Week 24

AP Statistics

Homework bundle for this week

本周作业合集

exercises.lol

Contents 目录

7. Inference for Quantitative Data: Means

Starter Quiz · 课前小测

AP Statistics Name: ______________ Score: ________

1. Inference about a mean uses the t-distribution (not z) mainly because...
   - ☐ the population σ is unknown and estimated by s
   - ☐ means are always skewed
   - ☐ z is illegal
   - ☐ the sample is too big
2. For a one-sample t-interval with n = 25, what are the degrees of freedom (n − 1)?

   ____
3. A 95% interval for a mean is (145.9, 154.1). The claim $\mu = 160$ is...
   - ☐ not plausible (outside the interval)
   - ☐ plausible (inside)
   - ☐ proven true
   - ☐ impossible to judge
4. The null hypothesis for a one-mean test has the form...
   - ☐ H0: $\mu = \mu 0$
   - ☐ H0: x-bar = 0
   - ☐ Ha: $\mu \neq \mu 0$
   - ☐ H0: s = 0
5. With p-value = 0.032 and $\alpha = 0.05$, the decision is...
   - ☐ reject H0
   - ☐ fail to reject H0
   - ☐ accept H0
   - ☐ raise α
6. Measuring the same runners before and after training is which design?
   - ☐ paired (matched pairs)
   - ☐ two independent samples
   - ☐ a one-proportion study
   - ☐ a census

7. s1=6,n1=25 and s2=8,n2=25. Compute SE = $\sqrt{}(36/25 + 64/25)$.

____

8. Sampling variability is a mistake in data collection that must be fixed.

☐ True ☐ False

9. s = 10, n = 25. Compute the standard error of the mean s/√n.

____

10. '95% confidence' means there is a 95% probability that μ is in this particular interval.

☐ True ☐ False

AP Statistics

1. **the population σ is unknown and estimated by s**

   Estimating σ with s adds uncertainty $\rightarrow$ use t.

2. **24**

   df $=$ n $-$ 1 $=$ 25 $-$ 1 $=$ 24.

3. **not plausible (outside the interval)**

   160 lies outside $\rightarrow$ evidence against it.

4. **H0: $\mu = \mu$0**

   H0 states a specific claimed mean μ0.

5. **reject H0**

   0.032 $\leq$ 0.05 $\rightarrow$ reject H0.

6. **paired (matched pairs)**

   Same subjects measured twice $\rightarrow$ paired.

7. **2**

   √(1.44 + 2.56) = √4 = 2.

8. **False**

   Variability is natural and quantifiable — not a blunder.

9. **2**

   10/√25 = 10/5 = 2.

10. **False**

    μ is fixed; confidence describes the method over many samples.

AP STATISTICS • EXERCISE SHEET

7.1 Introducing Statistics: Should I Worry About Error?

Name: ______________ Class: __________ Date: ____________ **Total: 8 marks**

KEY WORDS distribution 分布 • degrees of freedom 自由度

Objective

Build the skills to answer exam questions on **inference for a mean and the t-distribution**.

You must be able to:

- explain why inference for a mean uses the **t-distribution** t 分布 when σ is unknown
- state that the sample standard deviation s estimates σ
- describe how the t-distribution approaches the normal as n grows

1 Worked examples

Study these first. Each one shows the method for a question type used later.

■ Why t, not z

When the population standard deviation σ is **unknown** (almost always), we estimate it with the **sample** standard deviation s. This extra uncertainty means we use the **t-distribution** instead of the normal.

■ The t-distribution

It is symmetric but has **heavier tails** than the normal, with **degrees of freedom** $= n - 1$. As n grows, it approaches the normal distribution.

2 Practice

2.1 State why we use the t-distribution instead of z for means. [1]

2.2 State what replaces σ when it is unknown. [1]

2.3 State what happens to the t-distribution as n increases. [1]

3 Exam-style questions

3.1 When the population σ is unknown, inference for a mean uses the [1]

- **A** z-distribution
- **B** t-distribution
- **C** binomial distribution
- **D** uniform distribution

3.2 The t-distribution is [1]

- **A** narrower than the normal
- **B** wider than the normal (heavier tails)
- **C** always identical to the normal
- **D** skewed

3.3 A sample of size 20 estimates a mean, with σ unknown.

(a) Name the distribution used. [1]

(b) State what estimates σ. [1]

(c) State the degrees of freedom. [1]

Solutions

2.1 because σ is unknown and estimated by s, adding uncertainty.

2.2 the sample standard deviation s.

2.3 it approaches the normal distribution.

3.1 **B**.

3.2 **B**.

3.3 (a) the t-distribution. (b) the sample standard deviation s. (c) $n - 1 = 19$.

AP STATISTICS • EXERCISE SHEET

7.2 Constructing a Confidence Interval for a Population Mean

Name: ________ Class: ________ Date: ________ **Total: 9 marks**

KEY WORDS degrees of freedom 自由度

Objective

Build the skills to answer exam questions on **a confidence interval for a population mean**.

You must be able to:

- construct the interval $\bar{x} \pm t^* \dfrac{s}{\sqrt{n}}$
- use degrees of freedom $n - 1$
- compute the margin of error

1 Worked examples

Study these first. Each one shows the method for a question type used later.

■ The confidence interval

$$\bar{x} \pm t^* \frac{s}{\sqrt{n}},$$

where $\dfrac{s}{\sqrt{n}}$ is the **standard error**, the **margin of error** is $t^* \dfrac{s}{\sqrt{n}}$, and the degrees of freedom are $n - 1$.

■ Example

$\bar{x} = 50$, $s = 10$, $n = 25$, $t^* = 2.06$: $\mathrm{SE} = \dfrac{10}{5} = 2$, $\mathrm{ME} = 2.06 \times 2 = 4.12$.

2 Practice

2.1 Write the confidence-interval formula for a mean. [1]

2.2 State the degrees of freedom for a one-sample t interval with $n = 25$. [1]

2.3 $\bar{x} = 50$, $s = 10$, $n = 25$, $t^* = 2.06$. Find the margin of error. [2]

3 Exam-style questions

3.1 A confidence interval for a mean uses the ________ critical value. [1]

- **A** z^*
- **B** t^*
- **C** p
- **D** χ^2

3.2 The degrees of freedom for a one-sample t interval are [1]

- **A** n
- **B** $n - 1$
- **C** $n + 1$
- **D** $2n$

3.3 $\bar{x} = 80$, $s = 12$, $n = 36$, $t^* = 2.03$.

(a) Find the standard error $\frac{s}{\sqrt{n}}$. [1]

(b) Find the margin of error. [2]

Solutions

2.1 $\bar{x} \pm t^* \dfrac{s}{\sqrt{n}}$.

2.2 $n - 1 = 24$.

2.3 $\mathrm{ME} = 2.06 \times \dfrac{10}{\sqrt{25}} = 2.06 \times 2 = 4.12$.

3.1 **B**.

3.2 **B**.

3.3 (a) $\dfrac{12}{\sqrt{36}} = 2$. (b) $\mathrm{ME} = 2.03 \times 2 = 4.06$.

Begin your response to **QUESTION 6** on this page.

SECTION II, Part B

Suggested Time—25 minutes

1 Question

Directions: Show all your work. Indicate clearly the methods you use, because you will be scored on the correctness of your methods as well as on the accuracy and completeness of your results and explanations.

