

2. Holly, a **botanist**, read an online report that stated that adding coffee grounds to the soil of rosebushes will produce more roses. Holly decides to conduct an experiment to investigate this claim. She grows 30 rosebushes in a greenhouse with a controlled environment so the conditions will be the same for all rosebushes.

When the rosebushes are a month old, she randomly assigns 15 rosebushes to have one-half cup of coffee grounds added to their soil weekly. The other 15 rosebushes will not have coffee grounds added to their soil.

After three months, she counts the number of roses on each rosebush.

botanist: a scientist who studies the biology of plants

Part A

Based on the information provided about Holly’s experiment, identify each of the following.

- i. Treatments
- ii. Experimental units
- iii. Response variable

Part B

In the context of Holly’s experiment, describe how the treatments can be randomly assigned to the experimental units so that each treatment has the same number of units.

Part C

After the experiment, Holly determines that her results were statistically significant at the $\alpha = 0.05$ level of significance. Explain the meaning of “statistically significant” in this context.

Descriptive Statistics

$$\bar{x} = \frac{1}{n} \sum x_i = \frac{\sum x_i}{n}$$

$$\hat{y} = a + bx$$

$$r = \frac{1}{n-1} \sum \left(\frac{x_i - \bar{x}}{s_x} \right) \left(\frac{y_i - \bar{y}}{s_y} \right)$$

$$s_x = \sqrt{\frac{1}{n-1} \sum (x_i - \bar{x})^2} = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n-1}}$$

$$\bar{y} = a + b\bar{x}$$

$$b = r \frac{s_y}{s_x}$$

Probability and Distributions

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Probability Distribution	Mean	Standard Deviation
Discrete random variable, X	$\mu_X = E(X) = \sum x_i P(x_i)$	$\sigma_X = \sqrt{\sum (x_i - \mu_X)^2 P(x_i)}$
If X has a binomial distribution with parameters n and p , then: $P(X = x) = \binom{n}{x} p^x (1 - p)^{n-x}$ where $x = 0, 1, 2, 3, \dots, n$	$\mu_X = np$	$\sigma_X = \sqrt{np(1 - p)}$
If X has a geometric distribution with parameter p , then: $P(X = x) = (1 - p)^{x-1} p$ where $x = 1, 2, 3, \dots$	$\mu_X = \frac{1}{p}$	$\sigma_X = \frac{\sqrt{1 - p}}{p}$

Descriptive Statistics

$$\bar{x} = \frac{1}{n} \sum x_i = \frac{\sum x_i}{n}$$

$$\hat{y} = a + bx$$

$$r = \frac{1}{n-1} \sum \left(\frac{x_i - \bar{x}}{s_x} \right) \left(\frac{y_i - \bar{y}}{s_y} \right)$$

$$s_x = \sqrt{\frac{1}{n-1} \sum (x_i - \bar{x})^2} = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n-1}}$$

$$\bar{y} = a + b\bar{x}$$

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If X has a binomial distribution with parameters n and p , then: $P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$ where $x = 0, 1, 2, 3, \dots, n$	$\mu_X = np$	$\sigma_X = \sqrt{np(1-p)}$
If X has a geometric distribution with parameter p , then: $P(X = x) = (1-p)^{x-1} p$ where $x = 1, 2, 3, \dots$	$\mu_X = \frac{1}{p}$	$\sigma_X = \frac{\sqrt{1-p}}{p}$

Sampling Distributions and Inferential Statistics

Standardized test statistic: $\frac{\text{statistic} - \text{parameter}}{\text{standard error of the statistic}}$
Confidence interval: $\text{statistic} \pm (\text{critical value})(\text{standard error of statistic})$

Chi-square statistic: $\chi^2 = \sum \frac{(\text{observed} - \text{expected})^2}{\text{expected}}$

Sampling distributions for proportions:

