containers to treatments), both descriptions are scored according to the three components and the lower score is used.
- Responses that assign experimental units only to groups and not to treatments (e.g., randomly select five containers and put them in group 1) do not satisfy component 3.
- If the response randomly assigns insects to containers, the containers must be assigned to a treatment to satisfy component 3. In this case, the assignment of treatment to container does not need to be at random to satisfy component 3.

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Question 2 (continued)

4 Complete Response

Three parts essentially correct

3 Substantial Response

Two parts essentially correct and one part partially correct

2 Developing Response

Two parts essentially correct and no parts partially correct

OR

One part essentially correct and one or two parts partially correct

OR

Three parts partially correct

1 Minimal Response

One part essentially correct

OR

No parts essentially correct and two parts partially correct

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Question 3

Intent of Question

The primary goals of this question were to assess a student's ability to (1) use information from a two-way table of relative frequencies to compute joint, marginal, and conditional probabilities; (2) recognize whether two events are independent; and (3) compute a probability for a binomial distribution.

Solution

Part (a):

(i) $P(\text{never and woman}) = 0.0636$

(ii)

$$\begin{aligned} P(\text{never or woman}) &= P(\text{never}) + P(\text{woman}) - P(\text{never and woman}) \\ &= 0.12 + 0.53 - 0.0636 \\ &= 0.5864 \end{aligned}$$

(iii) $P(\text{never} \mid \text{woman}) = \dfrac{P(\text{never and woman})}{P(\text{woman})} = \dfrac{0.0636}{0.53} = 0.12$

Part (b):

Yes, the event of being a person who responds never is independent of the event of being a woman because

$P(\text{never} \mid \text{woman}) = P(\text{never}) = 0.12.$

Part (c):

Define X as the number of people in a random sample of five people who always take their medicine as prescribed. Then X has a binomial distribution with $n = 5$ and $p = 0.54,$ and

$$\text{P}(X \geq 4) = \binom{5}{4}(0.54)^4(0.46)^1 + \binom{5}{5}(0.54)^5(0.46)^0 \approx 0.19557 + 0.04592 \approx 0.24149.$$

Scoring

Parts (a), (b), and (c) are each scored as essentially correct (E), partially correct (P), or incorrect (I).

Part (a) is scored as follows:

Essentially correct (E) if the response reports correct values of the probabilities for (i), (ii), and (iii).

Partially correct (P) if only one or two of the probabilities are correct.

Incorrect (I) if none of the probabilities are correct.

Notes:

- Assuming independence for events never and woman in (i) without referencing the result in part (b) does not satisfy (i).
- Alternative solutions for (ii) include $0.0564 + 0.0636 + 0.1384 + 0.3280 = 0.5864$ and $0.0564 + 0.53 = 0.5864$.

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Question 3 (continued)

Part (b) is scored as follows:

Essentially correct (E) if the response indicates that the events are independent, gives an explanation of independence using the events in the problem, *AND* provides appropriate justification using numbers from the table.

Note: Examples of valid explanations with appropriate justifications include:

- $P(\text{woman and never}) = 0.0636$ is the same as $P(\text{woman}) \times P(\text{never}) = (0.53)(0.12) = 0.0636$.
- $P(\text{never} \mid \text{woman}) = \dfrac{0.0636}{0.53} = 0.12$ is the same as $P(\text{never}) = 0.12$.
- $P(\text{woman} \mid \text{never}) = \dfrac{0.0636}{0.12} = 0.53$ is the same as $P(\text{woman}) = 0.53$.

Partially correct (P) if the response indicates that the events are independent *AND* gives an explanation of independence using the events in the problem but does not provide justification using numbers from the table

OR

if the response uses a correct method of illustrating that events are independent but makes an arithmetic mistake or a transcription mistake that results in concluding that these two events are not independent.

Incorrect (I) if the response does not satisfy requirements for E or P.

Part (c) is scored as follows:

Essentially correct (E) if the response satisfies the following three components:

1. Clearly indicates a binomial distribution with $n = 5$ and $p = 0.54$.
2. Indicates the correct boundary value and direction of the event.
3. Reports the correct probability.

Partially correct (P) if the response satisfies component 1 but it does not satisfy one or both of the other two components

OR

if the response does not satisfy component 1 but both of the other two components are satisfied.

Incorrect (I) if the response does not meet the criteria for E or P.

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Question 3 (continued)

Notes:

- The response $B(5, 0.54)$ satisfies component 1.
- Components 1 and 2 are satisfied by displaying the correct formula for computing the binomial probability using the correct values for n and p, e.g.,

  $$\binom{5}{4}(0.54)^4(0.46)^1 + \binom{5}{5}(0.54)^5(0.46)^0$$

- Only component 1 is satisfied if the correct binomial distribution is used in an incorrect probability formula, e.g.,

  $$\binom{5}{4}(0.54)^4(0.46)^1$$

- For component 2, the boundary value and direction may be described in words, e.g., *P*(at least four people).
- Component 2 may be satisfied by displaying a bar graph of a binomial distribution with the appropriate bars shaded.
- The response of $1 - \text{binomcdf}(n = 5,\ p = 0.54,\ \text{upper bound} = 3) = 0.24$ is scored E since n, p, and the boundary value are clearly identified.
  The response of $1 - \text{binomcdf}(n = 5, p = 0.54, 3) = 0.24$ is scored P since n and p are clearly identified and the boundary value is not identified.
  The response of $1 - \text{binomcdf}(5, 0.54, 3) = 0.24$ is scored I.
- A normal approximation to the binomial is not appropriate since $np = 5 \times 0.54 = 2.7$ and $2.7 < 5$.
  A response using the normal approximation can score at most P. To score P, the response must include all of the following:
  - an indication that the probability calculated is a normal approximation for the binomial probability
  - a correct mean and standard deviation based on the binomial parameters
  - clear indication of boundary and direction with a *z*-score or diagram
  - the probability computed correctly.

  An example of a response which meets these four criteria is $P\left(Z \geq \frac{4 - np}{\sqrt{np(1-p)}}\right) = P\left(Z \geq \frac{4 - (5)(0.54)}{\sqrt{(5)(0.54)(0.46)}}\right) \approx 0.1217,$ and the binomial distribution is mentioned.

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Question 3 (continued)

4 Complete Response

Three parts essentially correct

3 Substantial Response

Two parts essentially correct and one part partially correct

2 Developing Response

Two parts essentially correct and no parts partially correct

OR

One part essentially correct and one or two parts partially correct

OR

Three parts partially correct

1 Minimal Response

One part essentially correct

OR

No parts essentially correct and two parts partially correct

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Question 4

Intent of Question:

The primary goals of this question were to assess a student's ability to perform an appropriate hypothesis test to address a particular question. More specific goals were to assess students' ability to state appropriate hypotheses, identify the appropriate statistical test procedure, check appropriate assumptions/conditions for inference; calculate a correct test statistic and *p*-value; and draw a correct conclusion, with justification, in the context of the study.

Solution

Section 1:

Let p_{14} represent the proportion of the population of kochia plants in the western United States that were resistant to glyphosate in 2014. Let p_{17} represent the proportion of the population of kochia plants in the western United States that were resistant to glyphosate in 2017.

The null hypothesis $H_0 : p_{17} - p_{14} = 0$ is to be tested against the alternative hypothesis $H_a : p_{17} - p_{14} > 0$.

An appropriate inference procedure is a two-sample *z*-test for a difference in proportions. The formula for the test statistic is:

$$z = \frac{\hat{p}_{17} - \hat{p}_{14}}{\sqrt{\left(\frac{\hat{p}_c(1-\hat{p}_c)}{n_{17}} + \frac{\hat{p}_c(1-\hat{p}_c)}{n_{14}}\right)}}$$

where $\hat{p}_c = \frac{n_{14}\hat{p}_{14} + n_{17}\hat{p}_{17}}{n_{14} + n_{17}}$ is a pooled estimate of the proportion of resistant plants for 2014 and 2017 combined.