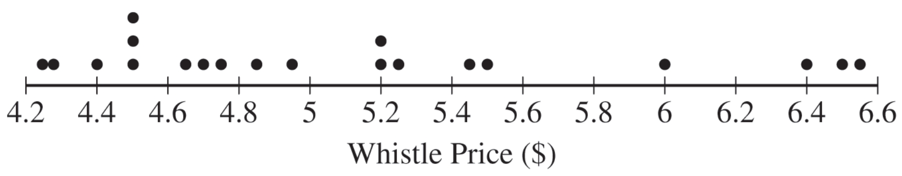
6. A company sells a certain type of whistle. The price of the whistle varies from store to store. Julio, a statistician at the company, wants to estimate the mean price, in dollars ($), of this type of whistle at all stores that sell the whistle.

(a) (i) Identify the appropriate inference procedure for Julio to use.

(ii) Describe the parameter for the inference procedure you identified in part (a-i) in context.

Julio called the managers of 20 randomly selected stores that sell the whistle and recorded the price of the whistle at each store. Following is a dotplot of Julio's data.

Price of the Whistle at 20 Stores



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Continue your response to **QUESTION 6** on this page.

The summary statistics for Julio's data are shown in the following table.

Summary Statistics for Julio's Data

Sample Size	Mean	Standard Deviation	Minimum	Q_1	Median	Q_3	Maximum
20	5.12	0.743	4.25	4.51	4.885	5.475	6.58

(b) Julio wants to examine some characteristics of the distribution of the sample of whistle prices.

(i) Describe the shape of the distribution of the sample of whistle prices. Justify your response using appropriate values from the summary statistics table.

(ii) Using the $1.5 \times \text{IQR}$ rule, determine whether there are any outliers in the sample of whistle prices. Justify your response.

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Continue your response to **QUESTION 6** on this page.

It can often be difficult to determine whether the distribution of sample data is skewed by looking at a graph of the data and the summary statistics, particularly when the sample size is small. Thus, statisticians sometimes measure how skewed a data set is. One such measure is Pearson's coefficient of skewness, which is calculated using the following formula.

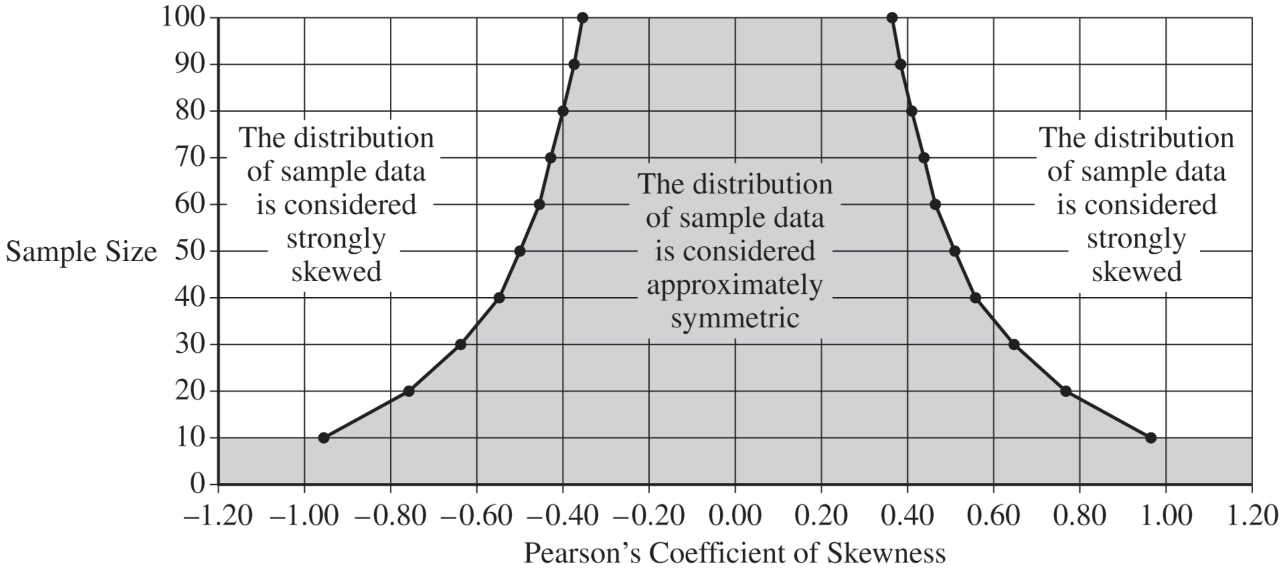
$$\text{Pearson's Coefficient of Skewness} = \frac{3(\bar{x} - m)}{s}$$

In the formula, $\bar{x}$ is the sample mean, m is the sample median, and s is the sample standard deviation.

(c) (i) Calculate Pearson's coefficient of skewness for Julio's sample of 20 whistle prices. Show your work.

The following graph shows conclusions that can be made about the shape of the distribution of sample data based on Pearson's coefficient of skewness and sample size.

Conclusion from Pearson's Coefficient of Skewness



(ii) Indicate the value of the Pearson's coefficient of skewness you calculated in part (c-i) for the appropriate sample size by marking it with an "X" on the preceding graph.

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Continue your response to **QUESTION 6** on this page.

(d) Consider your work in part (c).

(i) What should you conclude about the shape of the distribution of the sample of whistle prices? Justify your response.

Julio’s inference procedure in part (a-i) needs one of the following requirements to be satisfied to verify the normality condition.

- The sample size is greater than or equal to 30.
- If the sample size is less than 30, the distribution of the sample data is not strongly skewed and does not have outliers.

(ii) Using your response to (d-i) and the preceding requirements, is the normality condition satisfied for Julio’s data? Explain your response.

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STOP

END OF EXAM

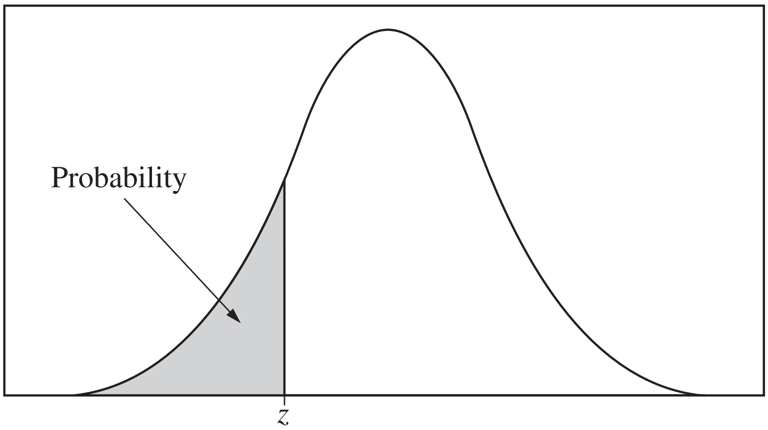


Table entry for z is the probability lying below z.

Table A Standard normal probabilities

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

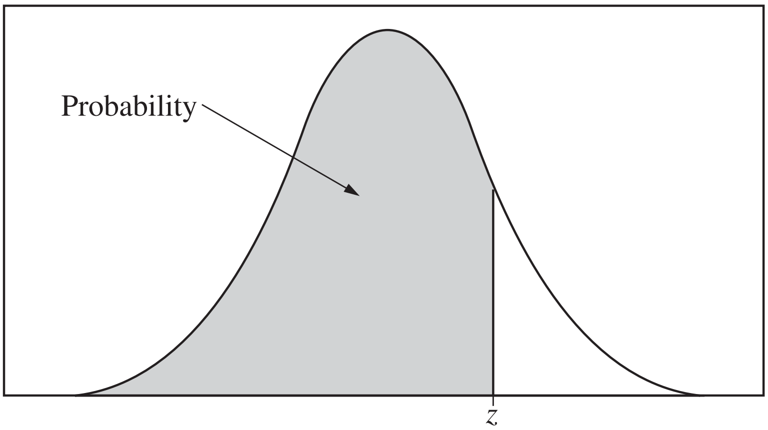


Table entry for z is the probability lying below z.

