

## Question 5: Multi-Focus

4 points

## General Scoring Notes

- Each part of the question (indicated by a letter) is initially scored by determining if it meets the criteria for essentially correct (E), partially correct (P), or incorrect (I). The response is then categorized based on the scores assigned to each letter part and awarded an integer score between 0 and 4 (see the table at the end of the question).
- The model solution represents an ideal response to each section of the question, and the scoring criteria identify the specific components of the model solution that are used to determine the score.

| Model Solution                                                                                                                                                                                                                          | Scoring                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            |
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| (a) The scatterplot reveals a strong, positive, roughly linear association between the chest circumference and weight of tule elk. There are no points that seriously deviate from the straight-line pattern of the points in the plot. | <p><b>Essentially correct (E)</b> if the response provides a description that includes at least three of components 1-4 and component 5:</p> <ol style="list-style-type: none"> <li>Direction of association (positive or increasing)</li> <li>Strength of association (strong)</li> <li>Form of association (linear or approximately linear)</li> <li>Unusual features (no points with large discrepancies from the straight-line pattern exhibited by most of the points on the plot)</li> <li>Context (association between chest circumference and weight of tule elk)</li> </ol> <p><b>Partially correct (P)</b> if the response satisfies only one or two components out of components 1-4 and component 5<br/> OR<br/> if the response satisfies at least two out of components 1-4 but does not satisfy component 5.</p> <p><b>Incorrect (I)</b> if the response does not meet the criteria for E or P.</p> |

## Additional Notes:

- To satisfy component 4 it is sufficient to simply indicate that there are no unusual features or no outliers.
- To satisfy component 5 it is minimally sufficient for the response to refer to the association or relationship between chest circumference or measurement and weight without explicitly mentioning tule elk.
- The quality of communication in part (a) should be considered if holistic scoring is required.

| Model Solution                                                                                                                                                                                                                                                                                                                                         | Scoring                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |
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| <p>(b) (i) The predicted weight of a male tule elk with a chest circumference of 145.9 cm is <math>-350.3 + 3.7455(145.9) \approx 196.17</math> kg.</p> <p>(ii) The residual for a male tule elk with a chest circumference of 145.9 cm with an actual weight of 204.3 kg is <math>204.3 - 196.17 \approx 8.13</math> kg.</p>                          | <p><b>Essentially correct (E)</b> if the response satisfies the following two components:</p> <ol style="list-style-type: none"> <li>Provides the correct value of 196.17 kg in part (b-i) with work shown</li> <li>Provides the correct value of 8.13 kg in part (b-ii), or a value consistent with the predicted weight calculated for component 1, with work shown</li> </ol> <p><b>Partially correct (P)</b> if the response satisfies only one of the two components required for an E<br/> OR<br/> the response gives both correct values with no work shown.</p> <p><b>Incorrect (I)</b> if the response does not meet the criteria for E or P.</p> |
| <p><b>Additional Notes:</b></p> <ul style="list-style-type: none"> <li>An arithmetic or transcription error in a response can be ignored if correct work is shown.</li> <li>A response that includes an undefined or incorrect variable to identify work shown (e.g., <math>x</math> or <math>\hat{p}</math>) does not satisfy component 1.</li> </ul> |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            |

| Model Solution                                                                                                                                                                                                                      | Scoring                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                |
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| <p>(c) The value of the slope of the least-squares regression line is 3.7455. This value indicates that the predicted weight of a tule elk increases by 3.7455 kilograms for each additional centimeter of chest circumference.</p> | <p><b>Essentially correct (E)</b> if the response satisfies the following three components:</p> <ol style="list-style-type: none"> <li>1. Identifies the value of the slope as 3.7455</li> <li>2. Provides an interpretation that references an increase of a number of kilograms of weight for each one-centimeter increase in chest circumference</li> <li>3. Indicates that the slope represents a change in a prediction using non-deterministic language such as “predicted,” “estimated,” “expected,” or “average”</li> </ol> <p><b>Partially correct (P)</b> if the response satisfies only two of the three components.</p> <p><b>Incorrect (I)</b> if the response does not meet the criteria for E or P.</p> |

**Additional Notes**

- The value of the slope, 3.7455, may be rounded to 3.746, 3.75, or 3.7, but not to 3 or 4, to satisfy the numerical requirement in component 1.
- A response that only contains 3.7455 in the interpretation satisfies component 1.
- A calculation of slope may satisfy component 1 provided that two points from the line are used in the calculation.
- Units of measurements must be correctly specified for both weight and length to satisfy component 2.
- It is not required to refer specifically to the “least-squares regression line.”

| Model Solution                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        | Scoring                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            |
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| <p>(d) (i) The degrees of freedom for the test of slope are <math>n - 2 = 30 - 2 = 28</math>. The <math>t</math>-table shows that for 28 degrees of freedom, the <math>p</math>-value for a one-sided test would be 0.001. Because this is a two-sided test, the <math>p</math>-value is <math>(2)(0.001) = 0.002</math>.</p> <p>(ii) Because the <math>p</math>-value = 0.002 is less than <math>\alpha = 0.05</math>, reject the null hypothesis. There is sufficient statistical evidence that the population slope for the linear regression of weight vs. chest circumference for male tule elk is different from 4.5 kg/cm.</p> | <p><b>Essentially correct (E)</b> if the response satisfies the following three components:</p> <ol style="list-style-type: none"> <li>1. Gives a correct <math>p</math>-value of 0.002, consistent with the two-sided alternative hypothesis</li> <li>2. Provides correct comparison of the <math>p</math>-value to alpha (<math>p</math>-value is less than/greater than alpha) AND provides a correct decision about the null and/or alternative hypothesis</li> <li>3. Provides a conclusion statement in context, consistent with, and in terms of, the alternative hypothesis using non-deterministic language</li> </ol> <p><b>Partially correct (P)</b> if the response satisfies only two of the three components.</p> <p><b>Incorrect (I)</b> if the response does not meet the criteria for E or P.</p> |

**Additional Notes**

- The response need not make an explicit decision about the null hypothesis (reject  $H_0$  or fail to reject  $H_0$ ).
- If the conclusion and justification are consistent with an incorrect  $p$ -value (or an incorrect value of the test statistic, or an incorrect confidence interval, or an incorrect critical value), the response may satisfy component 2.
- To satisfy the  $p$ -value comparison in component 2, the response can compare the value of the test statistic to the appropriate critical value (e.g., 2.048 or -2.048).
- If an explicit decision is stated and the conclusion statement is inconsistent with the decision, component 2 is not satisfied.
- The decision part of component 2 may be satisfied by implying the decision within the conclusion statement (sufficient evidence/insufficient evidence for the alternative hypothesis).
- If the response includes a statement that is equivalent to accepting the null hypothesis (e.g., “we conclude that the mean population slope is 4.5”), then component 2 is not satisfied.
- A student who doesn’t give a  $p$ -value, but gives a correct conclusion based on a  $p$ -value  $< \alpha$  with a consistent conclusion OR a  $p$ -value  $> \alpha$  with a consistent conclusion may be scored P, e.g.,
  - If  $p$  were less than alpha, then reject  $H_0$  and say there is sufficient evidence that the population slope for the linear regression of weight vs. chest circumference for male tule elk is different than 4.5 kg/cm.
  - If  $p$  were greater than alpha, then fail to reject  $H_0$  and say there is not sufficient evidence that the population slope for the linear regression of weight vs. chest circumference for male tule elk is different than 4.5 kg/cm.
- If the response includes an incorrect interpretation of the  $p$ -value (e.g., “this is the probability that the null hypothesis is true”), then component 2 is not satisfied.
- The quality of communication in part (d) should be considered if holistic scoring is required.

**Scoring for Question 5**

Each essentially correct (E) part counts as 1 point, and each partially correct (P) part counts as  $\frac{1}{2}$  point.

|                                                                                                                                                                                                                       | Score    |
|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------|
| <b>Complete Response</b>                                                                                                                                                                                              | <b>4</b> |
| <b>Substantial Response</b>                                                                                                                                                                                           | <b>3</b> |
| <b>Developing Response</b>                                                                                                                                                                                            | <b>2</b> |
| <b>Minimal Response</b>                                                                                                                                                                                               | <b>1</b> |
| If a response is between two scores (for example, 2 $\frac{1}{2}$ points), use a holistic approach to decide whether to score up or down, depending on the strength of the response and quality of the communication. |          |

**Question 1: Focus on Exploring Data****4 points****General Scoring Notes**

- Each part of the question (indicated by a letter) is initially scored by determining if it meets the criteria for essentially correct (E), partially correct (P), or incorrect (I). The response is then categorized based on the scores assigned to each letter part and awarded an integer score between 0 and 4 (see the table at the end of the question).
- The model solution represents an ideal response to each part of the question, and the scoring criteria identify the specific components of the model solution that are used to determine the score.

|     | Model Solution                                                                                                                                                                                                        | Scoring                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  |
|-----|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| (a) | The scatterplot reveals a strong, positive, roughly linear association between the mass and length of bullfrogs. There are no points that seriously deviate from the straight-line pattern of the points in the plot. | <p><b>Essentially correct (E)</b> if the response provides a description that includes at least three of components 1-4 and component 5:</p> <ol style="list-style-type: none"> <li>Direction of association (positive or increasing)</li> <li>Strength of association (strong)</li> <li>Form of association (linear or approximately linear)</li> <li>Unusual features (no points with large discrepancies from the pattern (straight line) exhibited by most of the points on the plot)</li> <li>Context (association between length and mass of bullfrogs)</li> </ol> <p><b>Partially correct (P)</b> if the response satisfies only one or two components out of components 1-4 and component 5<br/> OR<br/> if the response satisfies at least three out of components 1-4 but does not satisfy component 5.</p> <p><b>Incorrect (I)</b> if the response does not meet the criteria for E or P.</p> |

**Additional Notes:**

- To satisfy component 4, it is sufficient to simply indicate that there are no unusual features.
- To satisfy component 5, it is minimally sufficient for the response to refer to the association or relationship between mass and length without explicitly mentioning bullfrogs.
- The strength of the response in part (a) may be considered if holistic scoring is needed.