Section 2:

The first condition for applying the test is that the data are gathered from independent random samples from the populations of kochia plants in the western United States in 2014 and 2017. The question indicates that a random sample of 61 kochia plants was taken in 2014 and a second random sample of 52 kochia plants was taken in 2017. It is reasonable to assume that the 2017 sample of plants was in no way influenced by the 2014 sample of plants.

The second condition is that the sampling distribution of the test statistic is approximately normal. This condition is satisfied because the expected counts under the null hypothesis are all greater than 10. The pooled estimate of the proportion of resistant plants is $\hat{p}_c = \frac{(61)(0.197) + (52)(0.385)}{61 + 52} \approx 0.2835$. The estimates of the expected counts are
$61(0.2835) \approx 17.29,\ 61(1 - 0.2835) \approx 43.71,\ 52(0.2835) \approx 14.74,\ 52(1 - 0.2835) \approx 37.26,$ all of which are greater than 10.

Because sampling must have been done without replacement, the independence condition for each sample should be checked. Information on the population sizes of kochia plants is not given for either 2014 or 2017, but it is reasonable to assume that each population has millions of plants. Therefore it is reasonable to assume that the sample sizes are less than 10 percent of the respective population sizes.

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Question 4 (continued)

Using the pooled estimate of the proportion of resistant plants, $\hat{p}_c \approx 0.2835$, the value of the test statistic is:

$$z = \frac{0.385 - 0.197}{\sqrt{\left(\frac{(0.2835)(0.7165)}{61} + \frac{(0.2835)(0.7165)}{52}\right)}} \approx 2.21$$

The p-value is 0.0135.

Section 3:

Because the p-value is less than $\alpha = 0.05$, there is convincing statistical evidence to conclude that the proportion of resistant plants in the 2017 population of kochia plants is greater than the proportion of resistant plants in the 2014 population of kochia plants.

Scoring

Sections 1, 2, and 3 are each scored as essentially correct (E), partially correct (P), or incorrect (I).

Section 1 is scored as follows:

Essentially correct (E) if the response satisfies components 1 and 4 *AND* at least one of the remaining components:

1. Hypotheses imply equality of proportions in the null hypothesis and correct direction in the alternative hypothesis, which utilize an appropriate population parameter in words or symbols.
2. Identifies parameters that are population proportions.
3. Both parameters are correctly defined as proportions of resistant plants in 2014 and 2017.
4. The two-sample z-test for proportions is identified by name or formula

Partially correct (P) if the response does not meet the requirement for E, but at least two of the components are satisfied.

Incorrect if the response does not meet the criteria for E or P.

Notes:

- Correct ways to state the null hypothesis that satisfy component 1:
  $H_0 : p_{17} = p_{14}$ or $H_0 : p_{17} - p_{14} = 0$
  $H_0 : p_{17} \leq p_{14}$ or $H_0 : p_{17} - p_{14} \leq 0$
  $H_0 : p_{14} \geq p_{17}$ or $H_0 : p_{14} - p_{17} \geq 0$
  Correct ways to state the alternative hypothesis that satisfy component 1:
  $H_a : p_{17} > p_{14}$ or $H_a : p_{17} - p_{14} > 0$
  $H_a : p_{14} < p_{17}$ or $H_a : p_{14} - p_{17} < 0$
  Incorrect ways to state the null hypothesis that do not satisfy component 1:
  $H_0 : p_{17} < p_{14}$ or $H_0 : p_{17} - p_{14} < 0$
  $H_0 : p_{14} > p_{17}$ or $H_0 : p_{14} - p_{17} > 0$
  Incorrect ways to state the alternative hypothesis that do not satisfy component 1:
  $H_a : p_{17} \neq p_{14}$ or $H_a : p_{17} - p_{14} \neq 0$
  $H_a : p_{17} < p_{14}$ or $H_a : p_{17} - p_{14} < 0$

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Question 4 (continued)

- Examples for components 2 and 3:
  - Satisfies both components 2 and 3:
    - p_{17} is the proportion of resistant plants
      p_{14} is the proportion of resistant plants
  - Satisfies component 2 but not component 3:
    - p_1 is the proportion of resistant plants
      p_2 is the proportion of resistant plants
    - p_{17} is the proportion of plants
      p_{14} is the proportion of plants
    - p_{17}, p_{14}
    - p_1, p_2
- If the test is correctly identified by name, but then an incorrect formula is stated, this is considered to be a parallel response and component 4 is not satisfied.
- If the test identifies an unpooled two sample *z*-test for a difference in proportions as the correct test or formula, component 4 is satisfied.

Section 2 is scored as follows:

Essentially correct (E) if the response satisfies components 1 and 2 *AND* at least two of the remaining components:

1. Notes that the use of random samples in 2014 and 2017 satisfies the randomness condition.
2. Checks for approximate normality of the test statistic by showing that the expected numbers of resistant and non-resistant kochia plants are both larger than some commonly accepted criterion (e.g. 5 or 10) for both samples.
3. Notes that the populations of kochia plants must be extremely large in both years, thus satisfies the independence (10%) conditions.
4. Reports a correct value of the *z*-test statistic.
5. Reports a *p*-value that is consistent with the stated alternative hypothesis and reported test statistic.

Partially correct (P) if the response does not meet the criteria for E, but at least two of the five components are satisfied.

Incorrect if the response does not meet the criteria for E or P.

Notes:

- For the randomness component it is minimally acceptable to say "random samples—check" or "SRSs—check." The important concept is that the study used two independent random samples. Although it is not known if a SRS was taken versus another type of random sample, it is minimally acceptable to indicate SRSs since the sampling method is unknown. If the response implies that random assignment was used, the randomness component is not satisfied.

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Question 4 (continued)

- To satisfy component 2, the response must include actual numbers, or a formula with numbers plugged in, as well as a clear indication of comparison of the four quantities to some standard criterion, such as 5 or 10, or the statement that each such quantity is large enough. If a formula with numbers is used, simplification is NOT required.

  Examples of acceptable quantities (comparison still must be made):
  - 12, 49, 20, 32
  - 12.017, 48.983, 20.02, 31.98
  - $61(0.197)$, $61(1 - 0.197)$, $52(0.385)$, $52(1 - 0.385)$

  Examples of unacceptable quantities:
  - $n_{17}\hat{p}_{17}$, $n_{17}(1 - \hat{p}_{17})$, $n_{14}\hat{p}_{14}$, $n_{14}(1 - \hat{p}_{14})$
  - $n_{17}p_{17}$, $n_{17}(1 - p_{17})$, $n_{14}p_{14}$, $n_{14}(1 - p_{14})$
  - $n_{17}\hat{p}_c$, $n_{17}(1 - \hat{p}_c)$, $n_{14}\hat{p}_c$, $n_{14}(1 - \hat{p}_c)$
  - $61\hat{p}_c$, $61(1 - \hat{p}_c)$, $52\hat{p}_c$, $52(1 - \hat{p}_c)$
- The test statistics for the pooled and unpooled *z*-tests are 2.21 and 2.22 respectively, thus they are close to the same value. If the response provides the unpooled formula but then states a pooled test statistic, component 4 is satisfied. If the response provides the pooled formula but then states an unpooled test statistic, component 4 is satisfied.
- If the response uses a critical value approach rather than a *p*-value approach, then the correct critical value of -1.645 or 1.645, that is consistent with the alternative hypothesis, satisfies component 5.
- If the response did not satisfy component 1 in section 1 because a two-tailed alternative was stated or the direction of the alternative was incorrect, then the *p*-value in component 5 should be consistent with the stated alternative. If the response omits hypotheses or other incorrect hypotheses are stated, assume the correct alternative hypothesis is provided when scoring component 5.

Section 3 is scored as follows:

Essentially correct (E) if the response includes the following three components:
1. Provides justification of the conclusion based on a correct comparison between a stated *p*-value and an alpha value of 0.05.
2. Provides a correct conclusion consistent with the alternative hypothesis.
3. The conclusion is stated in context.

Partially correct (P)
if the response satisfies components 1 and 2

OR

if the response satisfies components 2 and 3

OR

if the response satisfies components 1 and 3 *AND*, based on the p-value from section 2, either
- the conclusion correctly rejects the null hypothesis but does not state that there is convincing evidence for the alternative hypothesis
  OR
- the conclusion correctly fails to reject the null hypothesis but does not state there is not convincing evidence for the alternative hypothesis.