Random Variable	Parameters of Sampling Distribution		Standard Error* of Sample Statistic
For one population: $\hat{p}$	$\mu_{\hat{p}} = p$	$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$	$s_{\hat{p}} = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$
For two populations: $\hat{p}_1 - \hat{p}_2$	$\mu_{\hat{p}_1 - \hat{p}_2} = p_1 - p_2$	$\sigma_{\hat{p}_1 - \hat{p}_2} = \sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}}$	$s_{\hat{p}_1 - \hat{p}_2} = \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$ When $p_1 = p_2$ is assumed: $s_{\hat{p}_1 - \hat{p}_2} = \sqrt{\hat{p}_c(1-\hat{p}_c) \left(\frac{1}{n_1} + \frac{1}{n_2} \right)}$ where $\hat{p}_c = \frac{X_1 + X_2}{n_1 + n_2}$

Sampling distributions for means:

Random Variable	Parameters of Sampling Distribution		Standard Error* of Sample Statistic
For one population: $\bar{X}$	$\mu_{\bar{X}} = \mu$	$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}}$	$s_{\bar{X}} = \frac{s}{\sqrt{n}}$
For two populations: $\bar{X}_1 - \bar{X}_2$	$\mu_{\bar{X}_1 - \bar{X}_2} = \mu_1 - \mu_2$	$\sigma_{\bar{X}_1 - \bar{X}_2} = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$	$s_{\bar{X}_1 - \bar{X}_2} = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$

Sampling distributions for simple linear regression:

Random Variable	Parameters of Sampling Distribution		Standard Error* of Sample Statistic
For slope: b	$\mu_b = \beta$	$\sigma_b = \frac{\sigma}{s_x \sqrt{n}}$, where $\sigma_x = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n}}$	$s_b = \frac{s}{s_x \sqrt{n-1}}$, where $s = \sqrt{\frac{\sum (y_i - \hat{y}_i)^2}{n-2}}$ and $s_x = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n-1}}$

Standard deviation is a measurement of variability from the theoretical population. Standard error is the estimate of the standard deviation. If the standard deviation of the statistic is assumed to be known, then the standard deviation should be used instead of the standard error.

2. Researchers are investigating the effectiveness of using a fungus to control the spread of an insect that destroys trees. The researchers will create four different concentrations of fungus mixtures: 0 milliliters per liter (ml/L), 1.25 ml/L, 2.5 ml/L, and 3.75 ml/L. An equal number of the insects will be placed into 20 individual containers. The group of insects in each container will be sprayed with one of the four mixtures, and the researchers will record the number of insects that are still alive in each container one week after spraying.
- (a) Identify the treatments, experimental units, and response variable of the experiment.
- Treatments:
- Experimental units:
- Response variable:
- (b) Does the experiment have a control group? Explain your answer.
- (c) Describe how the treatments can be randomly assigned to the experimental units so that each treatment has the same number of units.

STATISTICS
SECTION II
Part B
Question 6

Spend about 25 minutes on this part of the exam.
Percent of Section II score—25

Directions: Show all your work. Indicate clearly the methods you use, because you will be scored on the correctness of your methods as well as on the accuracy and completeness of your results and explanations.

6. Consider an experiment in which two men and two women will be randomly assigned to either a treatment group or a control group in such a way that each group has two people. The people are identified as Man 1, Man 2, Woman 1, and Woman 2. The six possible arrangements are shown below.

Arrangement A		Arrangement B		Arrangement C	
Treatment	Control	Treatment	Control	Treatment	Control
Man 1	Woman 1	Man 1	Man 2	Man 1	Man 2
Man 2	Woman 2	Woman 1	Woman 2	Woman 2	Woman 1

Arrangement D		Arrangement E		Arrangement F	
Treatment	Control	Treatment	Control	Treatment	Control
Woman 1	Man 1	Man 2	Man 1	Man 2	Man 1
Woman 2	Man 2	Woman 2	Woman 1	Woman 1	Woman 2

Two possible methods of assignment are being considered: the sequential coin flip method, as described in part (a), and the chip method, as described in part (b). For each method, the order of the assignment will be Man 1, Man 2, Woman 1, Woman 2.

- (a) For the sequential coin flip method, a fair coin is flipped until one group has two people. An outcome of tails assigns the person to the treatment group, and an outcome of heads assigns the person to the control group. As soon as one group has two people, the remaining people are automatically assigned to the other group.
- (i) Complete the table below by calculating the probability of each arrangement occurring if the sequential coin flip method is used.