| Model Solution                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     | Scoring                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            |
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| <b>(b)</b> The value of the slope of the least-squares regression line is 6.086. This value indicates that the predicted mass of a bullfrog increases by 6.086 grams for each additional millimeter of length.                                                                                                                                                                                                                                                                                                                                                                                                                                     | <p><b>Essentially correct (E)</b> if the response satisfies the following three components:</p> <ol style="list-style-type: none"> <li>1. Identifies the value of the slope as 6.086</li> <li>2. Provides an interpretation that references an increase of a number of grams of mass for each one-millimeter increase in length</li> <li>3. Indicates that the slope represents a change in a prediction using non-deterministic language such as “predicted,” “estimated,” “expected,” or “average”</li> </ol> <p><b>Partially correct (P)</b> if the response satisfies only two of the three components.</p> <p><b>Incorrect (I)</b> if the response does not meet the criteria for E or P.</p> |
| <b>Additional Notes:</b> <ul style="list-style-type: none"> <li>• The value of the slope, 6.086, may be rounded to 6.09 or 6.1, but not to 6, to satisfy the numerical requirement in component 1.</li> <li>• A response that only contains 6.086 in the interpretation satisfies component 1.</li> <li>• A calculation of slope may satisfy component 1, provided that two points from the line are used in the calculation.</li> <li>• Units of measurements must be correctly specified for both mass and length to satisfy component 2.</li> <li>• It is not required to refer specifically to the “least-squares regression line.”</li> </ul> |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |

| Model Solution                                                                                                                                                                                                                  | Scoring                                                                                                                                                                                                                                                                                                                                                                                                                          |
|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <b>(c)</b> The coefficient of determination is $r^2 \approx 0.819$ . This value indicates that 81.9% of the variation in bullfrog mass can be explained by variation in bullfrog length as described by the least-squares line. | <p><b>Essentially correct (E)</b> if the response provides a correct interpretation of <math>r^2</math> in context.</p> <p><b>Partially correct (P)</b> if the response provides a generic interpretation (no context)<br/> OR<br/> if the response provides a reasonable but incorrect interpretation of <math>r^2</math> in context.</p> <p><b>Incorrect (I)</b> if the response does not satisfy the criteria for E or P.</p> |

**Additional Notes:**

- Correct interpretations of  $r^2$  include the concept that part of the variation in the response (dependent or  $y$ ) variable is explained by the linear relationship with the explanatory (independent or  $x$ ) variable. The response can take any of several equivalent forms, such as:
  - The proportion of the total variability in the dependent (response) variable  $y$  that is explained by the independent (explanatory) variable  $x$ .
  - The proportion of variation in  $y$  that is accounted for by the linear model.
  - The proportionate reduction of the total variation of the  $y$ -values that is associated with the use of the independent variable  $x$ .
  - The proportionate reduction in the sum of the squares of vertical deviations obtained by using the least-squares line instead of the sample mean to predict values of  $y$ .
- Correct interpretation of  $r^2$  must explicitly relate to the dependent variable. Mention of the data, predicted values, or no mention of the dependent variable are incorrect interpretations. Common incorrect interpretations include:
  - The percent (or proportion or part of the total) variability in the predicted  $y$ -values that is explained by the linear relationship between  $y$  and  $x$ .
  - The percent (or proportion or part of the total) variability in the data that is explained by the linear relationship between  $y$  and  $x$ .
  - The percent (or proportion or part of the total) variability that is explained by the linear relationship between  $y$  and  $x$ .
  - The percent (or proportion or part of the total) variability in  $y$  that is on average explained by the linear relationship between  $y$  and  $x$ .
- A reasonable but incorrect interpretation of  $r^2$  with context might include the following responses:
  - 81.9% of the variation in mass and length can be accounted for by the least-squares regression line.
  - 81.9% of the variability in predicted mass is accounted for by the length.
- For context, the response variable ( $y$ ) must be identified as mass, and the explanatory variable ( $x$ ) must be identified as length.
- An interpretation of the correlation between mass and length,  $r = \sqrt{0.819} = 0.905$ , is not considered a reasonable interpretation of  $r^2$ .
- The value of the percentage (81.9%) or proportion (0.819) of variation does not need to be specified, but if an incorrect value is specified, the score is lowered by one level, from E to P or from P to I.
- The strength of the response in part (c) may be considered if the student provides a correct interpretation of  $r^2$ .

| Model Solution                                                                                                                                                                                                                                                                                                                                                       | Scoring                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       |
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| <p>(d) (i) The largest residual in absolute value belongs to the bullfrog with length 162 millimeters and mass 356 grams.</p> <p>(ii) The least-squares regression line overestimates the mass of the bullfrog with length 162 millimeters. Plot 2 shows that the point for the bullfrog with length 162 millimeters is below the least-squares regression line.</p> | <p><b>Essentially correct (E)</b> if the response satisfies the following two components:</p> <ol style="list-style-type: none"> <li>1. The response to part (d-i) identifies the correct bullfrog (length between 160 and 165 millimeters, mass between 350 and 375 grams)</li> <li>2. The response to part (d-ii) explicitly indicates whether the linear model overestimates or underestimates mass for the bullfrog identified in part (d-i) and provides a correct justification based on a comparison of the identified observation to the least-squares regression line</li> </ol> <p><b>Partially correct (P)</b> if the response satisfies only one of the two components.</p> <p><b>Incorrect (I)</b> if the response does not satisfy the criteria for E or P.</p> |

**Additional Notes:**

- The comparison of the observation to the regression line in the response to part (d-ii) is satisfied if the response does one of the following:
  - Correctly indicates if the observation is below (above) the least-squares regression line in Plot 2.
  - Notes that observed mass is smaller (larger) than the mass predicted by the least-squares regression line.
  - Marks the observation selected in part (d-i) on Plot 2 with an indication of the vertical distance from the least-squares regression line.
  - Notes the correct sign of the residual.
- Numerical values are not required in the response to part (d-ii). If a numerical value is given for the predicted mass, however, it must be reasonable. A numerical value for the predicted mass could be computed with the formula given in the stem, e.g.,  $-546 + (6.086)(162) = 439.9$  grams, for a bullfrog of length 162 millimeters, or a value can be read from the line shown in Plot 2. Any value between 425 and 450 should be considered a reasonable value. Showing work is not required.
- The word overestimate with the calculated predicted value of mass is enough to satisfy component 2.
- If the wrong observation is identified in part (d-i), the response to part (d) may be scored P if the response to part (d-ii) correctly compares that observation to the least-squares regression line and states the correct conclusion about overestimating or underestimating mass with justification.
- It is not required to refer specifically to the “least-squares regression line.”

**Scoring for Question 1**

Each essentially correct (E) part counts as 1 point, and each partially correct (P) part counts as ½ point.

|                                                                                                                                                                                                           | Score    |
|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------|
| <b>Complete Response</b>                                                                                                                                                                                  | <b>4</b> |
| <b>Substantial Response</b>                                                                                                                                                                               | <b>3</b> |
| <b>Developing Response</b>                                                                                                                                                                                | <b>2</b> |
| <b>Minimal Response</b>                                                                                                                                                                                   | <b>1</b> |
| If a response is between two scores (for example, 2 ½ points), use a holistic approach to decide whether to score up or down, depending on the strength of the response and quality of the communication. |          |

2017

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# AP Statistics

## Scoring Guidelines

## 2017 SCORING GUIDELINES

## Question 1

**Intent of Question**

The primary goals of this question were to assess a student's ability to (1) explain statistical terms used when describing the relationship between two variables; (2) interpret the slope of a linear regression equation; and (3) calculate a value of  $y$  when given a regression equation, a value of  $x$ , and a residual.

**Solution****Part (a):**

In the context of a scatterplot in which  $y$  represents weight and  $x$  represents length, the following are defined.

A positive relationship means that wolves with higher values of length also tend to have higher weights.

A linear relationship means that as length increases by one meter, weight tends to change by a constant amount, on average.

A strong relationship means that the data points fall close to a line (or curve).

**Part (b):**

The slope of 35.02 indicates that two wolves that differ by one meter in length are predicted to differ by 35.02 kilograms in weight, with the longer wolf having the greater weight.

**Part (c):**

In general, a residual is equal to actual weight minus predicted weight, or equivalently,

$$\text{actual weight} = \text{predicted weight} + \text{residual}.$$

For the wolf with length 1.4 meters and residual of  $-9.67$ , the predicted weight is

$$-16.46 + 35.02(1.4) = 32.568 \text{ kilograms}.$$

Therefore, the actual weight of the wolf is  $32.568 + (-9.67) = 22.898$  kilograms.

**Scoring**

Parts (a), (b), and (c) are scored as essentially correct (E), partially correct (P), or incorrect (I).

**Part (a)** is scored as follows:

Essentially correct (E) if the response includes the following four components:

1. A reasonable definition of positive
2. A reasonable definition of linear
3. A reasonable definition of strong
4. At least one definition in context

## 2017 SCORING GUIDELINES

## Question 1 (continued)

Partially correct (P) if the response includes only three of the four components.

Incorrect (I) if the response does not meet the conditions for E or P.

Notes:

- The description of a positive relationship should clearly indicate that relatively low values of one variable tend to appear with relatively low values of the other variable, and relatively high values of the first variable tend to appear with relatively high values of the other variable.
  - Examples of acceptable responses:
    - As length increases, so does weight.
    - Longer wolves weigh more.
    - The points on the graph go up as you move from left to right.
  - Examples of unacceptable responses:
    - As length goes up, weight changes.
    - Both length and weight get bigger.
    - The correlation is greater than 0.
- The description of a linear relationship can take one of two approaches: the data pattern (data points exhibit the pattern of a line in the graph) or the constant rate of change (as the explanatory variable changes, the response variable exhibits a constant rate of change).
  - Examples of acceptable responses:
    - The points generally follow a straight line.
    - The relationship between  $x$  and  $y$  is straight.
    - Length and width have a constant slope.
  - Examples of unacceptable responses:
    - The points all line up.
    - You can draw a straight line through the points.
    - There is a positive correlation.
    - Every increase in  $x$  yields a 35.02 increase in  $y$ .
- The description of strong should indicate how close points are to a line.
  - Examples of acceptable responses:
    - Observed values are close to predicted values.
    - Deviations from the least-squares regression line are small.
    - The correlation coefficient is close to 1.
  - Examples of unacceptable responses:
    - All the points are close together.
    - The scatterplots are clustered together.
    - There is a high positive correlation.
- Context can be shown by referring to length and weight or by using meters and kilograms.
- Sketches and graphs can be used to help clarify definitions, but a sketch alone cannot satisfy a definition component.

**Part (b)** is scored as follows:

Essentially correct (E) if the response includes the following three components:

1. The correct value of 35.02 for the slope.
2. An interpretation that includes an increase of a specified amount of weight for each unit increase in length.
3. An indication that the relationship is not exact by using words such as "on average" or "predicted weight."

## 2017 SCORING GUIDELINES

## Question 1 (continued)

Partially correct (P) if the response includes only two of the three components.

*Note:* If the response identifies the slope as  $-16.46$  (the intercept value), the second component is satisfied only if the response states that for each one-meter increase in length there is a *decrease* in predicted or average weight of 16.46 kilograms.

Incorrect (I) if the response does not meet the criteria for E or P.

Part (c) is scored as follows:

Essentially correct (E) if the response includes the following two components:

1. A correct computation for the predicted value 32.568 kilograms.
2. A correct computation for the actual weight 22.9 kilograms using the given residual and the predicted value.

Partially correct (P) if the response provides a correct computation for the predicted value but is not able to complete the correct calculation of the actual weight, including if the residual is defined in the wrong direction as (predicted weight)  $-$  (actual weight) to give an answer of 42.24 kilograms;

OR

if the response provides an incorrect value for the predicted weight, but then uses that value correctly to determine the actual weight as (predicted weight)  $+$  residual  $=$  (predicted weight)  $+$   $(-9.67)$ ;

OR

if the response provides a correct answer for the actual weight but does not give sufficient information to determine how it was calculated.

Incorrect (I) if the response does not meet the criteria for E or P.