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Question 4 (continued)

Incorrect (I) if the response does not satisfy the criteria for E or P.

Notes:

- If the conclusion is consistent with a reasonable, but incorrect, *p*-value from section 2, and is presented in context with justification based on comparison of the *p*-value to the level of significance, then section 3 is scored E.
- If the response implies that the outcome of the hypothesis test is a "proof" of either a true or false null, the score is lowered one level (that is, from E to P, or from P to I).
- If an incorrect interpretation of the *p*-value is given, the score is lowered one level (that is, from E to P, or from P to I).
- If the response uses a critical value approach rather than a *p*-value approach, then the correct critical value of -1.645 or 1.645 replaces the *p*-value in section 2, and comparison of the test statistic from section 2 to the critical value (e.g. $2.21 > 1.645$) satisfies component 1.
- If the response clearly states a reasonable level of significance that differs from 0.05 and provides a justification and conclusion in context based on that justification, the response is scored E.
- If the response provides the incorrect comparison between the stated *p*-value and the level of significance, but the conclusion is consistent with the given comparison and the alternative hypothesis, then component 2 is satisfied.
- If the response did not satisfy component 1 in section 1 because a two-tailed alternative was stated or the direction of the alternative was incorrect, then the conclusion component 2 should be consistent with the stated alternative. If the response states other incorrect hypotheses or omits hypotheses, assume the correct alternative hypothesis is provided when scoring component 2.

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Question 4 (continued)

Alternative Approach
Two-Sided Confidence Interval for Difference in Two Population Proportions

Section 1 is scored E, P, or I according to the guidelines in section 1 for a 2 sample *z*-test for proportions.

Notes:

- To satisfy component 4, the two-sample confidence interval for a difference in two proportions should be identified by name or formula by referring to the *z*-distribution and two proportions.

Section 2 is scored as follows:
Essentially correct (E) if the response satisfies components 1 and 2 *AND* at least one of the remaining components:

1. Notes that the use of random samples in 2014 and 2017 satisfies the randomness condition.
2. Checks for approximate normality of the test statistic by showing that the observed numbers of resistant and non-resistant kochia plants are both larger than some commonly accepted criterion (say 5 or 10) for both samples.
3. Notes that the populations of kochia plants must be extremely large in both years, thus satisfies the independence (10%) conditions.
4. Reports the correct confidence interval that is consistent with the stated alternative hypothesis.

Partially correct (P) if the response does not meet the criteria for E, but at least two of the four components are satisfied.

Incorrect (I) if the response does not satisfy the criteria for E or P.

Notes:

- Examples of correct 90% confidence intervals to address a one-sided alternative for $\alpha = 0.05$ are:
  $(-0.327, -0.049)$
  $(0.049, 0.327)$
- Examples of correct 95% confidence intervals to address a two-sided alternative for $\alpha = 0.05$ are:
  $(-0.353, -0.022)$
  $(0.022, 0.353)$

Section 3 is scored E, P or I according to the guidelines in section 3 for a 2 sample *z*-test for proportions.

Notes:

- Component 1 is satisfied if a confidence interval that is consistent with the alternative hypothesis is given and the appropriate interval endpoint(s) are compared to zero.

Overall Notes:

- If the response constructs two separate one-proportion *z*-intervals for 2014 and 2017, then sections 1 and 2 are scored as above, and section 3 is scored I.

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Question 4 (continued)

Alternative Approach
Chi-square test for homogeneity

The value of the Pearson chi-square test statistic (uncorrected) is 4.8821 with 1 degree of freedom and a *p*-value of 0.02714. This value is the same as the square of the pooled *z*-statistic, so it is the same test, but the *p*-value is for a two-sided alternative. This *p*-value could be divided by 2 to obtain an appropriate *p*-value for the one-sided alternative, but the sample data needs to be examined to determine the correct direction of the alternative.

Section 1 is scored E, P, or I according to the guidelines in section 1 for a 2 sample *z*-test for proportions.

Notes:

- Examples of unacceptable hypotheses:
  $H_0 : p_{17}$ and p_{14} are independent or $H_0 : p_{17}$ and p_{14} have no association
  $H_a : p_{17}$ and p_{14} are dependent or $H_a : p_{17}$ and p_{14} have an association
  $H_a : p_{17} \neq p_{14}$ or $H_a : p_{17} \geq p_{14}$ or $H_a : p_{14} \leq p_{17}$
- To satisfy component 4, a chi-square test for homogeneity is identified by name or formula.

Section 2 is scored E, P, or I according to the guidelines in section 2 for a 2 sample *z*-test for proportions.

Notes:

- To satisfy component 4, the reported value of the chi-square test statistic is 4.8821.
- If a one sided alternative hypothesis is given in section 1, then to satisfy component 5, the reported value of the *p*-value is 0.01357.
- If a two sided alternative hypothesis is given in section 1, then to satisfy component 5, the reported *p*-value is 0.02714.

Section 3 is scored E, P or I according to the guidelines in section 3 for a 2 sample *z*-test for proportions.

Notes:

- If the response clearly indicates that the two sample proportions were used to determine the correct one-sided direction and a *p*-value of 0.0135 was used as justification, section 3 is scored E.
- If a correct conclusion is reached based on the *p*-value of 0.02714 and a two-sided alternative, then section 3 is scored at most P.
- If the final response justifies the conclusion based on the *p*-value of 0.0135 but does not explicitly indicate how the correct direction was determined, section 3 is scored at most P.

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Question 4 (continued)

4 Complete Response

Three sections essentially correct

3 Substantial Response

Two sections essentially correct and one section partially correct

2 Developing Response

Two sections essentially correct and no sections partially correct

OR

One section essentially correct and one or two sections partially correct

OR

Three sections partially correct

1 Minimal Response

One section essentially correct

OR

No section essentially correct and two sections partially correct

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Question 5

Intent of Question

The primary goals of this question were to assess a student's ability to (1) evaluate a percentile of a normal distribution; (2) evaluate a probability for a normal distribution; and (3) compute an expected value for a random variable with two possible outcomes.

Solution

Part (a):

The 25th percentile of the standard normal distribution is -0.6745. Consequently the 25th percentile of a normal distribution with mean 30 months and standard deviation 8 months is $30 + 8(-0.6745) = 24.6$ months.

Part (b):

The probability that a randomly selected customer will need to request a replacement because the battery fails within 24 months from the date of purchase is

$$P(\text{life span} \le 24 \text{ months}) = P\left(Z \le \frac{24 - 30}{8}\right) = P(Z \le -0.75) \approx 0.2266.$$

Part (c):

The company's expected gain for each warranty purchased is

$$\begin{aligned}&(\$50) \times P(\text{life span} > 24 \text{ months}) + (-\$150) \times P(\text{life span} \le 24 \text{ months})\\ &= (\$50) \times (0.7734) + (-\$150) \times (0.2266) \approx \$4.68.\end{aligned}$$

Scoring

Parts (a), (b), and (c) are each scored as essentially correct (E), partially correct (P), or incorrect (I).

Part (a) is scored as follows:

Essentially correct (E) if the response satisfies the following three components:

1. Indicates the use of a normal distribution with a mean of 30 and a standard deviation of 8.
2. Sets up a correct approach for finding the 25th percentile of the battery life span distribution.
3. Reports the correct value of the 25th percentile (24.6) or a percentile value that is consistent with components 1 and 2.

Partially correct (P) if the response satisfies only two of the three components.

Incorrect (I) if the response does not meet the criteria for E or P.

Notes:

- Component 1 may be satisfied in either part (a) or (b).
- Incorrect statistical notation in specifying the distribution mean or standard deviation (for example, $\bar{x} = 30$ or $s = 8$) results in Component 1 not being met the first time it appears.