Table A ***(Continued)***

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

Table entry for p and C is the point t^* with probability p lying above it and probability C lying between $-t^*$ and t^*.

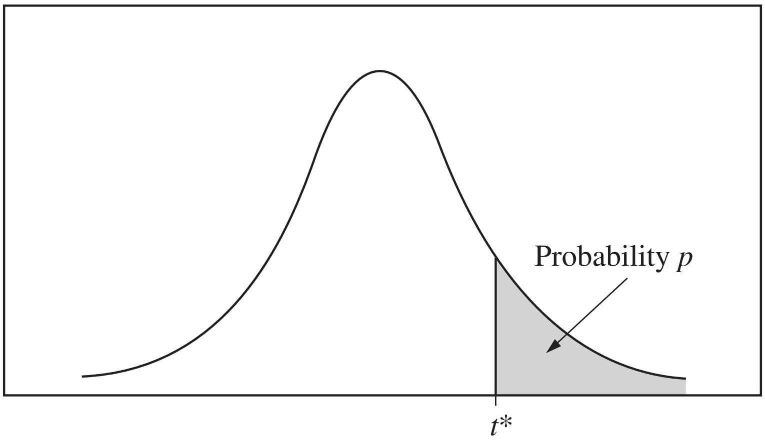


Table B ***t*** **distribution critical values**

	Tail probability p											
df	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.000	1.376	1.963	3.078	6.314	12.71	15.89	31.82	63.66	127.3	318.3	636.6
2	.816	1.061	1.386	1.886	2.920	4.303	4.849	6.965	9.925	14.09	22.33	31.60
3	.765	.978	1.250	1.638	2.353	3.182	3.482	4.541	5.841	7.453	10.21	12.92
4	.741	.941	1.190	1.533	2.132	2.776	2.999	3.747	4.604	5.598	7.173	8.610
5	.727	.920	1.156	1.476	2.015	2.571	2.757	3.365	4.032	4.773	5.893	6.869
6	.718	.906	1.134	1.440	1.943	2.447	2.612	3.143	3.707	4.317	5.208	5.959
7	.711	.896	1.119	1.415	1.895	2.365	2.517	2.998	3.499	4.029	4.785	5.408
8	.706	.889	1.108	1.397	1.860	2.306	2.449	2.896	3.355	3.833	4.501	5.041
9	.703	.883	1.100	1.383	1.833	2.262	2.398	2.821	3.250	3.690	4.297	4.781
10	.700	.879	1.093	1.372	1.812	2.228	2.359	2.764	3.169	3.581	4.144	4.587
11	.697	.876	1.088	1.363	1.796	2.201	2.328	2.718	3.106	3.497	4.025	4.437
12	.695	.873	1.083	1.356	1.782	2.179	2.303	2.681	3.055	3.428	3.930	4.318
13	.694	.870	1.079	1.350	1.771	2.160	2.282	2.650	3.012	3.372	3.852	4.221
14	.692	.868	1.076	1.345	1.761	2.145	2.264	2.624	2.977	3.326	3.787	4.140
15	.691	.866	1.074	1.341	1.753	2.131	2.249	2.602	2.947	3.286	3.733	4.073
16	.690	.865	1.071	1.337	1.746	2.120	2.235	2.583	2.921	3.252	3.686	4.015
17	.689	.863	1.069	1.333	1.740	2.110	2.224	2.567	2.898	3.222	3.646	3.965
18	.688	.862	1.067	1.330	1.734	2.101	2.214	2.552	2.878	3.197	3.611	3.922
19	.688	.861	1.066	1.328	1.729	2.093	2.205	2.539	2.861	3.174	3.579	3.883
20	.687	.860	1.064	1.325	1.725	2.086	2.197	2.528	2.845	3.153	3.552	3.850
21	.686	.859	1.063	1.323	1.721	2.080	2.189	2.518	2.831	3.135	3.527	3.819
22	.686	.858	1.061	1.321	1.717	2.074	2.183	2.508	2.819	3.119	3.505	3.792
23	.685	.858	1.060	1.319	1.714	2.069	2.177	2.500	2.807	3.104	3.485	3.768
24	.685	.857	1.059	1.318	1.711	2.064	2.172	2.492	2.797	3.091	3.467	3.745
25	.684	.856	1.058	1.316	1.708	2.060	2.167	2.485	2.787	3.078	3.450	3.725
26	.684	.856	1.058	1.315	1.706	2.056	2.162	2.479	2.779	3.067	3.435	3.707
27	.684	.855	1.057	1.314	1.703	2.052	2.158	2.473	2.771	3.057	3.421	3.690
28	.683	.855	1.056	1.313	1.701	2.048	2.154	2.467	2.763	3.047	3.408	3.674
29	.683	.854	1.055	1.311	1.699	2.045	2.150	2.462	2.756	3.038	3.396	3.659
30	.683	.854	1.055	1.310	1.697	2.042	2.147	2.457	2.750	3.030	3.385	3.646
40	.681	.851	1.050	1.303	1.684	2.021	2.123	2.423	2.704	2.971	3.307	3.551
50	.679	.849	1.047	1.299	1.676	2.009	2.109	2.403	2.678	2.937	3.261	3.496
60	.679	.848	1.045	1.296	1.671	2.000	2.099	2.390	2.660	2.915	3.232	3.460
80	.678	.846	1.043	1.292	1.664	1.990	2.088	2.374	2.639	2.887	3.195	3.416
100	.677	.845	1.042	1.290	1.660	1.984	2.081	2.364	2.626	2.871	3.174	3.390
1000	.675	.842	1.037	1.282	1.646	1.962	2.056	2.330	2.581	2.813	3.098	3.300
∞	.674	.841	1.036	1.282	1.645	1.960	2.054	2.326	2.576	2.807	3.091	3.291
	50%	60%	70%	80%	90%	95%	96%	98%	99%	99.5%	99.8%	99.9%
	Confidence level C											

Table entry for p is the point (χ^2) with probability p lying above it.