*Notes:*

- The expression  $-16.46 + 35.02(1.4)$  is enough to satisfy the first component.
- The equation  $-16.46 + 35.02(1.4) - 9.67 = 22.9$  satisfies both components.
- Arithmetic mistakes are overlooked if they do not lead to an unreasonable answer (such as a negative value). For example,  $32.568 + (-9.67) = 21.9$  satisfies the second component.

## 2017 SCORING GUIDELINES

## Question 1 (continued)

## 4 Complete Response

Three parts essentially correct

## 3 Substantial Response

Two parts essentially correct and one part partially correct

## 2 Developing Response

Two parts essentially correct and no parts partially correct

OR

One part essentially correct and one or two parts partially correct

OR

Three parts partially correct

## 1 Minimal Response

One part essentially correct

OR

No parts essentially correct and two parts partially correct

## 2017 SCORING GUIDELINES

## Question 2

**Intent of Question**

The primary goals of this question were to assess a student's ability to (1) construct and interpret a confidence interval for a population proportion and (2) use a confidence interval for a proportion to find a confidence interval for a dollar amount that can be calculated using that proportion.

**Solution****Part (a):**

Step 1: Identify the appropriate confidence interval by name or formula and check appropriate conditions.

The appropriate procedure is a one-sample  $z$ -interval for a population proportion  $p$ . In this case, the population is all customers of the restaurant who ask for a water cup, and  $p$  is the proportion of that population who will fill the cup with a soft drink. The appropriate formula is  $\hat{p} \pm z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$ .

Conditions:

1. Random sample
2. Large sample (number of successes  $n\hat{p} \geq 10$  and number of failures  $n(1-\hat{p}) \geq 10$ )

For condition 1 the stem of the problem states that a random sample of customers who asked for a water cup was used.

For condition 2 the number of successes (filled cup with soft drink) is 23 and the number of failures is 57, both of which are greater than 10.

Step 2: Correct mechanics

The sample proportion is  $\hat{p} = \frac{23}{80} = 0.2875$ . The confidence interval is

$$\begin{aligned} &0.2875 \pm 1.96 \sqrt{\frac{0.2875(1-0.2875)}{80}} \\ &= 0.2875 \pm 1.96(0.0506) \quad \text{or } 0.1883 \text{ to } 0.3867. \\ &= 0.2875 \pm 0.0992 \end{aligned}$$

Step 3: Interpretation

We can be 95 percent confident that in the population of all customers of the restaurant who ask for a water cup, the proportion who will fill it with a soft drink is between 0.1883 and 0.3867.

**Part (b):**

Using the confidence interval in part (a), a 95 percent interval estimate for the number of customers in June who asked for a water cup but then filled it with a soft drink is  $3,000 \times 0.1883$  to  $3,000 \times 0.3867$ , or 565 to 1,160. At a cost of \$0.25 per customer, a 95 percent interval estimate for the cost to the restaurant in June is \$141.25 to \$290.00.

## 2017 SCORING GUIDELINES

## Question 2 (continued)

**Scoring**

This question is scored in four sections. Section 1 consists of step 1 in part (a), section 2 consists of step 2 in part (a), section 3 consists of step 3 in part (a), and section 4 consists of part (b). Each section is scored as essentially correct (E), partially correct (P), or incorrect (I).

**Section 1** is scored as follows:

Essentially correct (E) if the one-sample  $z$ -interval for a proportion is identified (either by name or formula) *AND* both conditions (random sampling and large sample) are adequately addressed.

Partially correct (P) if the response identifies the correct procedure *BUT* adequately addresses only one of the two required conditions;

OR

if the response does not identify the correct procedure *BUT* adequately addresses both required conditions.

Incorrect (I) if the response does not meet the criteria for E or P.

Notes:

- Stating the large sample condition without verifying it is not sufficient. The response must use specific numerical values to adequately address the condition.
- If the response includes additional inappropriate conditions, such as  $n \geq 30$  or requiring a normal population, then the response earns at most P for section 1.
- Stating and checking a condition about the size of the sample relative to the size of the population is appropriate but not required.
- Any statement of hypotheses, description of the population, or definition of the parameter should be considered extraneous. However, if such statements are included and incorrect, they are considered as poor communication in terms of holistic scoring.

**Section 2** is scored as follows:

Essentially correct (E) if the response gives the correct 95 percent confidence interval. Supporting work is not required, but if included, it must be correct.

Partially correct (P) if the response gives a correct confidence interval with incorrect (but appropriate) supporting work shown;

OR

if the response gives an incorrect but reasonable confidence interval with appropriate supporting work shown — for instance, if a value other than 1.96 is used for the critical value or a value other than  $\frac{23}{80}$  is used for  $\hat{p}$ .

Incorrect (I) if the response does not meet the criteria for E or P.

Notes:

- Appropriate supporting working must have the form:  
proportion  $\pm$  (critical value)(SE/SD of proportion).

## 2017 SCORING GUIDELINES

## Question 2 (continued)

- A confidence interval that has both endpoints outside the interval from 0 to 1 is considered unreasonable.

Section 3 is scored as follows:

Essentially correct (E) if the response provides an appropriate interpretation of the interval that includes the following three components:

1. Conveys inference about a population proportion
2. Demonstrates a clear understanding that the parameter is the proportion of the water cup population that fills the cup with a soft drink
3. Mentions 95 percent confidence and interprets it correctly using words such as “We can be 95 percent confident” or “With 95 percent confidence”

Partially correct (P) if the response provides an appropriate interpretation of the interval that includes the first component *AND* only one of the other two components;

OR

if the response provides a correct interpretation of the confidence *level* in context without interpreting the specific interval.

Incorrect (I) if the response does not meet the criteria for E or P.

Notes:

- Clear indication of an inference to the random sample of 80 customers rather than to the population does not satisfy the first component and is scored I.
- Stating values that are unrealistic as proportions or percentages (including blanks) in the interpretation lowers the score one level (from E to P, or from P to I).
- When both the interpretation and the level of the interval are given, only the interpretation is scored. If the interpretation of the confidence level is incorrect, it is considered as poor communication in terms of holistic scoring.
- Any interpretation that implies the interval has a 95 percent chance (or possibility or probability) of capturing the population proportion is scored I.

Section 4 is scored as follows:

Essentially correct (E) if the response gives a correct interval estimate for the cost to the restaurant *AND* shows enough work to indicate how the interval was found.

Partially correct (P) if the response gives the correct interval estimate for the *number* of customers (565 to 1,160) who would fill the water cup with soda, including showing work, but does not multiply by \$0.25 to find the interval for the cost;

OR

if the response gives the correct interval estimate for the *expected cost* to the restaurant for an individual who asked for a water cup (\$0.05 to \$0.10), including showing work, but does not multiply by 3,000 to find the interval for the total cost;

OR

if the response makes a reasonable attempt to find the endpoints of the interval as  $(0.1883)(3,000)(0.25)$  and  $(0.3867)(3,000)(0.25)$ , but makes an error such as using the sample size of 80 instead of the population size of 3,000;

## 2017 SCORING GUIDELINES

## Question 2 (continued)

OR

if the response gives the correct interval without showing how it was found.

Incorrect (I) if the response does not meet the criteria for E or P.

Notes:

- The response in section 4 earns an E if it follows correctly from an incorrect interval found in part (a), even if the interval is unreasonable.
- Units (\$) are not required to earn an E.

Each essentially correct (E) section counts as 1 point, and a partially correct (P) section counts as  $\frac{1}{2}$  point.

**4 Complete Response**

**3 Substantial Response**

**2 Developing Response**

**1 Minimal Response**

If a response is between two scores (for example,  $2\frac{1}{2}$  points), use a holistic approach to decide whether to score up or down, depending on the strength of the response and communication.

## 2017 SCORING GUIDELINES

## Question 3

**Intent of Question**

The primary goals of this question were to assess a student's ability to (1) calculate a probability from a normal distribution; (2) calculate a weighted probability from two individual probabilities; and (3) calculate a conditional probability for dependent events when individual and joint probabilities are provided.

**Solution****Part (a):**

Let  $X$  denote the diameter of a randomly selected melon from Distributor J.  $X$  has an approximately normal distribution with mean 133 mm and standard deviation 5 mm.

The z-score for a diameter of 137 mm is  $z = \frac{137 - 133}{5} = \frac{4}{5} = 0.8$ .

Therefore,  $P(X > 137) = P(Z > 0.8) = 1 - 0.7881 = 0.2119$ .

**Part (b):**

Define events:

$J$ : melon is from Distributor J

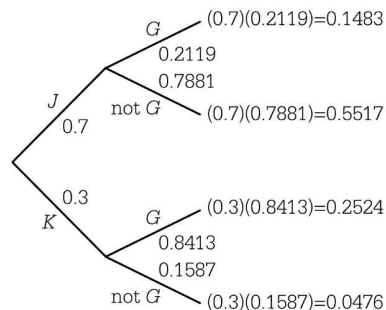
$K$ : melon is from Distributor K

$G$ : melon diameter is greater than 137 mm

$$\begin{aligned} P(G) &= P(G | J) \times P(J) + P(G | K) \times P(K) \\ &= (0.2119)(0.7) + (0.8413)(0.3) \\ &= 0.1483 + 0.2524 \\ &= 0.4007 \end{aligned}$$

For a randomly selected melon from the grocery store,

OR



From the tree diagram,  $P(G) = P(G \text{ and } J) + P(G \text{ and } K) = 0.1483 + 0.2524 = 0.4007$ .

## 2017 SCORING GUIDELINES

## Question 3 (continued)

**Part (c):**

Using the events defined in part (b), the requested probability is

$$P(J | G) = \frac{P(J \text{ and } G)}{P(G)} = \frac{P(G | J)P(J)}{P(G)} = \frac{(0.2119)(0.7)}{0.4007} = \frac{0.1483}{0.4007} = 0.3701.$$

**Scoring**

Parts (a), (b), and (c) are scored as essentially correct (E), partially correct (P), or incorrect (I).

**Part (a)** is scored as follows:

Essentially correct (E) if the response provides the following three components:

1. **Normality and parameters:** Indicates use of a normal (or approximately normal) distribution and clearly identifies the correct parameter values. (Showing correct components in a z-score, labeling the mean and standard deviation in a normalcdf calculator statement, or drawing a normal curve with sufficient values marked to indicate the mean and standard deviation are all sufficient methods for identifying the correct parameter values.)
2. **Boundary and direction:** Uses the correct boundary value of  $x = 137$  or  $z = 0.8$  and the correct direction. (Showing correct boundary and direction in a probability statement using "greater than," labeling the lower limit and upper limit in a normalcdf calculator statement, drawing a normal curve with 137 labeled and an indication that the area of interest is to the right of 137, or a conclusion in words using "greater than.")
3. **Probability:** Reports the correct normal probability consistent with the response's setup described in components 1 and 2.

Partially correct (P) if the response correctly provides only two of the three components.

Incorrect (I) if the response gives the answer with no work shown or otherwise does not satisfy the criteria for E or P.