Arrangement	A	B	C	D	E	F
Probability						

- (ii) For the sequential coin flip method, what is the probability that Man 1 and Man 2 are assigned to the same group?

The six arrangements are repeated below.

Arrangement A		Arrangement B		Arrangement C	
Treatment	Control	Treatment	Control	Treatment	Control
Man 1	Woman 1	Man 1	Man 2	Man 1	Man 2
Man 2	Woman 2	Woman 1	Woman 2	Woman 2	Woman 1

Arrangement D		Arrangement E		Arrangement F	
Treatment	Control	Treatment	Control	Treatment	Control
Woman 1	Man 1	Man 2	Man 1	Man 2	Man 1
Woman 2	Man 2	Woman 2	Woman 1	Woman 1	Woman 2

- (b) For the chip method, two chips are marked “treatment” and two chips are marked “control.” Each person selects one chip at random without replacement.
- (i) Complete the table below by calculating the probability of each arrangement occurring if the chip method is used.

Arrangement	A	B	C	D	E	F
Probability						

- (ii) For the chip method, what is the probability that Man 1 and Man 2 are assigned to the same group?
- (c) Sixteen participants consisting of 10 students and 6 teachers at an elementary school will be used for an experiment to determine lunch preference for the school population of students and teachers. As the participants enter the school cafeteria for lunch, they will be randomly assigned to receive one of two lunches so that 8 will receive a salad, and 8 will receive a grilled cheese sandwich. The students will enter the cafeteria first, and the teachers will enter next. Which method, the sequential coin flip method or the chip method, should be used to assign the treatments? Justify your choice.

STOP
END OF EXAM

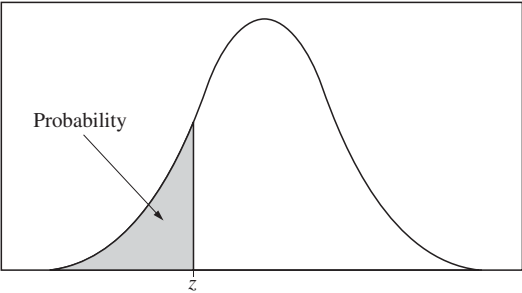


Table entry for z is the probability lying below z .

Table A Standard normal probabilities

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

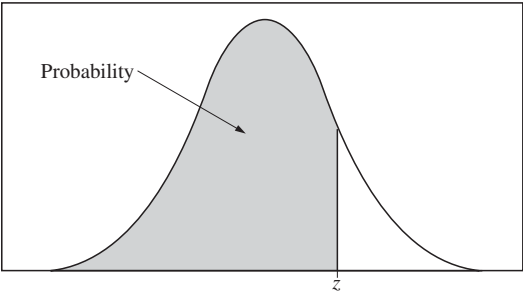


Table entry for z is the probability lying below z .

Table A (Continued)

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9997	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

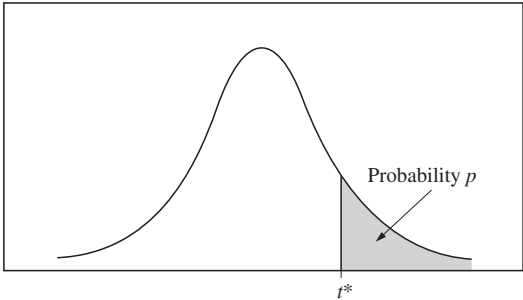


Table entry for p and C is the point t^* with probability p lying above it and probability C lying between $-t^*$ and t^* .