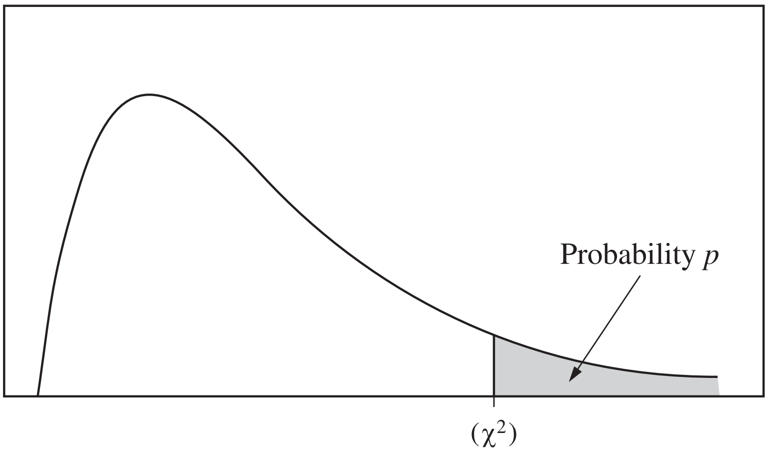


Table C χ^2 **critical values**

	Tail probability p											
df	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.32	1.64	2.07	2.71	3.84	5.02	5.41	6.63	7.88	9.14	10.83	12.12
2	2.77	3.22	3.79	4.61	5.99	7.38	7.82	9.21	10.60	11.98	13.82	15.20
3	4.11	4.64	5.32	6.25	7.81	9.35	9.84	11.34	12.84	14.32	16.27	17.73
4	5.39	5.99	6.74	7.78	9.49	11.14	11.67	13.28	14.86	16.42	18.47	20.00
5	6.63	7.29	8.12	9.24	11.07	12.83	13.39	15.09	16.75	18.39	20.51	22.11
6	7.84	8.56	9.45	10.64	12.59	14.45	15.03	16.81	18.55	20.25	22.46	24.10
7	9.04	9.80	10.75	12.02	14.07	16.01	16.62	18.48	20.28	22.04	24.32	26.02
8	10.22	11.03	12.03	13.36	15.51	17.53	18.17	20.09	21.95	23.77	26.12	27.87
9	11.39	12.24	13.29	14.68	16.92	19.02	19.68	21.67	23.59	25.46	27.88	29.67
10	12.55	13.44	14.53	15.99	18.31	20.48	21.16	23.21	25.19	27.11	29.59	31.42
11	13.70	14.63	15.77	17.28	19.68	21.92	22.62	24.72	26.76	28.73	31.26	33.14
12	14.85	15.81	16.99	18.55	21.03	23.34	24.05	26.22	28.30	30.32	32.91	34.82
13	15.98	16.98	18.20	19.81	22.36	24.74	25.47	27.69	29.82	31.88	34.53	36.48
14	17.12	18.15	19.41	21.06	23.68	26.12	26.87	29.14	31.32	33.43	36.12	38.11
15	18.25	19.31	20.60	22.31	25.00	27.49	28.26	30.58	32.80	34.95	37.70	39.72
16	19.37	20.47	21.79	23.54	26.30	28.85	29.63	32.00	34.27	36.46	39.25	41.31
17	20.49	21.61	22.98	24.77	27.59	30.19	31.00	33.41	35.72	37.95	40.79	42.88
18	21.60	22.76	24.16	25.99	28.87	31.53	32.35	34.81	37.16	39.42	42.31	44.43
19	22.72	23.90	25.33	27.20	30.14	32.85	33.69	36.19	38.58	40.88	43.82	45.97
20	23.83	25.04	26.50	28.41	31.41	34.17	35.02	37.57	40.00	42.34	45.31	47.50
21	24.93	26.17	27.66	29.62	32.67	35.48	36.34	38.93	41.40	43.78	46.80	49.01
22	26.04	27.30	28.82	30.81	33.92	36.78	37.66	40.29	42.80	45.20	48.27	50.51
23	27.14	28.43	29.98	32.01	35.17	38.08	38.97	41.64	44.18	46.62	49.73	52.00
24	28.24	29.55	31.13	33.20	36.42	39.36	40.27	42.98	45.56	48.03	51.18	53.48
25	29.34	30.68	32.28	34.38	37.65	40.65	41.57	44.31	46.93	49.44	52.62	54.95
26	30.43	31.79	33.43	35.56	38.89	41.92	42.86	45.64	48.29	50.83	54.05	56.41
27	31.53	32.91	34.57	36.74	40.11	43.19	44.14	46.96	49.64	52.22	55.48	57.86
28	32.62	34.03	35.71	37.92	41.34	44.46	45.42	48.28	50.99	53.59	56.89	59.30
29	33.71	35.14	36.85	39.09	42.56	45.72	46.69	49.59	52.34	54.97	58.30	60.73
30	34.80	36.25	37.99	40.26	43.77	46.98	47.96	50.89	53.67	56.33	59.70	62.16
40	45.62	47.27	49.24	51.81	55.76	59.34	60.44	63.69	66.77	69.70	73.40	76.09
50	56.33	58.16	60.35	63.17	67.50	71.42	72.61	76.15	79.49	82.66	86.66	89.56
60	66.98	68.97	71.34	74.40	79.08	83.30	84.58	88.38	91.95	95.34	99.61	102.7
80	88.13	90.41	93.11	96.58	101.9	106.6	108.1	112.3	116.3	120.1	124.8	128.3
100	109.1	111.7	114.7	118.5	124.3	129.6	131.1	135.8	140.2	144.3	149.4	153.2

7.3 Justifying a Claim About a Population Mean Based on a Confidence Interval

Name: ____________ Class: ________ Date: __________ **Total: 8 marks**

Objective

Build the skills to answer exam questions on **justifying a claim about a mean from a confidence interval**.

You must be able to:

- use a confidence interval to judge whether a claimed mean is **plausible**
- recognise a value inside the interval as plausible and one outside as not supported

1 Worked examples

Study these first. Each one shows the method for a question type used later.

■ Judging a claim about μ

The confidence interval gives the plausible values of the population mean:

- a claimed value **inside** the interval is **plausible**;
- a claimed value **outside** the interval is **not supported** by the data.

■ Example

A 95% interval of $(48, 52)$ contains 50, so "$\mu = 50$" is plausible; it does not contain 55, so "$\mu = 55$" is not supported.

2 Practice

2.1 State how to judge a claim about μ using a confidence interval. [1]

2.2 A 95% CI for μ is $(48, 52)$. State whether the claim "$\mu = 50$" is plausible. [1]

2.3 State what a claimed μ outside the interval implies. [1]

3 Exam-style questions

3.1 A claimed mean that lies inside the confidence interval is [1]

- **A** rejected
- **B** plausible
- **C** impossible
- **D** certainly correct

3.2 A 95% CI for μ is $(60, 70)$. The claim "$\mu = 55$" is [1]

- **A** plausible
- **B** not supported
- **C** certainly true
- **D** unknown

3.3 A 95% CI for the mean is $(10.2, 11.8)$.

(a) State whether "$\mu = 11$" is plausible. [1]

(b) State whether "$\mu = 12$" is plausible. [1]

(c) State the confidence level. [1]

Solutions

2.1 a value inside the interval is plausible; a value outside is not supported.

2.2 yes, plausible (50 is inside).

2.3 the data do not support that claim.

3.1 **B**.

3.2 **B**.

3.3 (a) yes. (b) no. (c) 95%.

5. According to a 2017 national survey in Country B, the mean number of bedrooms in newly built houses was 2.9. Rodney, a researcher, believes the mean number of bedrooms in newly built houses in the country was different in 2024 than it was in 2017. To investigate his belief, he took a large random sample of newly built houses in Country B in 2024 and recorded the number of bedrooms in each house. The distribution of the number of bedrooms for the sampled houses is summarized in the table.

Distribution of the Number of Bedrooms for the Houses Sampled in 2024

Number of Bedrooms	1	2	3	4	5	6
Proportion of Houses	0.12	0.22	0.28	0.22	0.14	0.02

A.

i. A house from the sample will be selected at random. What is the probability that the house had fewer than 3 bedrooms? Show your work.

ii. What is the mean number of bedrooms for the sample of newly built houses in 2024? Show your work.

B. Rodney will use a one-sample *t*-test for a population mean to test his belief.

i. In the context of Rodney's investigation, state the hypotheses for the test.

ii. Explain, in context, what a Type I error would be for Rodney's hypothesis test.

C. A different researcher, Keisha, suggests using a confidence interval to investigate whether the mean number of bedrooms in newly built houses in 2024 in Country B was different from 2.9.

Assume the conditions for inference have been met. Using Rodney's data, Keisha calculated a one-sample 97 percent confidence interval to estimate the population mean as $(3.01, 3.19)$. Based on the confidence interval, what conclusion can be made for Rodney's hypothesis test in part B at $\alpha = 0.03$? Justify your answer.

AP STATISTICS • EXERCISE SHEET

7.4 Setting Up a Test for a Population Mean

Name: ______________ Class: __________ Date: ____________ **Total: 10 marks**

KEY WORDS degrees of freedom 自由度

Objective

Build the skills to answer exam questions on **setting up a test for a population mean**.

You must be able to:

- state the hypotheses $H_0 : \mu = \mu_0$ and H_a
- write the test statistic $t = \dfrac{\bar{x} - \mu_0}{s/\sqrt{n}}$
- state the degrees of freedom $n - 1$

1 Worked examples

Study these first. Each one shows the method for a question type used later.