Notes:

- An inconsistency in calculations lowers the score for part (a) by one level (from E to P, or from P to I). For instance, if the response states the requested probability as  $P(X > 137)$ , but actually computes  $P(X < 137)$ .
- An error in statistical notation, such as using  $s$  instead of  $\sigma$  for the standard deviation or  $\bar{x}$  instead of  $\mu$  for the mean, does not satisfy component 1.
- A correct direction can be obtained by using a left direction and an upper bound of 137 and then subtracting the value of the cumulative probability from 1. However, simply showing the calculation  $1 - 0.7881 = 0.2119$  does NOT give a correct direction.
- Consider any steps of a hypothesis test as extraneous work.
- Standard notations such as  $N(133,5)$  or  $N(133,25)$  satisfy the first component.
- A sketch of the normal curve with 133 and 138 in the appropriate positions satisfies the first component.
- Use of a value other than 137 does not satisfy the second component.
- If the only error in part (a) is the reversal of the numerator for the z-score  $(133 - 137)$ , the response is scored as P.

## 2017 SCORING GUIDELINES

## Question 3 (continued)

- For calculator notation:
  - ncdf is sufficient to indicate use of the normal distribution.
  - Stating the lower and upper bounds is sufficient for identifying boundary and direction.
  - Minimum notation, such as “u” for upper, is sufficient for identifying boundary.
  - The minimum value for an upper bound must be at least 4 standard deviations above the mean.

**Part (b)** is scored as follows:

Essentially correct (E) if the probability is computed correctly, *AND* work is shown that includes correct numerical values using a formula, end results from a tree diagram, or some other appropriate method.

Partially correct (P) if the response provides a correct strategy for finding the probability, such as a formula or tree diagram, but uses one or more incorrect values or does not use the end result from the tree;

OR

if the response gives the correct probability but not enough work is shown to determine how it was found.

Incorrect if the response does not meet the criteria for E or P.

Notes:

- A response that uses 0.3 for  $J$  and 0.7 for  $K$  earns a P if no other mistakes are made.
- Tree diagrams:
  - If used, the multiplication is implied, but the addition must be indicated.
  - If not used, both the multiplication and the addition must be indicated.
- The shown work might be found in part (c).
- Probabilities can be shown in a table. Addition is implied if “Total” is labeled, as shown.

| Intersection | $G$    | Not $G$ | Total |
|--------------|--------|---------|-------|
| $J$          | 0.1483 | 0.5517  | 0.7   |
| $K$          | 0.2524 | 0.0576  | 0.3   |
| Total        | 0.4007 | 0.5993  |       |

**Part (c)** is scored as follows:

Essentially correct (E) if the probability is computed correctly *AND* work is shown that illustrates how the probability was found.

Partially correct (P) if the response provides a strategy for finding the probability for dependent variables, such as the appropriate formula or end results from the correct tree diagram, but uses one or more incorrect values or does not use the end result from the tree;

OR

if the response gives the correct probability but not enough work is shown to determine how it was found.

Incorrect (I) if the response does not meet the criteria for E or P.

## 2017 SCORING GUIDELINES

## Question 3 (continued)

Notes:

- Part (c) earns an E if a correct answer follows from incorrect values used from part (a) and/or part (b), provided all values are between 0 and 1, inclusive.
- A transcription error in a response can be ignored if correct work is shown.
- For any part, if the resulting probability or part of the calculation of the probability uses a value that is not between 0 and 1, inclusive, the score is lowered by one level (from E to P, or from P to I).
- In any part, unsupported answers are scored as I.

#### 4 Complete Response

Three parts essentially correct

#### 3 Substantial Response

Two parts essentially correct and one part partially correct

#### 2 Developing Response

Two parts essentially correct and no parts partially correct

OR

One part essentially correct and one or two parts partially correct

OR

Three parts partially correct

#### 1 Minimal Response

One part essentially correct

OR

No parts essentially correct and two parts partially correct

## 2017 SCORING GUIDELINES

## Question 4

**Intent of Question**

The primary goals of this question were to assess a student's ability to use boxplots to (1) compare multiple sets of data; (2) identify which set of data is most likely to have produced a particular summary value; and (3) determine which variable is most useful for classifying a new observation.

**Solution****Part (a):**

The median value for the percent of chemical Z in the pottery pieces is similar for all three sites, at about 7 percent. The ranges for the percent of chemical Z are much different for the three sites, with the smallest range of about 2 percent (from 6 percent to 8 percent) at Site II, a range of about 6 percent (from about 4 percent to 10 percent) at Site I, and the largest range of about 8 percent (from about 3 percent to 11 percent) at Site III.

**Part (b):**

- (i) The piece most likely originated at Site III. Although values outside of the range of data observed in the samples would be possible, using the available data results in approximate minimum and maximum sums of the percents for the three chemicals as shown in the table below. Site III is the only site in which 20.5 falls between the sums of the minimum and maximum values.

|          | Site I |     | Site II |     | Site III |      |
|----------|--------|-----|---------|-----|----------|------|
| Chemical | Min    | Max | Min     | Max | Min      | Max  |
| X        | 6      | 8   | 5       | 7   | 5        | 7.5  |
| Y        | 11     | 15  | 1.9     | 4   | 6        | 8    |
| Z        | 4      | 10  | 6       | 8   | 3        | 11   |
| Sum      | 21     | 33  | 12.9    | 19  | 14       | 26.5 |

- (ii) Chemical Y would be most useful, because the distribution of the percentages of total weights at the three sites do not overlap. The distributions of chemicals X and Z have substantial overlap.

**Scoring**

This question is scored in three sections. Section 1 consists of part (a), section 2 consists of part (b-i), and section 3 consists of part (b-ii). Each section is scored as essentially correct (E), partially correct (P), or incorrect (I).

**Section 1** is scored as follows:

Essentially correct (E) if the response includes the following three components:

1. Recognition that the medians or centers are almost the same for the three sites
2. Recognition that the variability (ranges, IQRs, spread) is different across the three sites
3. Context is included

Partially correct (P) if the response includes only two of the three components.

Incorrect (I) if the response includes at most one of the three components.

## 2017 SCORING GUIDELINES

## Question 4 (continued)

*Notes:*

- In all sections, comments about shape should be ignored because complete shape information is not obtainable from boxplots.
- Responses are not required to give numerical values. If responses provide numerical values, any reasonable approximation from the boxplots is acceptable.
- Because the boxplots are all symmetric, it is acceptable if the response discusses means instead of medians.
- Any discussion of chemical X and chemical Y is considered extraneous.
- Context is satisfied by any of the following references: site, chemical, weight, total weight, X, Y, or Z.

**Section 2** is scored as follows:

Essentially correct (E) if the response includes the following three components:

1. Site III is chosen.
2. Sums of the minimum and maximum are computed for the three chemicals at each site.
3. A reasonable numerical justification is given involving sums of a statistical measure across the three chemicals to choose Site III.

Partially correct (P) if the response includes only two of the three components.

Incorrect (I) if the response includes at most one of the three components.

*Notes:*

- If the response computes only the sum of the minimums for Site I and the sum of the maximums for Site II and recognizes that this is sufficient, the response is scored E.
- If an alternative measure is used that involves sums of the three chemicals, such as the sum of the medians or the sums of the first quartiles and sums of the third quartiles, instead of the minimum and maximum sums, the second component is not satisfied, but the third component might be satisfied.
  - If the response explicitly or implicitly compares the alternate sum to the other two sites (for example, by indicating that the sum is the closest to 20.5 percent or by listing the sums for all three sites) the response is scored P.
  - If the response does not have an implicit or explicit comparison, the response is scored I.
- If either Site I or Site II is identified as the correct choice, no matter how that choice is justified, the response is scored I.
- The approximate sums of the medians are 27.5 for Site I, 16 for Site II, and 20 for Site III.

**Section 3** is scored as follows:

Essentially correct (E) if the response chooses chemical Y **AND** gives a reasonable justification based on the fact that the distributions of chemical Y are distinctive across sites.

Partially correct (P) if the response chooses chemical Y **AND** provides justification based on the boxplots, but does not clearly explain that the distributions of chemical Y are distinctive across sites;

OR

if the response correctly discusses that the distributions of chemical Y are distinctive across sites, but never explicitly chooses chemical Y as the best choice, for instance, by stating only that there is substantial overlap across sites for chemicals X and Z but no overlap for chemical Y.

## 2017 SCORING GUIDELINES

## Question 4 (continued)

Incorrect (I) if the response does not meet the criteria for E or P.

Notes:

- To justify that the distributions of chemical Y are distinctive across sites, the justification must address both location and variability of the boxplots; for example, by stating that the boxplots do not overlap for chemical Y.
- If the response chooses chemical X or Z OR chooses chemical Y with no reasonable justification, the response is scored I.
- The justification that the distributions of chemical Y are distinctive across sites:
  - The following are acceptable because both location and variability are addressed. Such responses are scored E.
    - The boxplots for chemical Y do not overlap, or the boxplots for chemicals X and Z overlap.
    - All values of Site I are high, all values of Site II are low, and all values of Site III are in the middle.
    - The ranges never intersect.
    - The boxplots share no data.
    - Has completely different percentages at each site.
  - The following are incomplete justifications and are scored P.
    - The boxplots vary.
    - Chemical Y varies the most.
    - Chemical Y has the greatest variation.
    - The variation between/among sites is the largest.
    - The boxplots are different.
    - The medians/means differ.
    - The medians/means are most variable.
    - There is a difference in the percentages of chemical Y for each site.
    - The distribution of percents differs the most among the sites.

## 2017 SCORING GUIDELINES

## Question 4 (continued)

## 4 Complete Response

Three sections essentially correct

## 3 Substantial Response

Two sections essentially correct and one section partially correct

## 2 Developing Response

Two sections essentially correct and no sections partially correct

OR

One section essentially correct and one or two sections partially correct

OR

Three sections partially correct

## 1 Minimal Response

One section essentially correct

OR

No sections essentially correct and two sections partially correct

## 2017 SCORING GUIDELINES

## Question 5

**Intent of Question**

The primary goal of this question was to assess a student's ability to identify, set up, perform, and interpret the results of an appropriate hypothesis test to address a particular question. More specific goals were to assess a student's ability to (1) state appropriate hypotheses; (2) identify the appropriate statistical test procedure and check appropriate conditions for inference; (3) calculate the appropriate test statistic and  $p$ -value; and (4) draw an appropriate conclusion, with justification, in the context of the study.

**Solution**

Step 1: State a correct pair of hypotheses.

The null hypothesis is that age group at diagnosis and gender are independent (that is, they are not associated) for the population of people currently being treated for schizophrenia.

The alternative hypothesis is that age group at diagnosis and gender are not independent for the population of people currently being treated for schizophrenia.

Step 2: Identify a correct test procedure (by name or formula) and check appropriate conditions.

The appropriate test is a chi-square test of independence.

The conditions for this test are satisfied because:

1. The question states that the sample was randomly selected.
2. The expected counts for the eight cells of the table are at least 5, as seen in the following table, with expected counts shown below observed counts.

|       | Age at Diagnosis |             |             |            | Total |
|-------|------------------|-------------|-------------|------------|-------|
|       | 20 to 29         | 30 to 39    | 40 to 49    | 50 to 59   |       |
| Women | 46<br>56.91      | 40<br>36.22 | 21<br>17.25 | 12<br>8.62 | 119   |
| Men   | 53<br>42.09      | 23<br>26.78 | 9<br>12.75  | 3<br>6.38  | 88    |

Step 3: Find the value of the test statistic and the  $p$ -value.

$$\begin{aligned} \text{The test statistic is calculated as } \chi^2 &= \sum \frac{(O - E)^2}{E}, \text{ or} \\ \chi^2 &= 2.093 + 0.395 + 0.817 + 1.322 \\ &\quad + 2.830 + 0.534 + 1.105 + 1.788 \\ &= 10.884. \end{aligned}$$

The  $p$ -value is  $P(\chi^2 \geq 10.884) = 0.012$ , based on  $(4 - 1) \times (2 - 1) = 3$  degrees of freedom.