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Question 5 (continued)

- Component 1 may be satisfied by one of the following:
  - Graphical: Displaying a graph of a normal density function with the horizontal axis clearly labeled using the mean and standard deviation for the battery life span distribution.
  - Calculator syntax: Labeling the mean and standard deviation in an inverse normal cdf calculator statement, for example, invNorm$(0.25, \mu = 30, \sigma = 8)$.
  - z-score: Showing correct components in a standard z-score calculation, for example, $z = \frac{x - 30}{8}$, or $-0.6745 = \frac{x - 30}{8}$, or $x = 30 + 8(-0.6745)$.
  - Notation: Using standard notation for a normal distribution, for example, $N(30, 8)$ or Normal$(30, 64)$.
- Component 2 may be satisfied by one of the following:
  - Graphical: Identifying the lower-tail area corresponding to the probability of 0.25 in a graph of a normal density function.
  - Calculator syntax: Stating the correct percentile in an inverse normal cdf calculator statement, for example, invNorm$(0.25, \mu = 30, \sigma = 8)$.
  - z-score: Equating the z-score of the 25th percentile of the battery life span distribution to -0.6745, for example, $-0.6745 = \frac{x - 30}{8}$ or $x = 30 + 8(-0.6745)$.

Part (b) is scored as follows:

Essentially correct (E) if the response satisfies the following three components:
1. Indicates the use of a normal distribution with a mean of 30 and a standard deviation of 8.
2. Specifies the correct event, including the correct boundary value and direction.
3. Reports the correct probability (0.2266) or a probability consistent with components 1 and 2.

Partially correct (P) if response satisfies only two of the three components.

Incorrect (I) if the response does not meet the criteria for E or P.

Notes:
- Component 1 may be satisfied in either part (a) or (b).
  - Incorrect statistical notation in specifying the distribution mean or standard deviation (for example, $\bar{x} = 30$ or $s = 8$) results in Component 1 not being met the first time it appears.
- Component 1 may be satisfied by one of the following:
  - Graphical: Displaying a graph of a normal density function with the horizontal axis clearly labeled using the mean and standard deviation for the battery life span distribution.
  - Calculator syntax: Labeling the mean and standard deviation in a normal cdf calculator statement, for example, normalcdf$(0, \text{upper} = 24, \mu = 30, \sigma = 8)$.
  - z-score: Showing correct components in a standard z-score calculation, for example, $z = \frac{x - 30}{8}$, or $z = \frac{24 - 30}{8}$, or $\frac{24 - 30}{8} = -0.75$.
  - Notation: Using standard notation for a normal distribution, for example, $N(30, 8)$ or Normal$(30, 64)$

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Question 5 (continued)

- Component 2 may be satisfied by one of the following:
  - o Graphical: A normal density graph labeled with boundary and direction corresponding to the region of interest.
  - o Calculator syntax: Labeling the upper bound in a normal cdf calculator statement. The lower bound does not need to be labeled but must be less than or equal to 0, for example, normalcdf(0, upper = 24, $\mu = 30$, $\sigma = 8$).
  - o In words: Specifying the correct event, boundary value, and direction, in words: for example, P(battery life span $\leq$ 24 months), or "probability requires battery replacement within 24 months."
  - o Random variable: Specifying the boundary value and direction using a random variable, for example, $P(X \leq 24)$ or $P(Z \leq -0.75)$.

Part (c) is scored as follows:

Essentially correct (E) if the response satisfies the following two components:

1. Provides an expected value calculation with two appropriate probabilities that add to 1 and are paired with the correct outcomes.
2. Reports the correct expected value (4.68), or a reasonable expected value consistent with work shown.

Partially correct (P) if the response satisfies only one of the two components.

Incorrect (I) if the response does not satisfy the criteria for E or P.

Notes:

- For component 1, appropriate probabilities that add to 1 are:
  - o probabilities consistent with the value computed in part (b); or
  - o the correct probabilities; or
  - o if there is no probability provided in part (b), probabilities explicitly defined in context in part (c).
- If the correct probability is used, due to rounding considerations, any number between 4.00 and 4.70 is acceptable as a correct expected value for component 2.
- A reasonable expected value for component 2 can be any number between −\$150 and +\$50 that is consistent with work shown.

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Question 5 (continued)

4 Complete Response

Three parts essentially correct

3 Substantial Response

Two parts essentially correct and one part partially correct

2 Developing Response

Two parts essentially correct and no parts partially correct

OR

One part essentially correct and one or two parts partially correct

OR

Three parts partially correct

1 Minimal Response

One part essentially correct

OR

No parts essentially correct and two parts partially correct

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Question 6

Intent of Question

The primary goals of this question were to assess a student's ability to (1) recognize the population to which results from a random sample may be generalized; (2) describe a disadvantage of using a sample mean rather than a sample median to indicate typical values when the sample distribution is skewed; (3) describe how the theoretical sampling distribution of the sample median could be constructed; (4) construct an approximate confidence interval for a population median using results from a bootstrap procedure; and (5) interpret a confidence interval.

Solution

Part (a):

Because random sampling was used, the results of the sample may be generalized to the population of rental prices for one-bedroom apartments in the city that are listed on this particular website at the time the sample was taken.

Part (b):

Because the distribution of the 50 rental prices in the sample is skewed to the right, the sample median provides a better indicator of typical rental prices than the sample mean. Some very large rental prices results in a sample mean that is substantially larger than the more typical rental prices. As a result the sample mean would overestimate the typical rental price, whereas the sample median would be a more accurate representation of typical rental prices.

Part (c):

To determine the sampling distribution of median rental prices for random samples of 50 one-bedroom apartments from this population, Emma would need to obtain every possible sample of 50 one-bedroom apartments from this website and compute the median of each sample. The collection of all possible sample medians is the theoretical sampling distribution for sample median.

Part (d):

(i) $(0.05)(15,000) = 750$ and $(0.95)(15,000) = 14,250$. The 5th percentile is a value, say $x_{0.05}$, such that at least 750 values in the table are less than or equal to $x_{0.05}$ and at least 14,250 are greater than or equal to $x_{0.05}$. Cumulate frequencies starting with the smallest sample median listed in the table and going toward the largest (going down columns) until you first reach 750 values, to obtain $x_{0.05} = \$2,500$.

(ii) Similarly, $x_{0.95} = \$2,950$.

Part (e):

The percentage of bootstrap medians between (and including) the values found in part (d) for the 5th and 95th percentiles is

$$\frac{14,404}{15,000} \times 100\% \approx 96.03\%$$

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Question 6 (continued)

Part (f):

From the results in part (d) and part (e), an approximate 96 percent confidence interval for the median rental price of all one-bedroom apartments listed on this website for this city is (\$2,500, \$2,950). We are approximately 96 percent confident that the median rental price of all one-bedroom apartments listed on this website for this city is between \$2,500 and \$2,950.

Scoring

This question is scored in four sections. Section 1 consists of parts (a) and (b), section 2 consists of part (c), section 3 consists of parts (d) and (e), and section 4 consists of part (f). Sections 1, 2, 3, and 4 are each scored as essentially correct (E), partially correct (P), or incorrect (I).

Section 1 is scored as follows:

Essentially correct (E) if the response satisfies the following three components:
1. The correct population (listings of one-bedroom apartments on the website) is identified in part (a).
2. In part (b), identifying that using the sample mean instead of the sample median overestimates the typical rental price.
3. The disadvantage of using the sample mean that is reported in part (b) is correctly linked to some feature of the distribution (e.g. skewness) that is evident in the histogram.

Partially correct (P) if the response satisfies only two of the three components.

Incorrect (I) if the response does not meet the criteria for E or P.

Note: Responses that refer to the mean being larger than the median in a skewed right distribution alone is not sufficient to satisfy component 2.

Section 2 is scored as follows:

Essentially correct (E) if the response satisfies the following two components:
1. Indicates that Emma would need to obtain every possible sample of 50 one-bedroom apartments.
2. Indicates that Emma would need to compute the median rental price for each sample.

Partially correct (P) if the response satisfies only one of the two components.

Incorrect (I) if the response does not satisfy the criteria for E or P.

Section 3 is scored as follows:

Essentially correct (E) if the response satisfies the following two components:
1. Correct values for the 5th percentile and the 95th percentile are reported in part (d).
2. The correct percentage of bootstrap samples that produced sample medians at or between the two values, if they are plausible, reported in part (d) is reported in part (e).