■ **Hypotheses**

- **Null** $H_0 : \mu = \mu_0$.
- **Alternative** $H_a : \mu \neq \mu_0$ (or $<$, or $>$).

■ **Test statistic**

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}, \qquad \mathrm{df} = n - 1.$$

■ **Example**

$\bar{x} = 52$, $\mu_0 = 50$, $s = 8$, $n = 16$: $t = \dfrac{52 - 50}{8/4} = \dfrac{2}{2} = 1.0$.

2 Practice

2.1 State the null hypothesis for a test about a mean. [1]

2.2 Write the test-statistic formula for a mean. [1]

2.3 $\bar{x} = 52$, $\mu_0 = 50$, $s = 8$, $n = 16$. Find the test statistic t. [2]

3 Exam-style questions

3.1 The test statistic for a mean is [1]

- **A** $\dfrac{\bar{x} - \mu_0}{s/\sqrt{n}}$
- **B** $\dfrac{\hat{p} - p_0}{\text{SE}}$
- **C** $\bar{x} \cdot s$
- **D** $\dfrac{\mu_0}{n}$

3.2 The degrees of freedom for a one-sample mean test are [1]

- **A** n
- **B** $n - 1$
- **C** $n + 1$
- **D** 0

3.3 A test of $\mu \neq 100$ has $\bar{x} = 104$, $s = 10$, $n = 25$.

(a) Write H_0. [1]

(b) Compute t. [2]

(c) State the degrees of freedom. [1]

Solutions

2.1 $H_0 : \mu = \mu_0$.

2.2 $t = \dfrac{\bar{x} - \mu_0}{s/\sqrt{n}}$.

2.3 $t = \dfrac{52 - 50}{8/\sqrt{16}} = \dfrac{2}{2} = 1.0$.

3.1 **A**.

3.2 **B**.

3.3 (a) $H_0 : \mu = 100$. (b) $t = \dfrac{104 - 100}{10/\sqrt{25}} = \dfrac{4}{2} = 2.0$. (c) 24.

5. According to a 2017 national survey in Country B, the mean number of bedrooms in newly built houses was 2.9. Rodney, a researcher, believes the mean number of bedrooms in newly built houses in the country was different in 2024 than it was in 2017. To investigate his belief, he took a large random sample of newly built houses in Country B in 2024 and recorded the number of bedrooms in each house. The distribution of the number of bedrooms for the sampled houses is summarized in the table.

Distribution of the Number of Bedrooms for the Houses Sampled in 2024

Number of Bedrooms	1	2	3	4	5	6
Proportion of Houses	0.12	0.22	0.28	0.22	0.14	0.02

A.

i. A house from the sample will be selected at random. What is the probability that the house had fewer than 3 bedrooms? Show your work.

ii. What is the mean number of bedrooms for the sample of newly built houses in 2024? Show your work.

B. Rodney will use a one-sample *t*-test for a population mean to test his belief.

i. In the context of Rodney's investigation, state the hypotheses for the test.

ii. Explain, in context, what a Type I error would be for Rodney's hypothesis test.

C. A different researcher, Keisha, suggests using a confidence interval to investigate whether the mean number of bedrooms in newly built houses in 2024 in Country B was different from 2.9.

Assume the conditions for inference have been met. Using Rodney's data, Keisha calculated a one-sample 97 percent confidence interval to estimate the population mean as $(3.01, 3.19)$. Based on the confidence interval, what conclusion can be made for Rodney's hypothesis test in part B at $\alpha = 0.03$? Justify your answer.

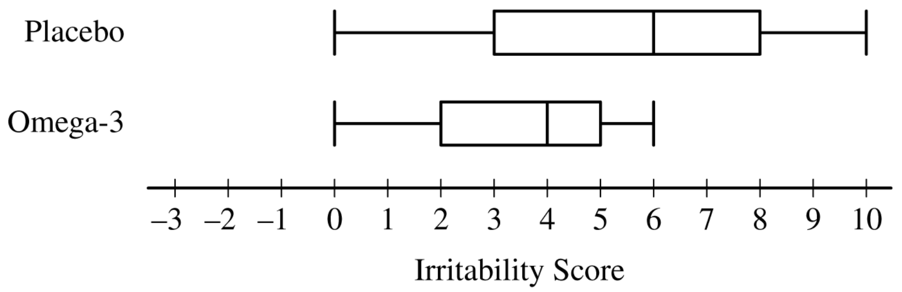
. A medical researcher completed a study comparing an omega-3 fatty acids supplement to a placebo in the treatment of irritability in patients with a certain medical condition. Nineteen patients with the medical condition volunteered to participate in the study. The study was conducted using the following weekly schedule.

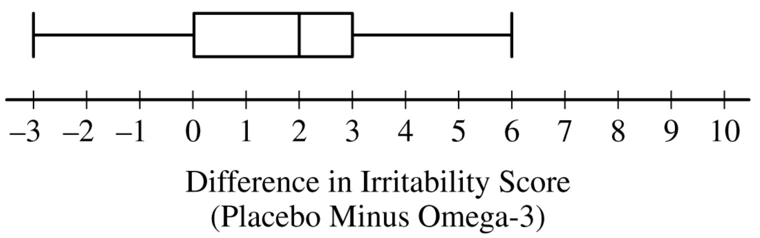
- Week 1: Each patient took a randomly assigned treatment, omega-3 supplement or placebo.
- Week 2: The patients did not take either the omega-3 supplement or the placebo. This was necessary to reduce the possibility of any carryover effect from the assigned treatment taken during week 1.
- Week 3: Each patient took the treatment, omega-3 supplement or placebo, that they did not take during week 1.

At the end of week 1 and week 3, each patient's irritability was given a score on a scale of 0 to 10, with 0 representing no irritability and 10 representing the highest level of irritability.

For each patient, the two irritability scores and the difference in their scores (placebo minus omega-3) were recorded. The results are summarized in the table and boxplots.

	n	Mean	Standard Deviation
Placebo	19	5.421	2.987
Omega-3	19	3.632	1.739
Difference (placebo minus omega-3)	19	1.789	2.485





GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 4** on this page.

The researcher claims the omega-3 supplement will decrease the mean irritability score of all patients with the medical condition similar to the volunteers who participated in the study. Is there convincing statistical evidence to support the researcher's claim at a significance level of $\alpha = 0.05$? Complete the appropriate inference procedure to support your answer.

GO ON TO THE NEXT PAGE.

STATISTICS
SECTION II
Part B
Question 6

Spend about 25 minutes on this part of the exam.

Percent of Section II score—25

Directions: Show all your work. Indicate clearly the methods you use, because you will be scored on the correctness of your methods as well as on the accuracy and completeness of your results and explanations.

6. Systolic blood pressure is the amount of pressure that blood exerts on blood vessels while the heart is beating. The mean systolic blood pressure for people in the United States is reported to be 122 millimeters of mercury (mmHg) with a standard deviation of 15 mmHg.

   The wellness department of a large corporation is investigating whether the mean systolic blood pressure of its employees is greater than the reported national mean. A random sample of 100 employees will be selected, the systolic blood pressure of each employee in the sample will be measured, and the sample mean will be calculated.

   Let μ represent the mean systolic blood pressure of all employees at the corporation. Consider the following hypotheses.

$$H_0 : \mu = 122$$
$$H_a : \mu > 122$$

   (a) Describe a Type II error in the context of the hypothesis test.