## 2017 SCORING GUIDELINES

## Question 5 (continued)

Step 4: State the conclusion in context, with linkage to the  $p$ -value.

Because the  $p$ -value is very small (for instance much smaller than  $\alpha = 0.05$ ), we reject the null hypothesis and conclude that the sample data provide strong evidence that there is an association between age group at diagnosis and gender for the population currently being treated for schizophrenia.

**Scoring**

This question is scored in three sections. Section 1 consists of steps 1 and 2 (stating the correct hypotheses, identifying the appropriate test procedure, and checking the technical conditions); section 2 consists of step 3 (performing the correct mechanics); and section 3 consists of step 4 (stating a correct conclusion with justification). Sections 1, 2, and 3 are scored as essentially correct (E), partially correct (P), or incorrect (I).

Section 1 is scored as follows:

Essentially correct (E) if the response correctly includes the following three components:

1. States BOTH hypotheses correctly with context included in at least one of them
2. Identifies a chi-square test of independence by name or formula
3. Verifies appropriate conditions that minimally include the condition for the expected counts and do not include any incorrect conditions (such as normality)

Partially correct (P) if the response includes only two of the three components.

Incorrect (I) if the response includes at most one of the three components.

Notes:

- Stating the expected count condition is not sufficient; the condition must be checked by reporting the expected counts, or minimally by showing that the smallest expected count is at least 5.
- The random sample condition was stated in the stem so need not be explicitly checked.
- If the null and alternative hypothesis are correctly stated in terms of population proportions, component 1 is satisfied. For example:  
 $H_0 : p_1 = p_2 = p_3 = p_4$ , where  $p_i$  is the population proportion of women at each indicated age group, 1, 2, 3, or 4, who are currently being treated for schizophrenia.  
 $H_a$  : At least one of the population proportions,  $p_1, p_2, p_3, p_4$ , differs from the other three.  
OR  
 $H_a$  : The population proportions for the four age groups are not all the same.

Section 2 is scored as follows.

Essentially correct (E) if the response correctly calculates the following two values:

1. The value of the chi-square test statistic
2. The  $p$ -value, critical value, or  $p$ -value range from chi-square table

Partially correct (P) if the response correctly calculates only one of the two values.

## 2017 SCORING GUIDELINES

## Question 5 (continued)

Incorrect (I) if the response does not satisfy the criteria for E or P.

Notes:

- If the response makes an error in one calculation, subsequent calculations are considered correct if they follow correctly from the initial miscalculation.
- With 3 degrees of freedom, the correct critical value is 7.81 for a significance level of 0.05 and 11.34 for a significance level of 0.01.
- Work does not have to be shown for calculations of test statistic or  $p$ -value. However, if incorrect work (other than minor arithmetic/transcription errors) is shown it is considered to be an incorrect calculation of the respective component, even if the correct value is given.
- If a response provides a test statistic that is not a chi-square test statistic, section 2 is scored I.

Section 3 is scored as follows.

Essentially correct (E) if the response includes the following three components:

1. A correct conclusion about the alternative hypothesis.
2. Justification of the conclusion based on linkage between the  $p$ -value and a reasonable alpha (or linkage between test statistic and critical value).
3. The conclusion is stated in context.

Partially correct (P) if the response includes only two of the three components.

Incorrect (I) if the response includes at most one of the three components.

Notes:

- If the response provides a correct decision, in context, with linkage to the  $p$ -value, but the decision is stated in terms of the null hypothesis with no conclusion about the alternative hypothesis, component 1 is not satisfied.
- Incorrect statistical statements are considered incorrect conclusions for the hypothesis test and do not satisfy component 1.
- If the conclusion is consistent with the  $p$ -value from section 2, and also in context with justification based on linkage to the  $p$ -value, section 3 is scored E.
- If no alpha level is given, the solution must be explicit about the linkage by giving a correct interpretation of the  $p$ -value or explaining how the conclusion follows from the  $p$ -value. For example, stating that because the  $p$ -value is small, we reject the null hypothesis or stating that because the  $p$ -value is large, we do not reject the null hypothesis.
- A decision about the null hypothesis (reject  $H_0$  or fail to reject  $H_0$ ) is not required, but if such a statement is given the scoring of the decision is considered in component 2.

## 2017 SCORING GUIDELINES

## Question 5 (continued)

## 4 Complete Response

Three sections essentially correct

## 3 Substantial Response

Two sections essentially correct and one section partially correct

## 2 Developing Response

Two sections essentially correct and no sections partially correct

OR

One section essentially correct and one or two sections partially correct

OR

Three sections partially correct

## 1 Minimal Response

One section essentially correct

OR

No sections essentially correct and two sections partially correct

OR

Section 1 partially correct and the other two sections incorrect

## 2017 SCORING GUIDELINES

## Question 6

**Intent of Question**

The primary goals of this question were to assess a student's ability to (1) calculate probabilities associated with treatment and control group memberships for two different methods of random assignment and (2) justify which method of random assignment is more appropriate in a given situation.

**Solution****Part (a):**

- (i) Let T (tail) represent being assigned to the treatment group and H (head) represent being assigned to the control group for each coin flip. The process stops when either the treatment group or the control group has two members. The outcomes and their probabilities are as follows.

| Arrangement   | A                                                  | B                                                                          | C                                                                          | D                                                  | E                                                                          | F                                                                          |
|---------------|----------------------------------------------------|----------------------------------------------------------------------------|----------------------------------------------------------------------------|----------------------------------------------------|----------------------------------------------------------------------------|----------------------------------------------------------------------------|
| Chip outcomes | TT                                                 | THT                                                                        | THH                                                                        | HH                                                 | HTH                                                                        | HTT                                                                        |
| Calculation   | $\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$ | $\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$ | $\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$ | $\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$ | $\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$ | $\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$ |
| Probability   | $\frac{1}{4}$                                      | $\frac{1}{8}$                                                              | $\frac{1}{8}$                                                              | $\frac{1}{4}$                                      | $\frac{1}{8}$                                                              | $\frac{1}{8}$                                                              |

- (ii) Man 1 and Man 2 are assigned to the same group for arrangements A and D, so the probability is

$$P(A) + P(D) = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}.$$

**Part (b):**

- (i) Let T represent being assigned to the treatment group and C represent being assigned to the control group for each chip drawn. The process stops when either the treatment group or the control group has two members. The probabilities differ from the coin flip method because chips are drawn *without* replacement. The outcomes and their probabilities are as follows.

| Arrangement   | A                                                  | B                                                                          | C                                                                          | D                                                  | E                                                                          | F                                                                          |
|---------------|----------------------------------------------------|----------------------------------------------------------------------------|----------------------------------------------------------------------------|----------------------------------------------------|----------------------------------------------------------------------------|----------------------------------------------------------------------------|
| Chip outcomes | TT                                                 | TCT                                                                        | TCC                                                                        | CC                                                 | CTC                                                                        | CTT                                                                        |
| Calculation   | $\left(\frac{2}{4}\right)\left(\frac{1}{3}\right)$ | $\left(\frac{2}{4}\right)\left(\frac{2}{3}\right)\left(\frac{1}{2}\right)$ | $\left(\frac{2}{4}\right)\left(\frac{2}{3}\right)\left(\frac{1}{2}\right)$ | $\left(\frac{2}{4}\right)\left(\frac{1}{3}\right)$ | $\left(\frac{2}{4}\right)\left(\frac{2}{3}\right)\left(\frac{1}{2}\right)$ | $\left(\frac{2}{4}\right)\left(\frac{2}{3}\right)\left(\frac{1}{2}\right)$ |
| Probability   | $\frac{1}{6}$                                      | $\frac{1}{6}$                                                              | $\frac{1}{6}$                                                              | $\frac{1}{6}$                                      | $\frac{1}{6}$                                                              | $\frac{1}{6}$                                                              |

- (ii) Man 1 and Man 2 are assigned to the same group for arrangements A and D, so the probability is

$$P(A) + P(D) = \frac{1}{6} + \frac{1}{6} = \frac{1}{3}.$$

## 2017 SCORING GUIDELINES

## Question 6 (continued)

**Part (c):**

Use the chip method. The chip method gives equal probability to all possible arrangements, but the coin method does not, as shown in the tables from parts (a-i) and (b-i). Furthermore, the coin method is more likely to result in imbalanced treatment groups with regard to students and teachers, based on the probabilities in parts (a-ii) and (b-ii). If food preferences for teachers are different than for students, the imbalance is a problem. For example, if one treatment group consists entirely of students, it would be impossible to know if a difference in the response variable is due to the treatment (type of meal) or the role of the person at the school (teacher or student).

**Scoring**

Parts (a), (b), and (c) are each scored as essentially correct (E), partially correct (P), or incorrect (I).

**Part (a)** is scored as follows:

Essentially correct (E) if the response includes the following three components:

1. The correct probability for each arrangement in the table of (i).
2. Appropriate justification is shown in (i).
3. The correct probability is given in (ii).

Partially correct (P) if the response includes only two of the three components.

Incorrect (I) if the response does not meet the criteria for E or P.

**Notes:**

- For component 2, examples of appropriate justification include the following:
  - Shows calculation of a correct probability for at least one of arrangements A and D *AND* shows calculation of a correct probability for at least one of arrangements B, C, E, and F. Calculations must be linked to a specific arrangement with a label, by proximity to an arrangement in the table, or by following the structure of the table or display of arrangements.
  - Lists correct coin outcomes for at least one of arrangements A and D *AND* lists correct coin outcomes for at least one of arrangements B, C, E, and F (for example, A: TT and B: THT). Lists of outcomes must be linked to a specific arrangement with a label, by proximity to an arrangement in the table, or by following the structure of the table or display of arrangements.
  - Shows a tree diagram that ends with six branches corresponding to the six arrangements. Probabilities or coin outcomes must be listed on the branches, but arrangements do not need to be identified.
  - Shows a tree diagram that ends with eight (or 16) branches where the branches leading to at least one of arrangements A and D are identified, and the branches leading to at least one of arrangements B, C, E, and F are identified. Probabilities or coin outcomes must be listed on the branches.
- If an incorrect probability is given in part (ii), component 3 is satisfied if the probability is between 0 and 1 and:
  - is the result of a minor arithmetic error; *OR*
  - is the result of adding the probabilities of arrangements A and D from the table in (i).

## 2017 SCORING GUIDELINES

## Question 6 (continued)

Part (b) is scored as follows:

Essentially correct (E) if the response includes the following two components:

1. The correct probability for each arrangement in the table of (i).
2. The correct probability is given in (ii).

Partially correct (P) if the response includes only one of the three components.

Incorrect (I) if the response does not meet the criteria for E or P.