Partially correct (P) if the response satisfies only one of the two components.

Incorrect (I) if the response does not satisfy the criteria for E or P.

Note: Plausible values for part (d) will be considered values between 2,345 and 3,062.5.

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Question 6 (continued)

Section 4 is scored as follows:

Essentially correct if the response in part (f) satisfies the following three components:

1. Uses $2500 and $2950 or the values of the percentiles reported in part (d) as the endpoints of the confidence interval.
2. Indicates an approximate 90 or 96 percent level of confidence or a level of confidence consistent with part (e).
3. Makes a correct statement in context indicating that the confidence interval is for the median.

Partially correct if the response satisfies only two of the three components.

Incorrect if the response does not satisfy the criteria for E or P.

Note: Since rental prices from the population are discrete values, the true confidence level of the interval from part (d) is unknown. A correctly calculated part (e) is a way to estimate the confidence level; from Emma's sample the confidence level is estimated to be approximately 96 percent. The process described in part (d) for calculating the interval will result in a confidence level of at least 90 percent. For these reasons, confidence levels of either 90 or 96 percent satisfy component 2.

Each essentially correct (E) section counts as 1 point, and a partially correct (P) section counts as ½ point.

4 **Complete Response**

3 **Substantial Response**

2 **Developing Response**

1 **Minimal Response**

If a response is between two scores (for example, 2½ points), use a holistic approach to decide whether to score up or down depending on the strength of the response and communication.

AP® Statistics
2015 Scoring Guidelines

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Question 1

Intent of Question

The primary goals of this question were to assess a student's ability to (1) compare features of two distributions of data displayed in boxplots and (2) identify statistical measures that are important in making decisions based on data sets.

Solution

Part (a):

The median salary is approximately the same for both corporations. The range and interquartile range of the salaries are greater for Corporation A than for Corporation B. The two highest salaries at Corporation A are outliers while Corporation B has no outliers.

Part (b):

(i) Five years after starting, at least 3 out of 30 (10%) of the salaries at Corporation A are greater than the maximum salary at Corporation B. If I accept the offer from Corporation A, I might be able to make a higher salary at Corporation A than at Corporation B.

(ii) Five years after starting, the minimum salary at Corporation B is greater than at Corporation A. In fact, at Corporation A it looks like some people are still making the starting salary of $36,000 and never received a raise in the five years since they were hired. So if I work at Corporation A, I might never receive a raise in salary.

Scoring

Parts (a) and (b) are scored as essentially correct (E), partially correct (P), or incorrect (I).

Part (a) is scored as follows:

Essentially correct (E) if the response includes the following four components:
1. A correct comparison of center.
2. A correct comparison of spread.
3. A discussion of the outliers for Corporation A.
4. The response is in context.

Partially correct (P) if the response includes only three of the four components.

Incorrect (I) if the response includes at most two of the four components.

Note: Any mention of shape should be ignored because complete shape information cannot be determined from a boxplot.

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Question 1 (continued)

Part (b) is scored as follows:

Essentially correct (E) if the response includes the following four components:
1. In part (b-i) a relevant statistical measure is identified (or described) or a relevant statistical comparison is provided that supports the choice of Corporation A.
2. In part (b-i) an explanation is provided for why the measure or comparison is relevant.
3. In part (b-ii) a relevant statistical measure is identified (or described) or a relevant statistical comparison is provided that supports the choice of Corporation B.
4. In part (b-ii) an explanation is provided for why the measure or comparison is relevant.

Partially correct (P) if the response includes only two or three of the four components.

Incorrect (I) if the response includes none or one of the four components.

Note: If a response does not provide a statistical measure or comparison in part (b-i) or (b-ii), the second and fourth components can still be satisfied if an acceptable explanation is provided that would follow from a relevant statistical measure or comparison. For example, if the response in part (b-i) only states "At Corporation A, I have the potential to earn a higher salary," the second component is satisfied.

4 Complete Response

Both parts essentially correct

3 Substantial Response

One part essentially correct and one part partially correct

2 Developing Response

One part essentially correct and one part incorrect

OR

Both parts partially correct

1 Minimal Response

One part partially correct and one part incorrect

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Question 2

Intent of Question

The primary goals of this question were to assess a student's ability to (1) use confidence intervals to test a question about a proportion and (2) understand the relationship between sample size and margin of error in a confidence interval for a proportion.

Solution

Part (a):

(i) No. The confidence interval is $(0.09, 0.21)$, which includes the value of 0.20. Therefore, it is plausible that the computer program is generating discounts with a probability of 0.20, and the confidence interval does not provide convincing statistical evidence that the program is not working as intended.

(ii) No. The confidence interval includes values from 0.09 to 0.21, so any value in that interval is a plausible value for the probability that the computer is using to generate discounts.

Part (b):

The formula for computing the margin of error for a proportion includes the square root of the sample size in the denominator. For a random sample that is four times the size of the original sample, the margin of error can be determined by dividing the margin of error of the original sample by two. Therefore, the new margin of error is 0.03.

Part (c):

Using the margin of error of 0.03 obtained from the second sample, the confidence interval for p is 0.15 ± 0.03 or $(0.12, 0.18)$. The interval does not include 0.20, and therefore, there is convincing evidence that the computer program is not working as intended and is not generating discounts with a probability of 0.20.

Scoring

This question is scored in four sections. Section 1 consists of part (a-i); section 2 consists of part (a-ii); section 3 consists of part (b); and section 4 consists of part (c). Sections 1, 2, 3, and 4 are scored as essentially correct (E), partially correct (P), or incorrect (I).

Section 1 is scored as follows:

Essentially correct (E) if the response states that because the interval contains 0.20, it does not provide convincing statistical evidence that the computer program is not working as intended.

Partially correct (P) if the response indicates that it is necessary to check whether the value of 0.20 is in the computed interval, but there are errors in implementation. Examples of errors include:

- The response notes that 0.20 is within the interval but does not draw a conclusion.
- The response has an arithmetic error in the computation of the endpoints of the interval but provides a correct conclusion with justification that is consistent with the computed interval.

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Question 2 (continued)

Incorrect (I) if the response does not recognize how to use the confidence interval to check whether the computer is working correctly;

OR

if the response states that the interval shows that the proportion is equal to 0.20;

OR

if the response notes that 0.20 is within the interval and concludes that the program is not working as intended;

OR

if the response otherwise does not meet the criteria for E or P.

Section 2 is scored as follows:

Essentially correct (E) if the response concludes that there is not convincing statistical evidence that the computer program generates the discount with a probability of 0.20 *AND* justifies the conclusion by noting that there are values other than 0.20 in the interval.

Partially correct (P) if the response correctly concludes that there is not convincing evidence that the computer program generates the discount with a probability of 0.20, but provides incomplete reasoning to justify the conclusion.

Examples of incomplete reasoning include:

- stating that 0.20 is a plausible value for the proportion of discounts without giving further explanation;
- indicating that having 0.20 in the interval does not prove that 0.20 is the true proportion; and
- providing a generic statistical argument, such as reference to the fact that the null hypothesis should never be accepted, or stating that not rejecting the null hypothesis is not proof that 0.20 is the true proportion of bills discounted.

Note: If an incorrect interval is computed in part (a-i) that does not contain 0.20, and in part (a-ii) the response concludes that the program is not generating discounts with a probability of 0.20 because 0.20 is not in the interval, section 2 is scored as P.

Incorrect (I) if the response states that there is evidence that the computer program generates the discount with a probability of 0.20;

OR

if the response correctly concludes that there is not convincing evidence that the computer program generates the discount with a probability of 0.20 *AND* provides incorrect or no justification;

OR

if the response otherwise does not meet the criteria for E or P.

Section 3 is scored as follows:

Essentially correct (E) if the response gives the correct value of 0.03 as the new margin of error by using the correct formula or by providing a correct explanation that recognizes that quadrupling the sample size divides the margin of error from the original sample by two.

Note: If the response relies on the margin of error formula and calculates a value that would round to 0.030, the response is scored as E.