   (b) Assume that σ, the standard deviation of the systolic blood pressure of all employees at the corporation, is 15 mmHg. If $\mu = 122$, the sampling distribution of $\overline{x}$ for samples of size 100 is approximately normal with a mean of 122 mmHg and a standard deviation of 1.5 mmHg. What values of the sample mean $\overline{x}$ would represent sufficient evidence to reject the null hypothesis at the significance level of $\alpha = 0.05$?

   The actual mean systolic blood pressure of all employees at the corporation is 125 mmHg, not the hypothesized value of 122 mmHg, and the standard deviation is 15 mmHg.

   (c) Using the actual mean of 125 mmHg and the results from part (b), determine the probability that the null hypothesis will be rejected.

   (d) What statistical term is used for the probability found in part (c) ?

   (e) Suppose the size of the sample of employees to be selected is greater than 100. Would the probability of rejecting the null hypothesis be greater than, less than, or equal to the probability calculated in part (c) ? Explain your reasoning.

STOP

END OF EXAM

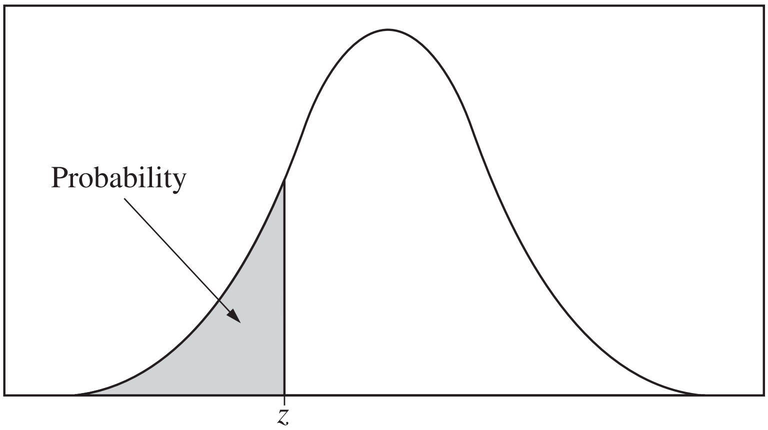


Table entry for z is the probability lying below z.

Table A Standard normal probabilities

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

Table entry for z is the probability lying below z.

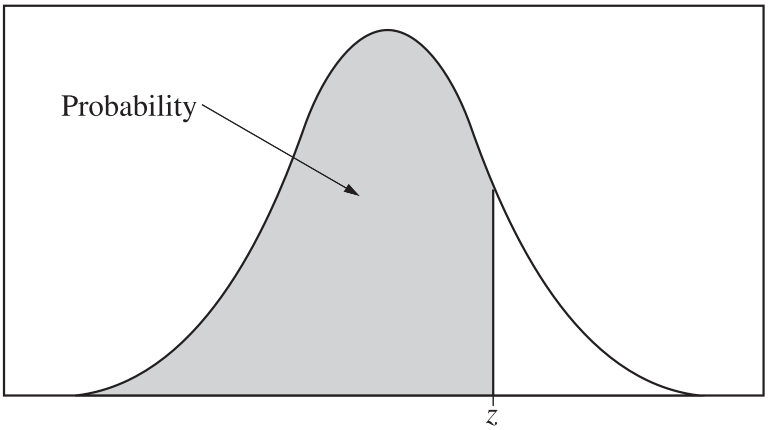


Table A ***(Continued)***

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

Table entry for p and C is the point t^* with probability p lying above it and probability C lying between $-t^*$ and t^*.

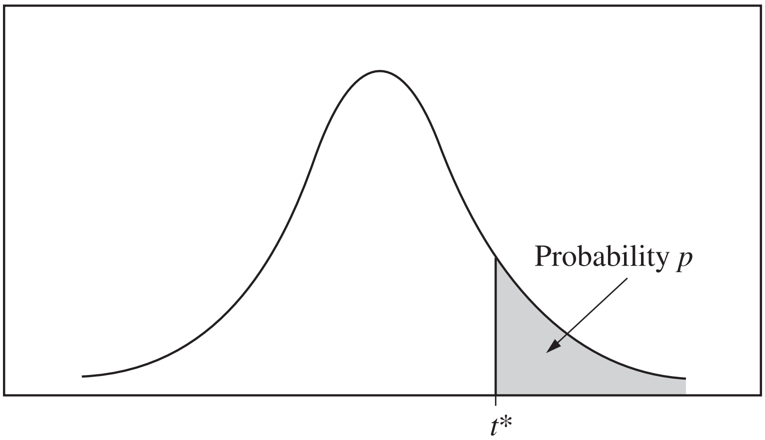


Table B *t* **distribution critical values**

	Tail probability p											
df	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.000	1.376	1.963	3.078	6.314	12.71	15.89	31.82	63.66	127.3	318.3	636.6
2	.816	1.061	1.386	1.886	2.920	4.303	4.849	6.965	9.925	14.09	22.33	31.60
3	.765	.978	1.250	1.638	2.353	3.182	3.482	4.541	5.841	7.453	10.21	12.92
4	.741	.941	1.190	1.533	2.132	2.776	2.999	3.747	4.604	5.598	7.173	8.610
5	.727	.920	1.156	1.476	2.015	2.571	2.757	3.365	4.032	4.773	5.893	6.869
6	.718	.906	1.134	1.440	1.943	2.447	2.612	3.143	3.707	4.317	5.208	5.959
7	.711	.896	1.119	1.415	1.895	2.365	2.517	2.998	3.499	4.029	4.785	5.408
8	.706	.889	1.108	1.397	1.860	2.306	2.449	2.896	3.355	3.833	4.501	5.041
9	.703	.883	1.100	1.383	1.833	2.262	2.398	2.821	3.250	3.690	4.297	4.781
10	.700	.879	1.093	1.372	1.812	2.228	2.359	2.764	3.169	3.581	4.144	4.587
11	.697	.876	1.088	1.363	1.796	2.201	2.328	2.718	3.106	3.497	4.025	4.437
12	.695	.873	1.083	1.356	1.782	2.179	2.303	2.681	3.055	3.428	3.930	4.318
13	.694	.870	1.079	1.350	1.771	2.160	2.282	2.650	3.012	3.372	3.852	4.221
14	.692	.868	1.076	1.345	1.761	2.145	2.264	2.624	2.977	3.326	3.787	4.140
15	.691	.866	1.074	1.341	1.753	2.131	2.249	2.602	2.947	3.286	3.733	4.073
16	.690	.865	1.071	1.337	1.746	2.120	2.235	2.583	2.921	3.252	3.686	4.015
17	.689	.863	1.069	1.333	1.740	2.110	2.224	2.567	2.898	3.222	3.646	3.965
18	.688	.862	1.067	1.330	1.734	2.101	2.214	2.552	2.878	3.197	3.611	3.922
19	.688	.861	1.066	1.328	1.729	2.093	2.205	2.539	2.861	3.174	3.579	3.883
20	.687	.860	1.064	1.325	1.725	2.086	2.197	2.528	2.845	3.153	3.552	3.850
21	.686	.859	1.063	1.323	1.721	2.080	2.189	2.518	2.831	3.135	3.527	3.819
22	.686	.858	1.061	1.321	1.717	2.074	2.183	2.508	2.819	3.119	3.505	3.792
23	.685	.858	1.060	1.319	1.714	2.069	2.177	2.500	2.807	3.104	3.485	3.768
24	.685	.857	1.059	1.318	1.711	2.064	2.172	2.492	2.797	3.091	3.467	3.745
25	.684	.856	1.058	1.316	1.708	2.060	2.167	2.485	2.787	3.078	3.450	3.725
26	.684	.856	1.058	1.315	1.706	2.056	2.162	2.479	2.779	3.067	3.435	3.707
27	.684	.855	1.057	1.314	1.703	2.052	2.158	2.473	2.771	3.057	3.421	3.690
28	.683	.855	1.056	1.313	1.701	2.048	2.154	2.467	2.763	3.047	3.408	3.674
29	.683	.854	1.055	1.311	1.699	2.045	2.150	2.462	2.756	3.038	3.396	3.659
30	.683	.854	1.055	1.310	1.697	2.042	2.147	2.457	2.750	3.030	3.385	3.646
40	.681	.851	1.050	1.303	1.684	2.021	2.123	2.423	2.704	2.971	3.307	3.551
50	.679	.849	1.047	1.299	1.676	2.009	2.109	2.403	2.678	2.937	3.261	3.496
60	.679	.848	1.045	1.296	1.671	2.000	2.099	2.390	2.660	2.915	3.232	3.460
80	.678	.846	1.043	1.292	1.664	1.990	2.088	2.374	2.639	2.887	3.195	3.416
100	.677	.845	1.042	1.290	1.660	1.984	2.081	2.364	2.626	2.871	3.174	3.390
1000	.675	.842	1.037	1.282	1.646	1.962	2.056	2.330	2.581	2.813	3.098	3.300
∞	.674	.841	1.036	1.282	1.645	1.960	2.054	2.326	2.576	2.807	3.091	3.291
	50%	60%	70%	80%	90%	95%	96%	98%	99%	99.5%	99.8%	99.9%
	Confidence level C											