Note: If an incorrect probability is given in (ii), component 2 is satisfied if the probability is between 0 and 1 and

- is the result of a minor arithmetic error; OR
- is the result of adding the probabilities of arrangements A and D from the table in (i); OR
- work is shown that demonstrates the same incorrect approach is used in both parts (a-ii) and (b-ii). For example, component 2 is satisfied if the response forgets to include  $P(D)$  and says

$P(A) = \frac{1}{4}$  in (a-ii) and  $P(A) = \frac{1}{6}$  in part (b-ii). However, unlabeled answers of  $\frac{1}{4}$  in (a-ii) and  $\frac{1}{6}$  in (b-ii) would not satisfy component 2 because it is unclear if the same approach is being used.

Part (c) is scored as follows:

Essentially correct (E) if the response chooses the chip method AND includes the following three components:

1. Provides a statistical benefit to the chip method. For example, stating that all arrangements are equally likely.
2. Provides a statistical drawback to the coin method. For example, stating that the coin method is more likely to result in imbalanced treatment groups.
3. States that the responses (opinions, food preferences) of teachers and students might be different.

Partially correct if the response chooses the chip method and includes only one or two of the three components.

Incorrect (I) if the response does not meet the criteria for E or P.

Notes:

- A correct comparative statement can be used to satisfy both components 1 and 2. For example, stating that the coin method is *more likely* to result in a disproportionate number of students in one treatment group, or that the chip method has a *greater chance* of providing roughly equivalent treatment groups.
- Benefits and drawbacks must be about the probabilities of the arrangements (from the tables in (i)), or the probabilities of imbalanced (balanced) groups (from the questions in (ii)), or both.
- If the response chooses the coin method, look back to see if the coin method should be the preferred method based on incorrect work from parts (a) and (b). If so, score part (c) using the corresponding three components.

## 2017 SCORING GUIDELINES

## Question 6 (continued)

- If the response says that it does not matter which method to use, look back to see if the tables are the same in parts (a-i) and (b-i) or if the probabilities are the same in parts (a-ii) and (b-ii). If so, part (c) is partially correct if the response justifies the decision by stating that the two tables or the two probabilities are the same. To be essentially correct, the response also needs to satisfy component 3.
- If the response does not choose a method, deduct one component from the number of correct components.

## 4 Complete Response

Three parts essentially correct

## 3 Substantial Response

Two parts essentially correct and one part partially correct

## 2 Developing Response

Two parts essentially correct and no parts partially correct

OR

One part essentially correct and one or two parts partially correct

OR

Three parts partially correct

## 1 Minimal Response

OR

One part essentially correct

No parts essentially correct and two parts partially correct

# AP<sup>®</sup> Statistics

## 2014 Scoring Guidelines

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## 2014 SCORING GUIDELINES

### Question 1

#### Intent of Question

The primary goals of this question were to assess a student's ability to (1) calculate conditional proportions from a two-way table; (2) comment on association between two categorical variables as displayed in a graph; and (3) draw an appropriate conclusion from the  $p$ -value of a chi-square test.

#### Solution

##### Part (a):

The proportion of on campus residents who participate in at least one extracurricular activity is  $\frac{17 + 7}{33} = \frac{24}{33} \approx 0.727$ . The proportion of off campus residents who participate in at least one extracurricular activity is  $\frac{25 + 12}{67} = \frac{37}{67} \approx 0.552$ .

##### Part (b):

The graph reveals that on campus residents in this sample are more likely to participate in extra-curricular activities than off campus residents. The proportions who participate in two or more extra-curricular activities are similar between the two groups but slightly greater for on campus residents (on campus: 0.212, off campus: 0.179). On campus residents have a greater proportion who participate in one activity (on campus: 0.515, off campus: 0.373) and a smaller proportion who participate in no extracurricular activities (on campus: 0.273, off campus: 0.448) than off campus residents.

##### Part (c):

The  $p$ -value of 0.23 is greater than conventional significance levels such as  $\alpha = 0.10$  or  $\alpha = 0.05$  or  $\alpha = 0.01$ . Therefore, the  $p$ -value indicates that the sample data do not provide strong enough evidence to conclude that participation in extracurricular activities differs between on and off campus residents in the population of all students at the university.

## 2014 SCORING GUIDELINES

## Question 1 (continued)

**Scoring**

Parts (a), (b), and (c) were scored as essentially correct (E), partially correct (P), or incorrect (I).

**Part (a)** is scored as follows:

Essentially correct (E) if the response correctly performs *both* calculations with work shown.

Partially correct (P) if the response correctly performs one of the two calculations with work shown;

OR

if the response provides both correct answers with no work shown;

OR

if the response calculates the proportion of students involved in exactly one extracurricular activity rather than the proportion of students involved in at least one extracurricular activity for both groups, with work shown.

Incorrect (I) if the response does not meet the criteria for E or P.

*Note:* Answers reported as fractions rather than decimals are acceptable.

**Part (b)** is scored as follows:

Essentially correct (E) if the response correctly compares proportions between the two groups of students for at least two of the three categories.

Partially correct (P) if the response correctly lists proportions for at least two categories for the two groups but does not make an explicit comparison between the two groups;

OR

if the response correctly compares the relative values of the proportions between the two groups of students for only one of the three categories.

Incorrect (I) if the response does not meet the criteria for E or P.

*Notes:*

- A response without any reference to percentages or proportions is scored as at most P, (for example, a response that attempts to compare counts).
- A response that treats bar graphs as distributions of a quantitative variable lowers the score one level (that is, from E to P, or from P to I) in part (b).

## 2014 SCORING GUIDELINES

## Question 1 (continued)

**Part (c)** is scored as follows:

Essentially correct (E) if the response states a correct conclusion in the context of the study *AND* provides correct justification/decision of that conclusion based on linkage to the  $p$ -value.

Partially correct (P) if the response provides no conclusion in context but does provide correct justification/decision based on linkage to the  $p$ -value;

OR

if the response provides a correct conclusion in context but with incorrect or missing justification/decision based on linkage to the  $p$ -value;

OR

if the response provides the conclusion in context and correct justification/decision based on linkage to the  $p$ -value but states a conclusion equivalent to accepting the null hypothesis.

Incorrect (I) if the response does not meet the criteria for E or P.

*Notes:*

- Justification based on the  $p$ -value can be given by stating a significance level and noting that the  $p$ -value is larger than the significance level *OR* by simply stating that the  $p$ -value is large.
- A conclusion that is equivalent to “accept the null hypothesis”, either as a stated decision or as a conclusion in context, cannot be scored as E. Such a response is scored as P if it includes both content and correct justification based on linkage to  $p$ -value. If such a response lacks either context or linkage, it is scored as I.

**4 Complete Response**

All three parts essentially correct

**3 Substantial Response**

Two parts essentially correct and one part partially correct

**2 Developing Response**

Two parts essentially correct and one part incorrect

OR

One part essentially correct and one or two parts partially correct

OR

Three parts partially correct

**1 Minimal Response**

One part essentially correct and two parts incorrect

OR

Two parts partially correct and one part incorrect

## 2014 SCORING GUIDELINES

## Question 2

**Intent of Question**

The primary goals of this question were to assess a student's ability to (1) calculate a probability; (2) assess whether a claim about randomness is questionable in light of a calculated probability; and (3) determine whether a description of a simulation method achieves a correct simulation of a random process.

**Solution****Part (a):**

The probability that all 3 people selected are women can be calculated using the multiplication rule, as follows:

$P(\text{all three selected are women})$

$= P(\text{first is a woman}) \times P(\text{second is a woman} | \text{first is a woman}) \times P(\text{third is a woman} | \text{first two are women})$

$$= \frac{3}{9} \times \frac{2}{8} \times \frac{1}{7} \approx 0.012$$

**Part (b):**

The probability calculated in part (a) does provide a reason to doubt the manager's claim that the selections were made at random. The calculation shows that there is only about a 1.2% chance that random selection would have resulted in three women being selected. The probability is small enough that it may cast doubt on the manager's claim that the selections were made at random.

**Part (c):**

No, the process does not correctly simulate the random selection of three women from a group of nine people of whom six are men and three are women. The random selection of three people among nine is done *without* replacement. However, in the simulation with the dice, the three dice rolls in any given trial are independent of one another, indicating a selection process that is done *with* replacement.

## 2014 SCORING GUIDELINES

## Question 2 (continued)

**Scoring**

Parts (a), (b), and (c) were scored as essentially correct (E), partially correct (P), or incorrect (I).

**Part (a)** is scored as follows:

Essentially correct (E) if the response correctly computes the probability of selecting the three women, and shows how the probability was computed.

Partially correct (P) if the response shows only one of the following:

Gives the correct probability of  $\frac{1}{84}$  (0.012 or 0.011 is acceptable) but does not show how it was computed;

OR

Correctly shows how the probability should be computed, but does not carry the computation through correctly;

OR

Correctly computes (showing work) only the numerator, or only the denominator of the correct answer. (For example,  $\frac{1}{9} \times \frac{1}{8} \times \frac{1}{7} \approx 0.002$ , or  $\frac{3}{9} \times \frac{2}{9} \times \frac{1}{9} \approx 0.008$ , or  $\frac{3}{9} \times \frac{3}{8} \times \frac{3}{7} \approx 0.054$ );

OR

Mistakenly assumes independence and calculates (showing work) the binomial probability

$$\frac{1}{3} \times \frac{1}{3} \times \frac{1}{3} = \frac{1}{27} \approx 0.037.$$

Incorrect (I) if the response does not meet the criteria for E or P.

**Part (b)** is scored as follows:

Essentially correct (E) if the response states that the probability from part (a) is small (or insufficiently small), makes an appropriate decision consistent with the probability being small (or insufficiently small), and does so in the context of this situation.

Partially correct (P) if the response shows only one of the following:

Otherwise satisfies the criteria for an E but does so without any context;

OR

States a significance level and makes a decision in context that is appropriate to the given probability in part (a) and the stated significance level, but does not explicitly compare the probability and the significance level;

OR

Otherwise satisfies the criteria for an E but does not explicitly make a decision about whether there is reason to doubt the manager's claim. (For example: "The probability of selecting the three women from among the nine employees is very small so it is unlikely to occur by chance.")

Incorrect (I) if the response does not meet the criteria for E or P.

## 2014 SCORING GUIDELINES

## Question 2 (continued)

Notes:

- Each of the following situations is one in which a response that otherwise would be scored as E should be scored as P, and a response that otherwise would be scored as P should be scored as I:
  - The response includes a statement that the small probability *proves* that the manager did not make the selection at random (or any equivalent wording).
  - The response includes a statement that clearly interprets the probability from part (a) to be the probability that the manager selected the three people at random.
- Each of the following situations is one in which the response is scored as I:
  - The decision is inconsistent with the justification (e.g., "The probability is very small, so there is no reason to doubt the manager's claim").
  - The response states or implies that because the selection of three women was *not impossible*, there is *no reason to doubt* the manager's claim.