Table B t distribution critical values

df	Tail probability p									
	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025
1	1.000	1.376	1.963	3.078	6.314	12.71	15.89	31.82	63.66	127.3
2	.816	1.061	1.386	1.886	2.920	4.303	4.849	6.965	9.925	14.09
3	.765	.978	1.250	1.638	2.353	3.182	3.482	4.541	5.841	7.453
4	.741	.941	1.190	1.533	2.132	2.776	2.999	3.747	4.604	5.598
5	.727	.920	1.156	1.476	2.015	2.571	2.757	3.365	4.032	4.773
6	.718	.906	1.134	1.440	1.943	2.447	2.612	3.143	3.707	4.317
7	.711	.896	1.119	1.415	1.895	2.365	2.517	2.998	3.499	4.029
8	.706	.889	1.108	1.397	1.860	2.306	2.449	2.896	3.355	3.833
9	.703	.883	1.100	1.383	1.833	2.262	2.398	2.821	3.250	3.690
10	.700	.879	1.093	1.372	1.812	2.228	2.359	2.764	3.169	3.581
11	.697	.876	1.088	1.363	1.796	2.201	2.328	2.718	3.106	3.497
12	.695	.873	1.083	1.356	1.782	2.179	2.303	2.681	3.055	3.428
13	.694	.870	1.079	1.350	1.771	2.160	2.282	2.650	3.012	3.372
14	.692	.868	1.076	1.345	1.761	2.145	2.264	2.624	2.977	3.326
15	.691	.866	1.074	1.341	1.753	2.131	2.249	2.602	2.947	3.286
16	.690	.865	1.071	1.337	1.746	2.120	2.235	2.583	2.921	3.252
17	.689	.863	1.069	1.333	1.740	2.110	2.224	2.567	2.898	3.222
18	.688	.862	1.067	1.330	1.734	2.101	2.214	2.552	2.878	3.197
19	.688	.861	1.066	1.328	1.729	2.093	2.205	2.539	2.861	3.174
20	.687	.860	1.064	1.325	1.725	2.086	2.197	2.528	2.845	3.153
21	.686	.859	1.063	1.323	1.721	2.080	2.189	2.518	2.831	3.135
22	.686	.858	1.061	1.321	1.717	2.074	2.183	2.508	2.819	3.119
23	.685	.858	1.060	1.319	1.714	2.069	2.177	2.500	2.807	3.104
24	.685	.857	1.059	1.318	1.711	2.064	2.172	2.492	2.797	3.091
25	.684	.856	1.058	1.316	1.708	2.060	2.167	2.485	2.787	3.078
26	.684	.856	1.058	1.315	1.706	2.056	2.162	2.479	2.779	3.067
27	.684	.855	1.057	1.314	1.703	2.052	2.158	2.473	2.771	3.057
28	.683	.855	1.056	1.313	1.701	2.048	2.154	2.467	2.763	3.047
29	.683	.854	1.055	1.311	1.699	2.045	2.150	2.462	2.756	3.038
30	.683	.854	1.055	1.310	1.697	2.042	2.147	2.457	2.750	3.030
40	.681	.851	1.050	1.303	1.684	2.021	2.123	2.423	2.704	2.971
50	.679	.849	1.047	1.299	1.676	2.009	2.109	2.403	2.678	2.937
60	.679	.848	1.045	1.296	1.671	2.000	2.099	2.390	2.660	2.915
80	.678	.846	1.043	1.292	1.664	1.990	2.088	2.374	2.639	2.887
100	.677	.845	1.042	1.290	1.660	1.984	2.081	2.364	2.626	2.871
1000	.675	.842	1.037	1.282	1.646	1.962	2.056	2.330	2.581	2.813
∞	.674	.841	1.036	1.282	1.645	1.960	2.054	2.326	2.576	2.807

50% 60% 70% 80% 90% 95% 96% 98% 99% 99.5% 99.8% 99.9%

Confidence level C

Table entry for p is the point (χ^2) with probability p lying above it.