Table entry for p is the point (χ^2) with probability p lying above it.

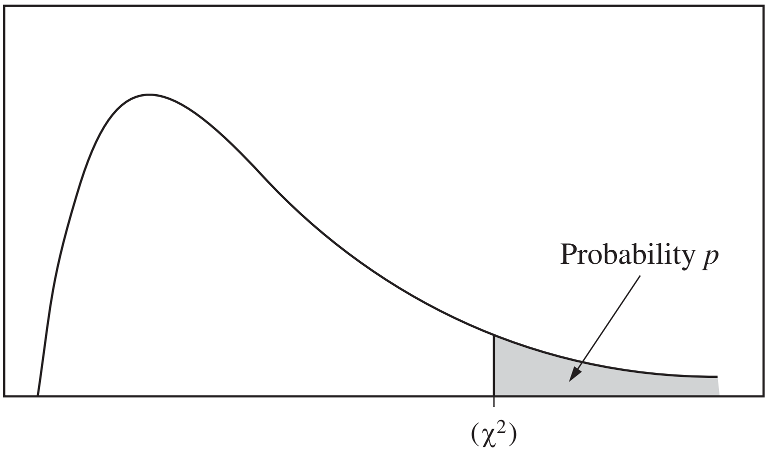


Table C χ^2 **critical values**

	Tail probability p											
df	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.32	1.64	2.07	2.71	3.84	5.02	5.41	6.63	7.88	9.14	10.83	12.12
2	2.77	3.22	3.79	4.61	5.99	7.38	7.82	9.21	10.60	11.98	13.82	15.20
3	4.11	4.64	5.32	6.25	7.81	9.35	9.84	11.34	12.84	14.32	16.27	17.73
4	5.39	5.99	6.74	7.78	9.49	11.14	11.67	13.28	14.86	16.42	18.47	20.00
5	6.63	7.29	8.12	9.24	11.07	12.83	13.39	15.09	16.75	18.39	20.51	22.11
6	7.84	8.56	9.45	10.64	12.59	14.45	15.03	16.81	18.55	20.25	22.46	24.10
7	9.04	9.80	10.75	12.02	14.07	16.01	16.62	18.48	20.28	22.04	24.32	26.02
8	10.22	11.03	12.03	13.36	15.51	17.53	18.17	20.09	21.95	23.77	26.12	27.87
9	11.39	12.24	13.29	14.68	16.92	19.02	19.68	21.67	23.59	25.46	27.88	29.67
10	12.55	13.44	14.53	15.99	18.31	20.48	21.16	23.21	25.19	27.11	29.59	31.42
11	13.70	14.63	15.77	17.28	19.68	21.92	22.62	24.72	26.76	28.73	31.26	33.14
12	14.85	15.81	16.99	18.55	21.03	23.34	24.05	26.22	28.30	30.32	32.91	34.82
13	15.98	16.98	18.20	19.81	22.36	24.74	25.47	27.69	29.82	31.88	34.53	36.48
14	17.12	18.15	19.41	21.06	23.68	26.12	26.87	29.14	31.32	33.43	36.12	38.11
15	18.25	19.31	20.60	22.31	25.00	27.49	28.26	30.58	32.80	34.95	37.70	39.72
16	19.37	20.47	21.79	23.54	26.30	28.85	29.63	32.00	34.27	36.46	39.25	41.31
17	20.49	21.61	22.98	24.77	27.59	30.19	31.00	33.41	35.72	37.95	40.79	42.88
18	21.60	22.76	24.16	25.99	28.87	31.53	32.35	34.81	37.16	39.42	42.31	44.43
19	22.72	23.90	25.33	27.20	30.14	32.85	33.69	36.19	38.58	40.88	43.82	45.97
20	23.83	25.04	26.50	28.41	31.41	34.17	35.02	37.57	40.00	42.34	45.31	47.50
21	24.93	26.17	27.66	29.62	32.67	35.48	36.34	38.93	41.40	43.78	46.80	49.01
22	26.04	27.30	28.82	30.81	33.92	36.78	37.66	40.29	42.80	45.20	48.27	50.51
23	27.14	28.43	29.98	32.01	35.17	38.08	38.97	41.64	44.18	46.62	49.73	52.00
24	28.24	29.55	31.13	33.20	36.42	39.36	40.27	42.98	45.56	48.03	51.18	53.48
25	29.34	30.68	32.28	34.38	37.65	40.65	41.57	44.31	46.93	49.44	52.62	54.95
26	30.43	31.79	33.43	35.56	38.89	41.92	42.86	45.64	48.29	50.83	54.05	56.41
27	31.53	32.91	34.57	36.74	40.11	43.19	44.14	46.96	49.64	52.22	55.48	57.86
28	32.62	34.03	35.71	37.92	41.34	44.46	45.42	48.28	50.99	53.59	56.89	59.30
29	33.71	35.14	36.85	39.09	42.56	45.72	46.69	49.59	52.34	54.97	58.30	60.73
30	34.80	36.25	37.99	40.26	43.77	46.98	47.96	50.89	53.67	56.33	59.70	62.16
40	45.62	47.27	49.24	51.81	55.76	59.34	60.44	63.69	66.77	69.70	73.40	76.09
50	56.33	58.16	60.35	63.17	67.50	71.42	72.61	76.15	79.49	82.66	86.66	89.56
60	66.98	68.97	71.34	74.40	79.08	83.30	84.58	88.38	91.95	95.34	99.61	102.7
80	88.13	90.41	93.11	96.58	101.9	106.6	108.1	112.3	116.3	120.1	124.8	128.3
100	109.1	111.7	114.7	118.5	124.3	129.6	131.1	135.8	140.2	144.3	149.4	153.2

7.5 Carrying Out a Test for a Population Mean

Name: ____________ Class: ________ Date: __________ **Total: 9 marks**

KEY WORDS distribution 分布

Objective

Build the skills to answer exam questions on **carrying out a test for a population mean**.

You must be able to:

- compute the t test statistic and find the **p-value** from the t-distribution
- compare the p-value to α and reach a decision
- state a conclusion in context

1 Worked examples

Study these first. Each one shows the method for a question type used later.

The procedure

1. State $H_0 : \mu = \mu_0$ and H_a; check conditions.
2. Compute $t = \dfrac{\bar{x} - \mu_0}{s/\sqrt{n}}$ with $\text{df} = n - 1$.
3. Find the **p-value** from the **t-distribution**.
4. Compare to α: reject H_0 if p-value $< \alpha$, else fail to reject.
5. State the conclusion **in context**.