Part (c) is scored as follows:

Essentially correct (E) if the response answers no *AND* states that the dice outcomes in the proposed simulation are independent *AND* states that the genders of the selected convention attendees are dependent. The table below shows statements that should be considered equivalent to the required statements of independence and dependence.

| <u>Independence</u> of dice outcomes                                                                                                                                                                                                                | <u>Dependence</u> of genders                                                                                                                                                                                                                           |
|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <ul style="list-style-type: none"> <li>The three dice outcomes are independent.</li> <li>The probability of rolling a 5 or a 6 is the same on all three dice.</li> <li>The dice simulation actually simulates sampling with replacement.</li> </ul> | <ul style="list-style-type: none"> <li>The genders of the three people are dependent (or not independent).</li> <li>The probability of selecting a woman changes after each selection.</li> <li>The people are sampled without replacement.</li> </ul> |

OR

Essentially correct if the response answers no *AND* computes the correct probability that a trial of the simulation will indicate the selection of three women  $\left(\frac{1}{3} \times \frac{1}{3} \times \frac{1}{3} \approx 0.037\right)$  *AND* states that the probability is different from the probability found in part (a).

Partially correct (P) if the response correctly answers no and either:

States only that the dice outcomes are independent or states only that the genders of the selected convention attendees are dependent, but not both;

OR

Otherwise meets the criteria for E but has poor communication. An example of poor communication is: "No, because it selects with replacement. It isn't possible for the same person to be selected twice." (There is an apparent shift between the two sentences from describing the simulation to describing the actual selection of people, but that is not made clear.)

## 2014 SCORING GUIDELINES

## Question 3

Intent of Question

The primary goals of this question were to assess a student's ability to (1) perform a probability calculation from a normal distribution; (2) explain an implication of examining the distribution of a sample mean rather than the distribution of a single measurement; and (3) perform a probability calculation involving independent events using the multiplication rule.

Solution

## Part (a):

Because the distribution of the daily number of absences is approximately normal with mean 120 students and standard deviation 10.5 students, the z-score for an absence total of 140 students is

$$z = \frac{140 - 120}{10.5} \approx 1.90. \text{ The table of standard normal probabilities or a calculator reveals that the}$$

probability that 140 or fewer students are absent is 0.9713. So the probability that more than 140 students are absent (and that the school will lose some state funding) is  $1 - 0.9713 = 0.0287$ .

## Part (b):

High School A would be *less* likely to lose state funding. With a random sample of 3 days, the distribution of the sample mean number of students absent would have less variability than that of a single day. With less variability, the distribution of the sample mean would concentrate more narrowly around the mean of 120 students, resulting in a smaller probability that the mean number of students absent would exceed 140.

In particular, the standard deviation of the sample mean number of absences,  $\bar{x}$ , is

$$\frac{\sigma}{\sqrt{n}} = \frac{10.5}{\sqrt{3}} = 6.062. \text{ So the z-score for a sample mean of 140 is } \frac{140 - 120}{6.062} \approx 3.30. \text{ The probability that}$$

High School A loses funding using the suggested plan would be  $1 - 0.9995 = 0.0005$ , as determined from the table of standard normal probabilities or from a calculator, which is less than a probability of 0.0287 obtained for the plan described in part (a).

## Part (c):

For any one typical school week, the probability is  $\frac{2}{5} = 0.4$  that the day selected is not Tuesday, not

Wednesday, or not Thursday. Therefore, because the days are selected independently across the three weeks, the probability that none of the three days selected would be a Tuesday or Wednesday or

Thursday is  $(0.4)^3 = 0.064$ .

## 2014 SCORING GUIDELINES

## Question 3 (continued)

## Scoring

Parts (a), (b), and (c) were scored as essentially correct (E), partially correct (P), or incorrect (I).

**Part (a)** is scored as follows:

Essentially correct (E) if the response provides the following three components:

1. Indicates use of a normal distribution and clearly identifies the correct parameter values (showing correct components of a z-score calculation is sufficient).
2. Uses the correct boundary value (140, 140.5, or 141 is acceptable).
3. Reports the correct normal probability consistent with components 1 and 2

OR

if the response reports a probability of 0.025 with justification based on the empirical rule for an acceptable boundary value (140, 140.5, or 141 is acceptable).

Partially correct (P) if the response correctly provides only two of the three components listed above.

OR

if the response provides an incorrect probability of 0.05 with justification based on the empirical rule for an acceptable boundary value (140, 140.5, or 141 is acceptable).

Incorrect (I) if the response does not satisfy the criteria for E or P.

*Note:* An inconsistency in calculations lowers the score for part (a) by one level (that is, from E to P or from P to I).

**Part (b)** is scored as follows:

Essentially correct (E) if the response provides the correct answer of less likely *AND* the following three components:

1. Clearly references the distribution of the sample mean.
2. Indicates that the variability of the distribution is smaller.
3. Indicates that the distribution is centered at 120.

OR

if the response provides the correct answer of less likely *AND* the following two components:

1. Correctly calculates the probability that the sample mean would exceed 140 (arithmetic errors are not penalized).
2. Correctly compares this probability to the probability in part (a).

Partially correct (P) if the response provides the correct answer of less likely *AND* only two of the following three components:

1. Clearly references the distribution of the sample mean.
2. Indicates that the variability of the distribution is smaller.
3. Indicates that the distribution is centered at 120.

OR

if the response provides the correct answer of less likely *AND* correctly calculates the probability that the sample mean would exceed 140 (arithmetic errors are not penalized) *BUT* does not correctly compare this probability with the probability in part (a).

## 2014 SCORING GUIDELINES

## Question 3 (continued)

Incorrect (I) if the response does not meet the criteria for E or P, including if the response provides the incorrect answer or provides the correct answer of less likely with no explanation or an incorrect explanation.

*Note:* An equivalent approach is to use the total number of absences for 3 days. The sampling distribution of the total number of absences for the 3 days is approximately normal, with mean

$3(120) = 360$  absences and standard deviation  $3(6.026) \approx 18.187$  absences. The z-score for a total of

$3(140) = 420$  absences is:  $\frac{420 - 360}{18.187} \approx 3.30$ . Such a response is scored E if the response provides the

correct answer of less likely and references the distribution of the sample total, and includes the correct mean and standard deviation.

**Part (c)** is scored as follows:

Essentially correct (E) if the response correctly calculates the probability *AND* shows sufficient work.

Partially correct (P) if the response reports the correct probability but shows no work or does not show sufficient work;

OR

if the response uses the multiplication rule involving three events but does so incorrectly and/or with an incorrect probability of not selecting a Tuesday, Wednesday, or Thursday.

Incorrect (I) if the response does not meet the criteria for E or P.

## 4 Complete Response

All three parts essentially correct

## 3 Substantial Response

Two parts essentially correct and one part partially correct

## 2 Developing Response

Two parts essentially correct and one part incorrect

OR

One part essentially correct and one or two parts partially correct

OR

Three parts partially correct

## 1 Minimal Response

One part essentially correct and two parts incorrect

OR

Two parts partially correct and one part incorrect

## 2014 SCORING GUIDELINES

## Question 4

Intent of Question

The primary goals of this question were to assess a student's ability to (1) describe why the median might be preferred to the mean in a particular context; (2) compare the relative merits of two sampling plans; and (3) describe a consequence of nonresponse in a particular study.

Solution**Part (a):**

The median is less affected by skewness and outliers than the mean. With a variable such as income, a small number of very large incomes could dramatically increase the mean but not the median. Therefore, the median would provide a better estimate of a typical income value.

**Part (b):**

Method 2 is better than Method 1. A sample obtained from Method 1 could be biased because of the voluntary nature of the response. It is plausible that class members with larger incomes might be more likely to return the form than class members with smaller incomes. The mean income for such a sample would overestimate the mean income of all class members. With Method 2, despite the smaller sample size, the random selection is likely to result in a sample that is more representative of the entire class and produce an unbiased estimate of mean yearly income of all class members.

## 2014 SCORING GUIDELINES

## Question 4 (continued)

Scoring

This question is scored in three sections. Part (b) has two components: (1) identifying a relevant characteristic for each sampling method; (2) indicating the effect of the biased method on the estimate of the mean income. Section 1 consists of Part (a); section 2 consists of part (b), component 1; and section 3 consists of part (b), component 2. Sections 1, 2, and 3 are scored as essentially correct (E), partially correct (P), or incorrect (I).

**Section 1** is scored as follows:

Essentially correct (E) if the response includes the following two components:

1. Describes how skewness or outliers affect the mean or do not affect the median.
2. Makes a conjecture about a relevant characteristic of the distribution of incomes, such as skewness or an outlier.

Partially correct (P) if the response includes only one of the two components listed above.

Incorrect (I) if the response does not meet the criteria for E or P.

*Notes:*

- For Component 1, examples of responses that are acceptable include:
  - The mean is affected by skewness (outliers).
  - The median is not affected by skewness (outliers).
  - The mean is greater (less) than the median when there is right (left) skewness or outliers.
- For Component 1, examples of responses that are not acceptable include:
  - Don't use the mean for skewed distributions or distributions with outliers.
  - Use the median for skewed distributions or distributions with outliers.
  - Responses that include an incorrect statement about means and/or medians, such as for right skewed distributions, the median will be higher than the mean.
- It is possible to satisfy *both* components with a single sentence, such as, "If there was a billionaire in the sample, the mean would be higher than the median."
- If a response argues that using the *mean* is a more appropriate way to estimate the typical income, then reduce the score in section 1 by one level (that is, from E to P or from P to I).

**Section 2** is scored as follows:

Essentially correct (E) if the response chooses Method 2 *AND* includes the following two components:

1. Identifies a relevant characteristic of Method 1.
2. Identifies a relevant characteristic of Method 2.

Partially correct (P) if the response chooses Method 2 *AND* includes only one of the two components listed above

*OR*

if the response includes both components but does not choose a method.

Incorrect (I) if the response chooses Method 1 *OR* otherwise does not meet the criteria for E or P.

## 2014 SCORING GUIDELINES

### Question 4 (continued)

#### Notes:

- Responses that do not explicitly choose Method 2 can still earn an E for section 2 if the choice is clearly implied. The choice of Method 2 is clearly implied if the response only discusses negative characteristics of Method 1 and only discusses positive characteristics of Method 2, such as, Method 1 is biased but Method 2 uses a random sample.
- Responses that *compare* the two methods can satisfy *both* components, such as, saying that Method 1 is more biased or that nonresponse will be less of a problem with Method 2.
- Responses that refer to the nonresponse bias as *voluntary response bias*, *response bias*, *undercoverage* can still earn an E.
- Discussions of conditions for inference should be considered extraneous and ignored.

**Section 3** is scored as follows:

Essentially correct (E) if the response includes the following two components:

1. Indicates the incomes of responders may be different from the incomes of nonresponders.
2. Indicates the biased sampling method may produce a misleading estimate/conclusion about the mean income, including direction, for example, "The sample mean is likely to be higher than the mean of the population."

Partially correct (P) if the response provides only one of the two components listed above.

Incorrect (I) if the response does not meet the criteria for E or P.

#### Notes:

- A single sentence can satisfy the first component of section 2 and the first component of section 3. (For example, "In method 1, rich people are more likely to respond.")
- For component 2, either direction is acceptable but the direction must be consistent with the identified bias. Saying only that Method 2 will be more accurate or more representative does not satisfy component 2.
- If a response addresses possible nonresponse bias in Method 2, the response can still satisfy both components of section 3.
- Responses that focus on the larger sample size in Method 1 can satisfy component 2 if such responses describe the effect as reducing the variability of the estimate. (For example, "I would use Method 1 since the larger sample size would give less variability of the mean.")
- Responses that focus on untruthful survey answers can satisfy component 2 if the effect on the estimate is appropriate. (For example, "People contacted in Method 2 might say they make more money than they actually do. This would make the estimated mean too high.")