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Question 2 (continued)

Partially correct (P) if the response uses the correct margin of error formula or provides a correct explanation that recognizes that quadrupling the sample size divides the margin of error from the original sample by two, but calculates an incorrect margin of error that is less than 0.06;
OR
if the response does not calculate a margin of error but provides a correct explanation that recognizes that quadrupling the sample size divides the margin of error from the original sample by two;
OR
if the response gives the correct margin of error without correct justification.

Incorrect (I) if the response does not recognize in any way that the new margin of error depends on the square root of the sample size;
OR
if the response calculates a margin of error that is greater than or equal to 0.06;
OR
if the response otherwise does not meet the criteria for E or P.

Section 4 is scored as follows:

Essentially correct (E) if the conclusion states that there is convincing evidence that the computer program is not working as intended because, based on the new margin of error, the interval does not contain 0.20;
OR
if the conclusion states the intended value of 0.20 is greater than the upper boundary of 0.18;
OR
if the conclusion states the margin of error is smaller than the difference between the sample proportion and the intended long-run proportion of 0.20.

Notes:

- If the margin of error was computed incorrectly in part (b), but a correct answer to part (c) is consistent with this incorrect margin of error, section 4 is scored as E.
- If no specific margin of error or confidence interval was given in parts (b) or (c), but the response provides a complete and correct conclusion with reference to a smaller margin of error (or narrower confidence interval), section 4 is scored as E.

Partially correct (P) if an interval is incorrectly constructed using the margin of error from part (b), but a correct conclusion is given and justified for the computed interval;
OR
if an interval is computed correctly using the margin of error from part (b) and an argument is made based on whether or not 0.20 is in the interval, but the conclusion is incorrect;
OR
if no specific margin of error or confidence interval has been given in parts (b) or (c), but the response concludes that because the margin of error has decreased (or the confidence interval is narrower), 0.20 is not in the interval.

Incorrect (I) if an interval is computed correctly using the margin of error from part (b) and a correct conclusion is given, but no argument is made based on whether or not 0.20 is in the interval;
OR
if the response otherwise does not meet the criteria for E or P.

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Question 2 (continued)

Notes:

- If a response includes a confidence level, the level can be ignored because no confidence level was provided.
- If the response bases a conclusion on the relative location of 0.20 within the interval (for example, 0.20 is near the edge of the interval), the response is scored as I.
- A response that provides additional incorrect explanation lowers the score in section 4 by one level (that is, from E to P, or from P to I).

Each essentially correct (E) section counts as 1 point. Each partially correct (P) section counts as ½ point.

4 Complete Response

3 Substantial Response

2 Developing Response

1 Minimal Response

If a response is between two scores (for example, 2½ points), use a holistic approach to decide whether to score up or down, depending on the overall strength of the response and communication.

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Question

Intent of Question

The primary goals of this question were to assess a student's ability to (1) perform a probability calculation from a discrete random variable; (2) calculate the expected value of a discrete random variable; (3) perform a conditional probability calculation from a discrete random variable; and (4) use probabilistic thinking to make a prediction about how an expected value will change given a condition about the random variable.

Solution

Part (a):

The probability that at least one ATM is working when the mall opens is:

$$P(X \geq 1) = 0.21 + 0.40 + 0.24 = 0.85.$$

Part (b):

The expected value of the number of ATMs that are working when the mall opens is:

$$E(X) = 0(0.15) + 1(0.21) + 2(0.40) + 3(0.24) = 1.73 \text{ machines.}$$

Part (c):

›robability that all three ATMs are working when the mall opens, given that at least one ATM is working is:

$$P(X = 3 \mid X \geq 1) = \frac{P(X = 3 \text{ and } X \geq 1)}{P(X \geq 1)} = \frac{P(X = 3)}{P(X \geq 1)} = \frac{0.24}{0.85} \approx 0.282$$

Part (d):

Given that at least one ATM is working when the mall opens, the expected value of the number of working ATMs would be greater than the expected value calculated in part (b). By eliminating the possibility of 0 working ATMs, the probabilities for 1, 2, and 3 working ATMs all increase proportionally, so the expected value must increase.

Scoring

(b), (c), and (d) are scored as essentially correct (E), partially correct (P), or incorrect (I).

Part (a) is scored as follows:

Essentially correct (E) if the probability is computed correctly with work shown.

Partially correct (P) if the correct answer is given, but no work is shown;
OR
ıppropriate work is shown but the answer is incorrect or missing;
OR

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Question 3 (continued)

one of the incorrect cumulative probabilities $P(X < 1)$, $P(X \leq 1)$, or $P(X > 1)$ is stated *AND* computed correctly.

Incorrect (I) if the response does not meet the criteria for E or P.

Note: The probability can be calculated as $1 - P(X = 0) = 1 - 0.15 = 0.85$.

Part (b) is scored as follows:

Essentially correct (E) if the expected value is computed correctly with work shown.

Partially correct (P) if the correct answer is given, but no work is shown;
OR
if appropriate work is shown but the answer is incorrect or missing.

Incorrect (I) if the response does not meet the criteria for E or P.

Part (c is scored as follows:

Essentially correct (E) if the probability is computed correctly, with work shown that includes correct numerical values for both the numerator and denominator.

Partially correct (P) if the response includes a numerator and denominator in calculating the conditional probability, with one (numerator or denominator) correct in numerical value and the other incorrect;
OR
if the correct answer is given, but no work is shown.

Incorrect (I) if the response does not meet the criteria for E or P.

Part (d) is scored as follows:

Essentially correct (E) if the response provides the correct answer (greater than) with a reasonable explanation based on the fact that with $X = 0$ eliminated, the probabilities for $X = 1$, $X = 2$, $X = 3$ *all* increase;
OR
if the response provides the correct answer (greater than) with a reasonable explanation based on fact that with $X = 0$ eliminated, the balance point of the distribution increases;
OR
if the response provides the correct answer (greater than) and the conditional expected value is computed correctly with work shown.

Partially correct (P) if the response provides the correct answer (greater than) with a weak explanation, such as "Yes, because 0 is eliminated";
OR

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Question 3 (continued)

if the response provides the correct answer (greater than) with a reasonable but incorrect attempt to calculate the conditional expected value using a revised probability distribution with only the values $X = 1$, $X = 2$, and $X = 3$, and corresponding probabilities greater than or equal to those in the original probability distribution.

Incorrect (I) if the response does not provide the correct answer (greater than);
OR
if the response provides the correct answer (greater than) with an incorrect explanation or no explanation.

Note: The conditional expected value is:

$$1\left(\frac{0.21}{0.85}\right) + 2\left(\frac{0.40}{0.85}\right) + 3\left(\frac{0.24}{0.85}\right) \approx 1(0.247) + 2(0.471) + 3(0.282) = 2.04 \text{ machines.}$$

Each essentially correct (E) part counts as 1 point. Each partially correct (P) part counts as ½ point.

4 **Complete Response**

3 **Substantial Response**

2 **Developing Response**

1 **Minimal Response**

If a response is between two scores (for example, 2½ points), use a holistic approach to decide whether score up or down, depending on the overall strength of the response and communication.

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Question 4

Intent of Question

The primary goal of this question was to assess a student's ability to identify, set up, perform, and interpret the results of an appropriate hypothesis test to address a particular question. More specific goals were to assess a student's ability to (1) state appropriate hypotheses; (2) identify the appropriate statistical test procedure and check appropriate conditions for inference; (3) calculate the appropriate test statistic and *p*-value; and (4) draw an appropriate conclusion, with justification, in the context of the study.

Solution

Step 1: States a correct pair of hypotheses.

Let p_{asp} represent the population proportion of adults similar to those in the study who would have developed colon cancer within the six years of the study if they had taken a low-dose aspirin each day. Similarly, let p_{plac} represent the population proportion of adults similar to those in the study who would have developed colon cancer within the six years of the study if they had taken a placebo each day.

The hypotheses to be tested are $H_0 : p_{asp} = p_{plac}$ versus $H_a : p_{asp} < p_{plac}$ or equivalently, $H_0 : p_{asp} - p_{plac} = 0$ versus $H_a : p_{asp} - p_{plac} < 0$.

Step 2: Identifies a correct test procedure (by name or by formula) and checks appropriate conditions.