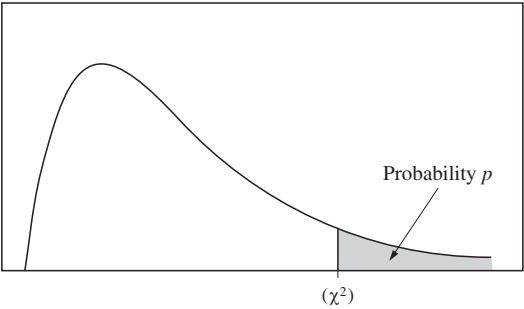


Table C χ^2 critical values

df	Tail probability p											
	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.32	1.64	2.07	2.71	3.84	5.02	5.41	6.63	7.88	9.14	10.83	12.12
2	2.77	3.22	3.79	4.61	5.99	7.38	7.82	9.21	10.60	11.98	13.82	15.20
3	4.11	4.64	5.32	6.25	7.81	9.35	9.84	11.34	12.84	14.32	16.27	17.73
4	5.39	5.99	6.74	7.78	9.49	11.14	11.67	13.28	14.86	16.42	18.47	20.00
5	6.63	7.29	8.12	9.24	11.07	12.83	13.39	15.09	16.75	18.39	20.51	22.11
6	7.84	8.56	9.45	10.64	12.59	14.45	15.03	16.81	18.55	20.25	22.46	24.10
7	9.04	9.80	10.75	12.02	14.07	16.01	16.62	18.48	20.28	22.04	24.32	26.02
8	10.22	11.03	12.03	13.36	15.51	17.53	18.17	20.09	21.95	23.77	26.12	27.87
9	11.39	12.24	13.29	14.68	16.92	19.02	19.68	21.67	23.59	25.46	27.88	29.67
10	12.55	13.44	14.53	15.99	18.31	20.48	21.16	23.21	25.19	27.11	29.59	31.42
11	13.70	14.63	15.77	17.28	19.68	21.92	22.62	24.72	26.76	28.73	31.26	33.14
12	14.85	15.81	16.99	18.55	21.03	23.34	24.05	26.22	28.30	30.32	32.91	34.82
13	15.98	16.98	18.20	19.81	22.36	24.74	25.47	27.69	29.82	31.88	34.53	36.48
14	17.12	18.15	19.41	21.06	23.68	26.12	26.87	29.14	31.32	33.43	36.12	38.11
15	18.25	19.31	20.60	22.31	25.00	27.49	28.26	30.58	32.80	34.95	37.70	39.72
16	19.37	20.47	21.79	23.54	26.30	28.85	29.63	32.00	34.27	36.46	39.25	41.31
17	20.49	21.61	22.98	24.77	27.59	30.19	31.00	33.41	35.72	37.95	40.79	42.88
18	21.60	22.76	24.16	25.99	28.87	31.53	32.35	34.81	37.16	39.42	42.31	44.43
19	22.72	23.90	25.33	27.20	30.14	32.85	33.69	36.19	38.58	40.88	43.82	45.97
20	23.83	25.04	26.50	28.41	31.41	34.17	35.02	37.57	40.00	42.34	45.31	47.50
21	24.93	26.17	27.66	29.62	32.67	35.48	36.34	38.93	41.40	43.78	46.80	49.01
22	26.04	27.30	28.82	30.81	33.92	36.78	37.66	40.29	42.80	45.20	48.27	50.51
23	27.14	28.43	29.98	32.01	35.17	38.08	38.97	41.64	44.18	46.62	49.73	52.00
24	28.24	29.55	31.13	33.20	36.42	39.36	40.27	42.98	45.56	48.03	51.18	53.48
25	29.34	30.68	32.28	34.38	37.65	40.65	41.57	44.31	46.93	49.44	52.62	54.95
26	30.43	31.79	33.43	35.56	38.89	41.92	42.86	45.64	48.29	50.83	54.05	56.41
27	31.53	32.91	34.57	36.74	40.11	43.19	44.14	46.96	49.64	52.22	55.48	57.86
28	32.62	34.03	35.71	37.92	41.34	44.46	45.42	48.28	50.99	53.59	56.89	59.30
29	33.71	35.14	36.85	39.09	42.56	45.72	46.69	49.59	52.34	54.97	58.30	60.73
30	34.80	36.25	37.99	40.26	43.77	46.98	47.96	50.89	53.67	56.33	59.70	62.16
40	45.62	47.27	49.24	51.81	55.76	59.34	60.44	63.69	66.77	69.70	73.40	76.09
50	56.33	58.16	60.35	63.17	67.50	71.42	72.61	76.15	79.49	82.66	86.66	89.56
60	66.98	68.97	71.34	74.40	79.08	83.30	84.58	88.38	91.95	95.34	99.61	102.7
80	88.13	90.41	93.11	96.58	101.9	106.6	108.1	112.3	116.3	120.1	124.8	128.3
100	109.1	111.7	114.7	118.5	124.3	129.6	131.1	135.8	140.2	144.3	149.4	153.2