

Question 3: Focus on Probability and Sampling Distributions

4 points

General Scoring Notes

- Each part of the question (indicated by a letter) is initially scored by determining if it meets the criteria for essentially correct (E), partially correct (P), or incorrect (I). The response is then categorized based on the scores assigned to each letter part and awarded an integer score between 0 and 4 (see the table at the end of the question).
- The model solution represents an ideal response to each part of the question, and the scoring criteria identify the specific components of the model solution that are used to determine the score.

Model Solution	Scoring
(a) The random variable X is the dollar value of the cash prize in a bath fizzy. (i) The proportion of bath fuzzies containing \$1 is equal to the $P(X = \\$1)$ and $P(X = \$1) = 1 - (0.2 + 0.05 + 0.05 + 0.01 + 0.01) = 0.68.$ (ii) The proportion of bath fuzzies that contain at least \$10 is equal to the $P(X \geq \\$10)$ and $P(X \geq \$10) = 0.05 + 0.05 + 0.01 + 0.01 = 0.12.$	Essentially correct (E) if the response satisfies at least three of the following four components: - Correctly calculates the proportion of bath fuzzies containing \$1 - Provides correct supporting work for the value calculated in component 1 - Correctly calculates the proportion of bath fuzzies containing at least \$10 - Provides correct supporting work for the value calculated in component 3 Partially correct (P) if the response satisfies only two of the four components. Incorrect (I) if the response does not meet the criteria for E or P.

Additional Notes:

- A response that provides a correct percentage, instead of a proportion, may satisfy components 1 and 3, such as 68 percent or 12 percent.
- An arithmetic or transcription error in a response can be ignored if correct work is shown.

Model Solution	Scoring
(b) Given a bath fizzy contains at least \$10, then the probability that it contains \$100 is $P(X = \$100 \mid X \geq \$10) = \frac{0.01}{0.12} \approx 0.0833.$	Essentially correct (E) if the response satisfies the following two components: - Correctly calculates the requested probability - Shows work consistent with their response to part (a-ii) Partially correct (P) if the response satisfies only one of the two components. Incorrect (I) if the response does not meet the criteria for E or P.

Additional Notes:

- A specific conditional probability statement is not required, but if correctly given should be considered a positive in holistic scoring.
- An arithmetic or transcription error in a response can be ignored if correct work is shown.

Model Solution	Scoring
(c) The expected value of the distribution of X is $E(X) = 1(0.68) + 5(0.2) + 10(0.05) + 20(0.05) + 50(0.01) + 100(0.01) = \$4.68.$ The expected value is the mean of the cash prizes that result from the long run of many, many trials of randomly selecting bath fuzzies and determining the amount each contains.	Essentially correct (E) if the response satisfies both components 1 and 2 AND at least two of components 3–5: - States the correct expected value - Shows appropriate work to calculate the expected value - Interpretation includes the concept of repeating the selection process over a long period of time - Interpretation includes the concept of an average or mean - Interpretation includes the context of receiving a cash prize Partially correct (P) if the response does not meet the criteria for E but satisfies two or three of components 1–4. Incorrect (I) if the response does not meet the criteria for E or P.

Additional Notes:

- Supporting work for finding the expected value must include at least two of the terms in the equation to show the pattern, such as $1(0.68) + 5(0.2) + \dots$
- Calculator notation does not satisfy component 2, such as $1 - \text{VAR STATS}(L1, L2)$.
- Responses that satisfy only components 1 and 5 or only components 2 and 5 receive a score of I but should be considered a positive in holistic scoring.
- An arithmetic or transcription error in a response can be ignored if correct work is shown.

Model Solution	Scoring
(d) The expected value of the distribution of X in euros is $4.68(0.89) \approx 4.17$ euros.	Essentially correct (E) if the response correctly calculates the number of euros for the expected value showing work and including units. Partially correct (P) if the response correctly calculates the number of euros for the expected value but is missing either work or units. Incorrect (I) if the response does not meet the criteria for E or P.

Additional Notes:

- A response that does not have at least one decimal place in the final response (e.g., rounded to an integer) should be scored no more than P.
- The response can either use the expected value calculated in part (c) or first convert all the values in the probability distribution to euros to find the new expected value.
- An arithmetic or transcription error in a response can be ignored if correct work is shown.

Scoring for Question 3

Each essentially correct (E) part counts as 1 point, and each partially correct (P) part counts as $\frac{1}{2}$ point.

	Score
Complete Response	4
Substantial Response	3
Developing Response	2
Minimal Response	1

If a response is between two scores (for example, $2\frac{1}{2}$ points), use a holistic approach to decide whether to score up or down, depending on the strength of the response and quality of the communication.