

3. A certain sports team has its team song performed by a different musician at the beginning of each of its games. The time it takes for the team song to be performed by different musicians can be modeled using a normal distribution with mean  $\mu = 109$  seconds and standard deviation  $\sigma = 16$  seconds. All performances of the team song are independent.

**Part A**

What is the probability that a randomly selected performance of the team song will take longer than 120 seconds? Show your work.

**Part B**

Ten performances of the team song will be randomly selected. Let the random variable  $X$  represent the number of games at which the performance of the team song takes longer than 120 seconds. Find  $P(X \geq 3)$ . Show your work.

**Part C**

Ben will attend all of the games for this sports team. Let the random variable  $Y$  represent the number of games Ben will attend until a performance of the team song takes longer than 120 seconds.

- i. Calculate the mean of  $Y$ . Show your work.
- ii. Calculate the standard deviation of  $Y$ . Show your work.

**Part D**

Interpret the standard deviation calculated in part C (ii) in context.

**3.** Ms. Fey is a manager at a restaurant. To improve the dining experience for her customers, she uses a digital music service to create a playlist of songs that will be played in the restaurant. The playlist contains 1,000 songs and consists of four different types of music in the following quantities: 200 country songs, 400 pop songs, 100 rock songs, and 300 jazz songs. The digital music service will select songs at random from the playlist to be played in the restaurant. Any song can be replayed at any time.

**A.**

- i. Suppose one song is selected at random to be played. What is the probability that the song is a rock song? Show your work.
- ii. Suppose two songs are selected at random to be played. What is the probability that both songs are rock songs? Show your work.

**B.** In every one-hour period, 20 songs will be played at random and any song can be replayed at any time. Ms. Fey is interested in how many rock songs will be played in a typical one-hour period.

- i. Define the random variable of interest to Ms. Fey, and state how the random variable is distributed.
- ii. What is the expected value for the random variable in part B (i)? Show your work.

**C.** Recall that in every one-hour period, 20 songs will be played at random and any song can be replayed at any time.

- i. Determine the probability that 4 or more rock songs in a particular one-hour period will be played. Show your work.
- ii. Suppose 4 rock songs are played during a particular one-hour period. Does this provide strong evidence that the song selection process was not truly random? Justify your answer without performing an inference procedure.

A machine at a manufacturing company is programmed to fill shampoo bottles such that the amount of shampoo in each bottle is normally distributed with mean 0.60 liter and standard deviation 0.04 liter. Let the random variable  $A$  represent the amount of shampoo, in liters, that is inserted into a bottle by the filling machine.

- (a) A bottle is considered to be underfilled if it has less than 0.50 liter of shampoo. Determine the probability that a randomly selected bottle of shampoo will be underfilled. Show your work.

After the bottles are filled, they are placed in boxes of 10 bottles per box. After the bottles are placed in the boxes, several boxes are placed in a crate for shipping to a beauty supply warehouse. The manufacturing company's contract with the beauty supply warehouse states that one box will be randomly selected from a crate. If 2 or more bottles in the selected box are underfilled, the entire crate will be rejected and sent back to the manufacturing company.

- (b) The beauty supply warehouse manager is interested in the probability that a crate shipped to the warehouse will be rejected. Assume that the amounts of shampoo in the bottles are independent of each other.

- (i) Define the random variable of interest for the warehouse manager and state how the random variable is distributed.

- (ii) Determine the probability that a crate will be rejected by the warehouse manager. Show your work.

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Continue your response to **QUESTION 3** on this page.

To reduce the number of crates rejected by the beauty supply warehouse manager, the manufacturing company is considering adjusting the programming of the filling machine so that the amount of shampoo in each bottle is normally distributed with mean 0.56 liter and standard deviation 0.03 liter.

- (c) Would you recommend that the manufacturing company use the original programming of the filling machine or the adjusted programming of the filling machine? Provide a statistical justification for your choice.

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- (b) Consider two rules for identifying outliers, method A and method B. Let method A represent the  $1.5 \times \text{IQR}$  rule, and let method B represent the 2 standard deviations rule.
- (i) Using method A, determine any data points that are potential outliers in the distribution of length of stay. Justify your answer.
- (ii) The mean length of stay for the sample is 7.42 days with a standard deviation of 2.37 days. Using method B, determine any data points that are potential outliers in the distribution of length of stay. Justify your answer.
- (c) Explain why method A might identify more data points as potential outliers than method B for a distribution that is strongly skewed to the right.