The appropriate procedure is a two-sample *z*-test for comparing proportions.

Because this is a randomized experiment, the first condition is that the volunteers were randomly assigned to one treatment group or the other. The condition is satisfied because we are told that the volunteers were randomly assigned to take a low-dose aspirin or a placebo.

The second condition is that the sample sizes are large, relative to the proportions involved. The condition is satisfied because all sample counts are large enough; that is, 15 with colon cancer in aspirin group, 26 with colon cancer in placebo group, $500 - 15 = 485$ cancer-free in aspirin group, and $500 - 26 = 474$ cancer-free in placebo group.

Step 3: Calculates the appropriate test statistic and *p*-value.

The sample proportions who developed colon cancer are $\hat{p}_{asp} = \frac{15}{500} = 0.030$ and $\hat{p}_{plac} = \frac{26}{500} = 0.052$.

The combined sample proportion who developed colon cancer is $\hat{p}_{combined} = \frac{15 + 26}{500 + 500} = 0.041$.

The test statistic is $z = \frac{0.030 - 0.052}{\sqrt{0.041(1 - 0.041)\left(\frac{1}{500} + \frac{1}{500}\right)}} \approx -1.75$ (-1.7542 from calculator).

The *p*-value is $P(Z \le -1.75) = 0.0401$ (0.0397 from calculator), where Z has a standard normal distribution.

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Question 4 (continued)

Step 4: States a correct conclusion in the context of the study, using the result of the statistical test.

Because the *p*-value is less than the given significance level of $\alpha = 0.05$, we reject the null hypothesis. The data provide convincing statistical evidence that the proportion of all adults similar to the volunteers who would develop colon cancer if they had taken a low-dose aspirin every day is less than the proportion of all adults similar to the volunteers who would develop colon cancer if they had not taken a low-dose aspirin every day.

Scoring

Steps 1, 2, 3, and 4 are scored as essentially correct (E), partially correct (P), or incorrect (I).

Step 1 is scored as follows:

Essentially correct (E) if the response identifies correct parameters *AND* both hypotheses are labeled and state the correct relationship between the parameters.

Partially correct (P) if the response identifies correct parameters *OR* states correct relationships, but not both.

Incorrect (I) if the response does not meet the criteria for E or P.

Note: Either defining the parameters in context, or simply using common parameter notation with subscripts clearly relevant to the context, such as p_{asp} and p_{plac}, is sufficient.

Step 2 is scored as follows:

Essentially correct (E) if the response correctly includes the following three components:
1. Identifies the correct test procedure (by name or by formula).
2. Notes that the use of random assignment satisfies the randomness condition.
3. Checks for approximate normality of the test statistic by citing that all four counts are larger than some standard criterion such as 5 or 10.

Partially correct (P) if the response correctly includes only two of the three components.

Incorrect (I) if the response correctly includes at most one of the three components.

Notes:
- For the randomness component, it is (minimally) acceptable to say "random assignment — check" but not acceptable to say "random — check" or "SRS — check." The important concept here is that it is random assignment, and not random sampling, that is required. If the response implies that the study used a random sample, the randomness component is not satisfied, regardless of whether random assignment is correctly addressed.
- The normality check may use the expected counts under the null hypothesis in place of observed counts.

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Question 4 (continued)

Step 3 is scored as follows:

Essentially correct (E) if the response correctly calculates both the test statistic and a *p*-value that is consistent with the stated alternative hypothesis.

Partially correct (P) if the response correctly calculates the test statistic but not the *p*-value;
OR
if the response calculates the test statistic incorrectly but then calculates the correct *p*-value for the computed test statistic;
OR
if the response reports the correct *p*-value but no calculations or test statistic are shown.

Incorrect (I) if the response fails to meet the criteria for E or P.

Note: The *p*-value is considered correct if it is consistent with the alternative stated in the response and the calculated test statistic, even if those are incorrect.

Step 4 is scored as follows:

Essentially correct (E) if the response provides a correct conclusion in context, with justification based on linkage between the *p*-value and the given $\alpha = 0.05$.

Partially correct (P) if the response provides a correct conclusion, with linkage to the *p*-value, but not in context;
OR
if the response provides a correct conclusion in context, but without justification based on linkage to the *p*-value.

Incorrect (I) if the response does not meet the criteria for E or P.

Notes:

- The conclusion must be related to the alternative hypothesis.
- If the *p*-value is incorrect, then step 4 is scored as E if the response includes proper linkage and a conclusion in context consistent with that *p*-value.
- If the *p*-value is less than 0.05, wording that states or implies that the alternative hypothesis is *proven* lowers the score one level (that is, from E to P or P to I) in step 4.
- If the *p*-value is incorrect and greater than 0.05, wording that states or implies that the null hypothesis is *accepted* lowers the score one level (that is, from E to P or P to I) in step 4.

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Question 4 (continued)

Each essentially correct (E) step counts as 1 point. Each partially correct (P) step counts as ½ point.

4 **Complete Response**

3 **Substantial Response**

2 **Developing Response**

1 **Minimal Response**

If a response is between two scores (for example, 2½ points), use a holistic approach to decide whether to score up or down, depending on the overall strength of the response and communication.

SCORING GUIDELINES

Question

Intent of Question

The primary goals of this question were to assess a student's ability to (1) use the information provided by a scatterplot to describe the relationship between two quantitative variables; (2) interpret and use the information given by lines displayed on a scatterplot; and (3) use a regression equation to estimate a predicted value of y for a given x value.

Solution

Part

There is a moderately strong, positive, linear relationship between height and arm span so that taller students tend to have longer arm spans.

Part (b):

The line in Graph 2 is the one that is helpful. For each student, the graph illustrates whether arm is equal to height (points on the line), arm span is greater than height (points above the line), or arm span is less than height (points below the line).

Classification	Square	Tall Rectangle	Short Rectangle
Frequency	3	4	5

Part (c):

The predicted arm span is $\hat{y} = 11.74 + 0.8247x = 11.74 + 0.8247(61) = 62.05$ inches.

Scoring

b), and (c) are scored as essentially correct (E), partially correct (P), or incorrect (I).

Part (a) is scored as follows:

Essentially correct (E) if the response includes the following four components:

1. The relationship is approximately linear.
2. The relationship is positive.
3. There is a moderate to strong association (or relationship).
4. The response is given in context.

Partially correct (P) if the response includes only two or three of the four components.

Incorrect (I) if the response includes at most one of the four components.

SCORING GUIDELINES

Question 5 (continued)

Notes:

- Correlation alone is not sufficient to indicate a linear relationship. However, linear correlation sufficient to indicate a linear relationship.
- Reporting that the correlation coefficient is 0.81, the correlation coefficient is close to 1.0, or some other value or range of values for the correlation coefficient is not sufficient to satisfy component 3.

Part (b) is scored as follows:

Essentially correct (E) if the response contains the following four components:

1. The line in Graph 2 is selected.
2. A reasonable explanation for selecting the line in Graph 2 that links the body shapes to the regions defined by the line is given. The explanation should indicate that square shapes (arm span is equal to height) are represented by points on the line, tall rectangle shapes (arm spa less than height) are represented by points below the line, and short rectangle shapes (arm span is greater than height) are represented by points above the line.
3. Correct counting of body types demonstrated by reporting correct frequencies (3, 4, and 5) reporting correct proportions $\left(\frac{1}{4} = \frac{3}{12}, \frac{1}{3} = \frac{4}{12}, \text{ and } \frac{5}{12}\right)$ for the square, tall rectangle, and short rectangle body types, respectively.
4. The correct frequencies (3, 4, and 5) are reported in the table.

Partially correct (P) if the response includes only two or three of the four components.

Incorrect (I) if the response includes at most one of the four components.

