

3. A certain sports team has its team song performed by a different musician at the beginning of each of its games. The time it takes for the team song to be performed by different musicians can be modeled using a normal distribution with mean $\mu = 109$ seconds and standard deviation $\sigma = 16$ seconds. All performances of the team song are independent.

Part A

What is the probability that a randomly selected performance of the team song will take longer than 120 seconds? Show your work.

Part B

Ten performances of the team song will be randomly selected. Let the random variable X represent the number of games at which the performance of the team song takes longer than 120 seconds. Find $P(X \geq 3)$. Show your work.

Part C

Ben will attend all of the games for this sports team. Let the random variable Y represent the number of games Ben will attend until a performance of the team song takes longer than 120 seconds.

- i. Calculate the mean of Y . Show your work.
- ii. Calculate the standard deviation of Y . Show your work.

Part D

Interpret the standard deviation calculated in part C (ii) in context.

4. In an online game, players move through a virtual world collecting geodes, a type of hollow rock. When broken open, these geodes contain crystals of different colors that are useful in the game. A red crystal is the most useful crystal in the game. The color of the crystal in each geode is independent and the probability that a geode contains a red crystal is 0.08.

(a) Sarah, a player, will collect and open geodes until a red crystal is found.

(i) Calculate the mean of the distribution of the number of geodes Sarah will open until a red crystal is found. Show your work.

(ii) Calculate the standard deviation of the distribution of the number of geodes Sarah will open until a red crystal is found. Show your work.

(b) Another player, Conrad, decides to play the game and will stop opening geodes after finding a red crystal or when 4 geodes have been opened, whichever comes first. Let Y = the number of geodes Conrad will open. The table shows the partially completed probability distribution for the random variable Y .

Number of geodes Conrad will open, y	1	2	3	4
Probability, $P(Y = y)$	0.08	0.0736		

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Continue your response to **QUESTION 4** on this page.

(i) Calculate $P(Y = 3)$. Show your work.

(ii) Calculate $P(Y = 4)$. Show your work.

(c) Consider the table and your results from part (b).

(i) Calculate the mean of the distribution of the number of geodes Conrad will open. Show your work.

(ii) Interpret the mean of the distribution of the number of geodes Conrad will open, which was calculated in part (c-i).

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4. A company manufactures model rockets that require igniters to launch. Once an igniter is used to launch a rocket, the igniter cannot be reused. Sometimes an igniter fails to operate correctly, and the rocket does not launch. The company estimates that the overall failure rate, defined as the percent of all igniters that fail to operate correctly, is 15 percent.

A company engineer develops a new igniter, called the super igniter, with the intent of lowering the failure rate. To test the performance of the super igniters, the engineer uses the following process.

Step 1: One super igniter is selected at random and used in a rocket.

Step 2: If the rocket launches, another super igniter is selected at random and used in a rocket.

Step 2 is repeated until the process stops. The process stops when a super igniter fails to operate correctly or 32 super igniters have successfully launched rockets, whichever comes first. Assume that super igniter failures are independent.

- (a) If the failure rate of the super igniters is 15 percent, what is the probability that the first 30 super igniters selected using the testing process successfully launch rockets?
- (b) Given that the first 30 super igniters successfully launch rockets, what is the probability that the first failure occurs on the thirty-first or the thirty-second super igniter tested if the failure rate of the super igniters is 15 percent?
- (c) Given that the first 30 super igniters successfully launch rockets, is it reasonable to believe that the failure rate of the super igniters is less than 15 percent? Explain.