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2. Researchers will conduct a year-long investigation of walking and cholesterol levels in adults. They will select a random sample of 100 adults from the target population to participate as subjects in the study.
- (a) One aspect of the study is to record the number of miles each subject walks per day. The researchers are deciding whether to have subjects wear an activity tracker to record the data or to have subjects keep a daily journal of the miles they walk each day. Describe what bias could be introduced by keeping the daily journal instead of wearing the activity tracker.

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During the course of the study, the subjects will have their cholesterol levels measured each month by a doctor. The researchers will perform a significance test at the end of the study to determine whether the average cholesterol level for subjects who walk fewer miles each day is greater than for those who walk more miles each day.

- (b) Selecting a random sample creates a reasonable representative sample of the target population. Explain the benefit of using a representative sample from the population.

- (c) Suppose the researchers conduct the test and find a statistically significant result. Would it be valid to claim that increased walking causes a decrease in average cholesterol levels for adults in the target population? Explain your reasoning.

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3. To increase morale among employees, a company began a program in which one employee is randomly selected each week to receive a gift card. Each of the company's 200 employees is equally likely to be selected each week, and the same employee could be selected more than once. Each week's selection is independent from every other week.
- (a) Consider the probability that a particular employee receives at least one gift card in a 52-week year.
- (i) Define the random variable of interest and state how the random variable is distributed.
- (ii) Determine the probability that a particular employee receives at least one gift card in a 52-week year. Show your work.

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(b) Calculate and interpret the expected value for the number of gift cards a particular employee will receive in a 52-week year. Show your work.

(c) Suppose that Agatha, an employee at the company, never receives a gift card for an entire 52-week year. Based on her experience, does Agatha have a strong argument that the selection process was not truly random? Explain your answer.

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3. A medical researcher surveyed a large group of men and women about whether they take medicine as prescribed. The responses were categorized as never, sometimes, or always. The relative frequency of each category is shown in the table.

|       | Never  | Sometimes | Always | Total  |
|-------|--------|-----------|--------|--------|
| Men   | 0.0564 | 0.2016    | 0.2120 | 0.4700 |
| Women | 0.0636 | 0.1384    | 0.3280 | 0.5300 |
| Total | 0.1200 | 0.3400    | 0.5400 | 1.0000 |

- (a) One person from those surveyed will be selected at random.
- What is the probability that the person selected will be someone whose response is never and who is a woman?
  - What is the probability that the person selected will be someone whose response is never or who is a woman?
  - What is the probability that the person selected will be someone whose response is never given that the person is a woman?
- (b) For the people surveyed, are the events of being a person whose response is never and being a woman independent? Justify your answer.
- (c) Assume that, in a large population, the probability that a person will always take medicine as prescribed is 0.54. If 5 people are selected at random from the population, what is the probability that at least 4 of the people selected will always take medicine as prescribed? Support your answer.

2. An environmental science teacher at a high school with a large population of students wanted to estimate the proportion of students at the school who regularly recycle plastic bottles. The teacher selected a random sample of students at the school to survey. Each selected student went into the teacher's office, one at a time, and was asked to respond yes or no to the following question.

Do you regularly recycle plastic bottles?

Based on the responses, a 95 percent confidence interval for the proportion of all students at the school who would respond yes to the question was calculated as  $(0.584, 0.816)$ .

- (a) How many students were in the sample selected by the environmental science teacher?
- (b) Given the method used by the environmental science teacher to collect the responses, explain how bias might have been introduced and describe how the bias might affect the point estimate of the proportion of all students at the school who would respond yes to the question.
- (c) The statistics teacher at the high school was concerned about the potential bias in the survey. To obtain a potentially less biased estimate of the proportion, the statistics teacher used an alternate method for collecting student responses. A random sample of 300 students was selected, and each student was given the following instructions on how to respond to the question.
- In private, flip a fair coin.
  - If heads, you must respond no, regardless of whether you regularly recycle.
  - If tails, please truthfully respond yes or no.
- (i) What is the expected number of students from the sample of 300 who would be required to respond no because the coin flip resulted in heads?
- (ii) The results of the sample showed that 213 of the 300 selected students responded no. Based on the results of the sample, give a point estimate for the proportion of all students at the high school who would respond yes to the question.
3. Approximately 3.5 percent of all children born in a certain region are from multiple births (that is, twins, triplets, etc.). Of the children born in the region who are from multiple births, 22 percent are left-handed. Of the children born in the region who are from single births, 11 percent are left-handed.
- (a) What is the probability that a randomly selected child born in the region is left-handed?
- (b) What is the probability that a randomly selected child born in the region is a child from a multiple birth, given that the child selected is left-handed?
- (c) A random sample of 20 children born in the region will be selected. What is the probability that the sample will have at least 3 children who are left-handed?