Notes:

- Selecting the regression line on Graph 1 cannot satisfy either component 1 or component 2.
- To satisfy component 1, it is sufficient to refer to the $y = x$ line without explicitly mentioning Graph 2.
- Use of incorrect labels, such as regression line or least-squares regression line, in referring to the $y = x$ line in Graph 2 is an incorrect use of terminology that should be strongly discouraged, but it is ignored in this scoring rubric.
- The explanation for selecting Graph 2 is acceptable if it explicitly includes at least two of the following: (i) square body shapes (arm span equal to height) are represented by points line, (ii) tall rectangle body shapes (arm span less than height) are represented by points *below* the line, or (iii) short rectangle body shapes (arm span greater than height) are represented by points *above* the line.
- Incorrectly counting the points on the $y = x$ line in Graph 2 is considered a minor error. Frequencies (2, 4, and 5) are accepted for both component 3 and component 4. Reporting corresponding proportions $\left(\frac{2}{11}, \frac{4}{11}, \frac{5}{11}\right)$ or $\left(\frac{2}{12}, \frac{4}{12}, \frac{5}{12}\right)$ in the table satisfies component does not satisfy component 4.
- Frequencies reported in the table as (2, 5, 4) or (3, 5, 4), interchanging the counts for tall and short rectangle body shapes, satisfies component 3, but does not satisfy component 4. Reporting corresponding proportions, $\left(\frac{2}{11}, \frac{5}{11}, \frac{4}{11}\right)$ $\left(\frac{3}{12}, \frac{5}{12}, \frac{4}{12}\right)$, in the table does not satisfy either component 3 or component 4.

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Question 5 (continued)

Part (c) is scored as follows:

Essentially correct (E) if the response contains the following three components:
1. Correct formula for predicting arm span with 61 inserted for x.
2. Correct value for the predicted arm span.
3. Units for the predicted arm span given as inches.

Partially correct (P) if the response includes only two of the three components.

Incorrect (I) if the response includes at most one of the three components.

Notes:
- Any value for the predicted arm span between 61.5 and 62.5 is acceptable; values outside th interval do not satisfy component 2. This allows for inaccuracy in obtaining a value for the predicted arm span directly from the line displayed on Graph 1 and for rounding in applying the prediction formula; but it excludes predictions based on the $y = x$ line and other unreasonable predictions.
- Reporting a prediction equation that is similar to the equation given in the stem, with 61 inserted for x, satisfies components 1 and 2 if the value of the predicted arm span is between 61.5 and 62.5 inches. Otherwise, neither of those two components is satisfied. This could occur if a student enters the data from the graph into a calculator to compute the least squares regression line, instead of using the formula provided in the stem.

4 Complete Response

All three parts essentially correct

3 Substantial Response

Two parts essentially correct and one part partially correct

2 Developing Response

Two parts essentially correct and one part incorrect

OR

One part essentially correct and two parts partially correct

OR

One part essentially correct, one part partially correct, and one part incorrect

OR

Three parts partially correct

1 Minimal Response

One part essentially correct and two parts incorrect

OR

Two parts partially correct and one part incorrect

2015 SCORING GUIDELINES

Question 6

Intent of Question

The primary goals of this question were to assess a student's ability to (1) describe how sample data would differ using two different sampling methods; (2) describe the sampling distribution of the sample mean for two different sampling methods; and (3) choose the sampling method that will result in the best estimate of the population mean.

Solution

Part (a):

No, a sample obtained using Method 2 will not be representative of all tortillas made that day. The sample obtained using Method 2 will only represent the tortillas from one production line, not from the entire population because the distributions of diameters for the two production lines are different.

Part (b):

Method 1 was most likely used to select this sample. The bimodal shape in the histogram of sample data indicates that tortillas were selected from both production lines, which is what would happen using Method 1. Method 2 would be likely to produce a unimodal distribution of diameters centered at either 5.9 inches or 6.1 inches.

Part (c):

Method 2 would result in less variability in the sample of 200 tortillas on a given day because the sample comes from only one production line. Because the distributions of diameters are not the same for the two production lines, selecting tortillas from both lines as in Method 1 would result in more variable sample data.

Part (d):

The sampling distribution of the sample mean diameter for samples obtained using Method 1 would be approximately normal with mean 6 inches and standard deviation $\frac{0.11}{\sqrt{200}} \approx 0.0078$ inch.

Part (e):

Method 1 would result in less variability in the sample means over the 365 days, because with Method 2, roughly half of the sample means will be around 5.9 inches and the other half will be around 6.1 inches. With Method 1, however, the sample means will all be very close to 6 inches, as indicated by the standard deviation in part (d).

Part (f):

Method 1 is more likely to produce a sample mean close to 6 inches. Because the sample mean is an unbiased estimator for both methods, the manager should pick the method that would result in less variability in the distribution of the sample mean. Based on the answer to part (e), Method 1 results in less variability in the distribution of the sample mean.

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Question 6 (continued)

Scoring

This question is scored in three sections. Section 1 consists of parts (a), (b), and (c), section 2 consists of part (d), and section 3 consists of parts (e) and (f). Sections 1, 2, and 3 are scored as essentially correct (E), partially correct (P), or incorrect (I).

Section 1 is scored as follows:

Essentially correct (E) if the response includes the following three components:

1. In part (a) the response says no *AND* either argues that the sample will only be selected from one production line and not the entire population (that is, the sample will only represent one production line) *OR* argues that the tortillas from the two productions lines are different.
2. In part (b) the response chooses Method 1 *AND* refers to a relevant characteristic of the histogram (shape, center, or variability) that matches what would be expected when using Method 1 (or that does not match what would be expected when using Method 2).
3. In part (c) the response chooses Method 2 *AND* either justifies by stating that the sample comes from only one production line (does not come from both production lines) *OR* justifies by comparing the possible range of diameters for the two methods.

Partially correct (P) if the response includes only two of the three components.

Incorrect (I) if the response includes at most one of the three components.

Note: If a response includes more than one justification in an individual part, score the weaker of the two justifications. For example, a response for part (a) that says "No, because only one line was selected and because the sample size was too small" does not satisfy the first component because the sample size argument is incorrect.

Section 2 is scored as follows:

Essentially correct (E) if the response includes the following three components:

1. The shape is approximately normal.
2. The mean is 6 inches.
3. The standard deviation is $\frac{0.11}{\sqrt{200}}$ inch.

Partially correct (P) if the response includes only two of the three components.

Incorrect (I) if the response includes at most one of the three components.

Note: It is not necessary to include units (inches) for the mean or standard deviation.

Section 3 is scored as follows:

Essentially correct (E) if the response includes the following two components:

1. In part (e) the response chooses Method 1 *AND* describes the sampling distribution of the sample mean for Method 2 as having some *sample means* close to the mean of production line A (5.9 inches) and the other *sample means* close to the mean of production line B (6.1 inches).

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Question 6 (continued)

2. In part (f) the response chooses Method 1 *AND*
   - refers to a correct answer in part (e);

   OR
   - describes the sampling distribution of the sample mean for Method 2 as having some *sample means* close to the mean of production line A (5.9, less than 6) and the other *sample means* close to the mean of production line B (6.1, greater than 6);

   OR
   - argues that on a single day, it would be preferable to get a sample with a mean around 6 rather than getting a sample that would have a mean around 5.9 (less than 6) or a mean around 6.1 (greater than 6).

Partially correct (P) if the response includes one of the two components.

Incorrect (I) if the response does not meet the criteria for E or P.

Notes:

- In part (e), the response must be clear that there is *more than one* mean being described (365 sample means).
  - Correct: The sample means will be around 5.9 or 6.1.
  - Not correct: The sample mean will be around 5.9 or 6.1.

- Parts (e) and (f) are not satisfied if the response does not imply the sample means *vary* from the population means in both methods (or does not imply that the sample mean varies from the population mean when making the single day argument in part (f)). However, if the mistake is made in part (e), do not penalize the response in part (f) for the same mistake.
  - Correct: The sample means will be around 5.9 or 6.1 instead of close to 6.
  - Not correct: The sample means will be 5.9 or 6.1 instead of close to 6.
  - Not correct: The sample means will be 6 instead of around 5.9 or 6.1.

4 Complete Response

All three sections essentially correct

3 Substantial Response

Two sections essentially correct and one section partially correct

2 Developing Response

Two sections essentially correct and one section incorrect

OR

One section essentially correct and either one or two sections partially correct

OR

All three sections partially correct

1 Minimal Response

One section essentially correct and two sections incorrect

OR

Two sections partially correct and one section incorrect