## 2014 SCORING GUIDELINES

### Question 4 (continued)

#### 4 Complete Response

All three sections essentially correct

#### 3 Substantial Response

Two sections essentially correct and one section partially correct

#### 2 Developing Response

Two sections essentially correct and one section incorrect

OR

One section essentially correct and one or two sections partially correct

OR

Three sections partially correct

#### 1 Minimal Response

One section essentially correct and two sections incorrect

OR

Two sections partially correct and one section incorrect

## 2014 SCORING GUIDELINES

## Question 5

**Intent of Question**

The primary goal of this question was to assess students' ability to identify, set up, perform, and interpret the results of an appropriate hypothesis test to address a particular question. More specific goals were to assess students' ability to (1) state appropriate hypotheses; (2) identify the appropriate statistical test procedure and check appropriate conditions for inference; (3) calculate the appropriate test statistic and  $p$ -value; and (4) draw an appropriate conclusion, with justification, in the context of the study.

**Solution**

Step 1: States a correct pair of hypotheses.

Let  $\mu_{\text{diff}}$  represent the population mean difference in purchase price (woman – man) for identically equipped cars of the same model, sold to both men and women by the same dealer, in the county.

The hypotheses to be tested are  $H_0 : \mu_{\text{diff}} = 0$  versus  $H_a : \mu_{\text{diff}} > 0$ .

Step 2: Identifies a correct test procedure (by name or by formula) and checks appropriate conditions.

The appropriate procedure is a paired  $t$ -test.

The conditions for the paired  $t$ -test are:

1. The sample is randomly selected from the population.
2. The population of price differences (woman – man) is normally distributed, or the sample size is large.

The first condition is met because the car models and the individuals were randomly selected. The sample size ( $n = 8$ ) is not large, so we need to investigate whether it is reasonable to assume that the population of price differences is normally distributed. The dotplot of sample price differences reveals a fairly symmetric distribution, so we will consider the second condition to be met.

Step 3: Correct mechanics, including the value of the test statistic and  $p$ -value (or rejection region).

$$\text{The test statistic is } t = \frac{585 - 0}{\frac{530.71}{\sqrt{8}}} \approx 3.12.$$

The  $p$ -value, based on a  $t$ -distribution with  $8 - 1 = 7$  degrees of freedom, is 0.008.

Step 4: States a correct conclusion in the context of the study, using the result of the statistical test.

Because the  $p$ -value is very small (for instance, smaller than  $\alpha = 0.05$ ), we reject the null hypothesis. The data provide convincing evidence that, on average, women pay more than men in the county for the same car model.

## 2014 SCORING GUIDELINES

## Question 5 (continued)

**Scoring**

Each of steps 1, 2, 3, and 4 were scored as essentially correct (E), partially correct (P), or incorrect (I).

**Step 1** is scored as follows:

Essentially correct (E) if the response identifies the correct parameter *AND* states correct hypotheses.

Partially correct (P) if the response identifies the correct parameter *OR* states correct hypotheses, but not both.

Incorrect (I) if the response does not meet the criteria for E or P.

*Note:* Defining the parameter symbol in context or simply using common parameter notation is sufficient.

**Step 2** is scored as follows:

Essentially correct (E) if the response identifies the correct test procedure (by name or by formula) *AND* checks *both* conditions correctly.

Partially correct (P) if the response correctly completes two of the three components (identification of procedure, check of randomness condition, check of normality condition).

Incorrect (I) if the response does not meet the criteria for E or P.

*Note:* The random sampling condition can be verified by referring to the random selection of car models or to the random selection of male and female car buyers.

**Step 3** is scored as follows:

Essentially correct (E) if the response correctly calculates both the test statistic and the  $p$ -value.

Partially correct (P) if the response correctly calculates the test statistic but not the  $p$ -value;  
*OR*

if the response calculates the test statistic incorrectly but then calculates the correct  $p$ -value for the computed test statistic.

Incorrect (I) if the response does not meet the criteria for E or P.

*Note:* If the response identifies a  $z$ -test for a mean as the correct procedure in step 2, then the response can earn a P in step 3 if both the test statistic and the  $p$ -value are calculated correctly.

## 2014 SCORING GUIDELINES

## Question 5 (continued)

Step 4 is scored as follows:

Essentially correct (E) if the response provides a correct conclusion in context, also providing justification based on linkage between the  $p$ -value and the conclusion.

Partially correct (P) if the response provides a correct conclusion with linkage to the  $p$ -value, but not in context;

OR

if the response provides a correct conclusion in context, but without justification based on linkage to the  $p$ -value.

Incorrect (I) if the response does not meet the criteria for E or P.

Notes:

- If the conclusion is consistent with an incorrect  $p$ -value from step 3 and also in context with justification based on linkage to the  $p$ -value, step 4 is scored as E.
- A response that performs a two-sample  $t$ -test with correct calculations should fail to reject  $H_0$ . A conclusion that is equivalent to “accept  $H_0$ ” (such as “we conclude that women pay the same amount as men, on average”), either as a stated decision or as a conclusion in context, cannot be scored as E. Such a response will be scored as P provided that the conclusion is in context with linkage. Such a response will be scored as I if it lacks either context or linkage.

Each essentially correct (E) step counts as 1 point. Each partially correct (P) step counts as  $\frac{1}{2}$  point.

- 4 Complete Response**
- 3 Substantial Response**
- 2 Developing Response**
- 1 Minimal Response**

If a response is between two scores (for example,  $2\frac{1}{2}$  points), use a holistic approach to decide whether to score up or down, depending on the overall strength of the response and communication.

## 2014 SCORING GUIDELINES

## Question 6

Intent of Question

The primary goals of this question were to assess a student's ability to (1) calculate and interpret a residual value; (2) answer questions about residual plots; (3) compare associations between two scatterplots; and (4) identify an appropriate explanatory variable to include in a regression model based on residuals from simpler regression models.

Solution**Part (a):**

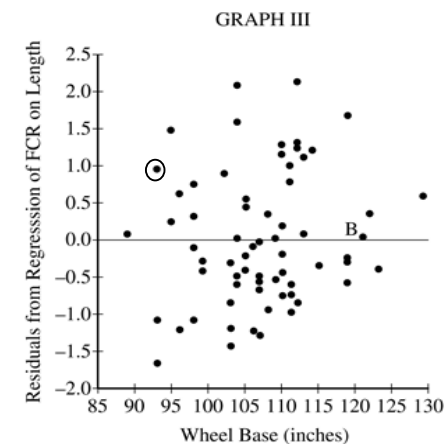
For a car with length 175 inches, the predicted value for the car's FCR, based on the least squares regression line, is

$$\text{predicted FCR} = -1.595789 + 0.0372614(175) \approx 4.92 \text{ gallons per 100 miles.}$$

The actual FCR for the car is 5.88, so the residual is  $5.88 - 4.92 = 0.96$ . The residual value means that the car's FCR is 0.96 gallons per 100 miles greater than would be predicted for a car of its length.

**Part (b):**

- (i) The point with a wheel base of 93 inches and a residual of 0.96 gallons per 100 miles is circled in graph III below.



- (ii) Point B corresponds to a car with an actual FCR that is very close to the FCR that would be predicted for a car with its length by the regression model which predicts FCR using the explanatory variable length.

## 2014 SCORING GUIDELINES

## Question 6 (continued)

**Part (c):**

Graph II reveals a moderate association that is positive and linear. In contrast, there is a weak association that is positive and linear in graph III. The association between engine size and residual (from predicting FCR based on length) is stronger than the association between wheel base and residual (from predicting FCR based on length).

**Part (d):**

Engine size is a better choice than wheel base for including with length in a regression model for predicting FCR. The stronger association between engine size and residual (from predicting FCR based on length) indicates that engine size is more useful than wheel base for reducing the variability in FCR values that remains unexplained (as indicated by residuals) after predicting FCR based on length.

**Scoring**

Parts (a), (b), (c), and (d) were scored as essentially correct (E), partially correct (P), or incorrect (I).

**Part (a)** is scored as follows:

Essentially correct (E) if the response provides the following two components:

1. A correct residual value with supporting calculation.
2. A correct interpretation of the residual value, in context.

Partially correct (P) if the response includes only one of the two components listed above.

Incorrect (I) if the response does not meet the criteria for E or P.

*Notes:*

- If the residual value is incorrect, the interpretation should be considered correct if it follows from the incorrect residual value.
- Correct interpretation of the residual must include the correct direction and magnitude of the FCR value away from the predicted FCR value.
- A calculated residual value which is slightly different from 0.96 due to the number of significant digits is acceptable.

**Part (b)** is scored as follows:

Essentially correct (E) if the response provides the following two components:

1. Circles the correct point in graph III.
2. Provides a reasonable interpretation of the car associated with point B having a residual near 0 that refers to predicting FCR based on length.

Partially correct (P) if the response correctly provides only one of the two components listed above.

Incorrect (I) if the response does not meet the criteria for E or P.

## 2014 SCORING GUIDELINES

## Question 6 (continued)

*Note:* A correct response for the second component must include reference to the observed FCR value of the car represented by point B, not the point B itself.

**Part (c)** is scored as follows:

Essentially correct (E) if the response correctly provides the following three components:

1. A description of form *AND* direction for both graphs.
2. A description of the strength of association for both graphs.
3. A comparison between the two graphs.

Partially correct (P) if the response correctly provides only two of the three components listed above.

Incorrect (I) if the response does not meet the criteria for E or P.

*Notes:*

- Part (c) is focused on the comparison of graph II and graph III. Inferences drawn from patterns in these graphs are considered in part (d).
- Linear is needed for form in graph II.
- Graph III may be described as having no association between wheel base and the residuals of FCR based on length, which is sufficient for describing the form, direction and strength of association of graph III.

**Part (d)** is scored as follows:

Essentially correct (E) if the response indicates the correct choice with a sound justification based on the following two components:

1. The strong(er) association.
2. Reducing the variability that remains unexplained in the model which predicts FCR based on length.

Partially correct (P) if the response indicates the correct choice and provides a justification based on only one of the two components which are listed above.

Incorrect (I) if the response indicates the incorrect choice;

*OR*

if the response indicates the correct choice but does not mention either of the two components which are listed above.

*Note:* Describing the variables in graph II and graph III as residuals is not required but can be used positively in holistic scoring. Incorrect descriptions of graph II or graph III or the variables in graphs are not acceptable.

**2014 SCORING GUIDELINES****Question 6 (continued)**

Each essentially correct (E) part counts as 1 point. Each partially correct (P) part counts as  $\frac{1}{2}$  point.

- 4      Complete Response**
- 3      Substantial Response**
- 2      Developing Response**
- 1      Minimal Response**

If a response is between two scores (for example,  $2\frac{1}{2}$  points), use a holistic approach to decide whether to score up or down, depending on the overall strength of the response and communication.