A worked test

Test $H_0 : \mu = 50$ against $H_a : \mu \neq 50$ with $\bar{x} = 52$, $s = 4$, $n = 16$. Then

$$t = \frac{52 - 50}{4/\sqrt{16}} = \frac{2}{1} = 2.0, \quad \text{df} = 15.$$

The two-sided p-value $\approx 0.06 > 0.05$, so we fail to reject H_0.

2 Practice

2.1 State the steps to carry out a t-test for a mean. [2]

2.2 State the distribution that gives the p-value. [1]

2.3 A t-test gives a p-value of 0.04 with $\alpha = 0.05$. State the decision. [1]

3 Exam-style questions

3.1 The p-value for a mean test comes from the [1]

- **A** normal distribution
- **B** t-distribution
- **C** binomial distribution
- **D** χ^2 distribution

3.2 A t-test rejects H_0 when the p-value is [1]

- **A** above α
- **B** below α
- **C** exactly zero
- **D** exactly one

3.3 A t-test of $\mu > 100$ gives a p-value of 0.02 at $\alpha = 0.05$.

(a) State the decision. [1]

(b) State whether the result is significant. [1]

(c) State the conclusion in context. [1]

Solutions

2.1 state hypotheses and check conditions; compute t; find the p-value; compare to α and conclude.

2.2 the t-distribution.

2.3 reject H_0 $(0.04 < 0.05)$.

3.1 **B**.

3.2 **B**.

3.3 (a) reject H_0. (b) yes. (c) there is convincing evidence that $\mu > 100$.

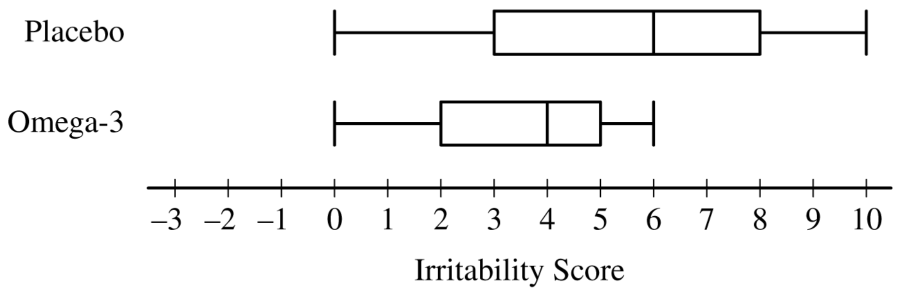
. A medical researcher completed a study comparing an omega-3 fatty acids supplement to a placebo in the treatment of irritability in patients with a certain medical condition. Nineteen patients with the medical condition volunteered to participate in the study. The study was conducted using the following weekly schedule.

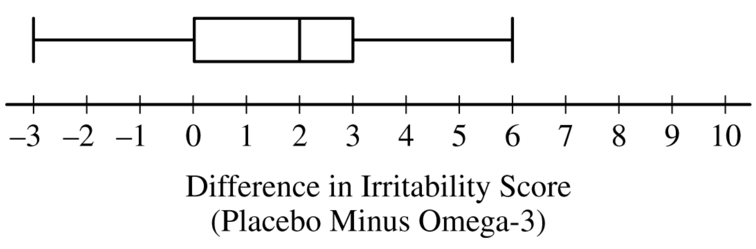
- Week 1: Each patient took a randomly assigned treatment, omega-3 supplement or placebo.
- Week 2: The patients did not take either the omega-3 supplement or the placebo. This was necessary to reduce the possibility of any carryover effect from the assigned treatment taken during week 1.
- Week 3: Each patient took the treatment, omega-3 supplement or placebo, that they did not take during week 1.

At the end of week 1 and week 3, each patient's irritability was given a score on a scale of 0 to 10, with 0 representing no irritability and 10 representing the highest level of irritability.

For each patient, the two irritability scores and the difference in their scores (placebo minus omega-3) were recorded. The results are summarized in the table and boxplots.

	n	Mean	Standard Deviation
Placebo	19	5.421	2.987
Omega-3	19	3.632	1.739
Difference (placebo minus omega-3)	19	1.789	2.485





GO ON TO THE NEXT PAGE.

The researcher claims the omega-3 supplement will decrease the mean irritability score of all patients with the medical condition similar to the volunteers who participated in the study. Is there convincing statistical evidence to support the researcher's claim at a significance level of $\alpha = 0.05$? Complete the appropriate inference procedure to support your answer.

GO ON TO THE NEXT PAGE.

STATISTICS
SECTION II
Part B
Question 6
Spend about 25 minutes on this part of the exam.
Percent of Section II score—25

Directions: Show all your work. Indicate clearly the methods you use, because you will be scored on the correctness of your methods as well as on the accuracy and completeness of your results and explanations.

6. Systolic blood pressure is the amount of pressure that blood exerts on blood vessels while the heart is beating. The mean systolic blood pressure for people in the United States is reported to be 122 millimeters of mercury (mmHg) with a standard deviation of 15 mmHg.

   The wellness department of a large corporation is investigating whether the mean systolic blood pressure of its employees is greater than the reported national mean. A random sample of 100 employees will be selected, the systolic blood pressure of each employee in the sample will be measured, and the sample mean will be calculated.

   Let μ represent the mean systolic blood pressure of all employees at the corporation. Consider the following hypotheses.

$$H_0 : \mu = 122$$
$$H_a : \mu > 122$$

   (a) Describe a Type II error in the context of the hypothesis test.

   (b) Assume that σ, the standard deviation of the systolic blood pressure of all employees at the corporation, is 15 mmHg. If $\mu = 122$, the sampling distribution of $\bar{x}$ for samples of size 100 is approximately normal with a mean of 122 mmHg and a standard deviation of 1.5 mmHg. What values of the sample mean $\bar{x}$ would represent sufficient evidence to reject the null hypothesis at the significance level of $\alpha = 0.05$?

   The actual mean systolic blood pressure of all employees at the corporation is 125 mmHg, not the hypothesized value of 122 mmHg, and the standard deviation is 15 mmHg.

   (c) Using the actual mean of 125 mmHg and the results from part (b), determine the probability that the null hypothesis will be rejected.

   (d) What statistical term is used for the probability found in part (c) ?

   (e) Suppose the size of the sample of employees to be selected is greater than 100. Would the probability of rejecting the null hypothesis be greater than, less than, or equal to the probability calculated in part (c) ? Explain your reasoning.

STOP

END OF EXAM

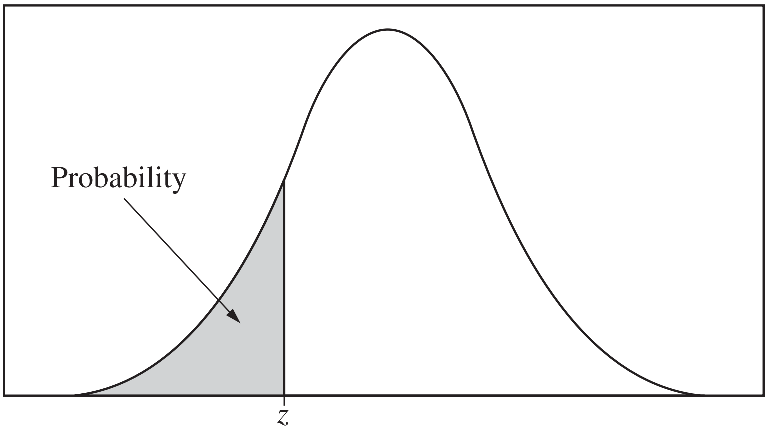


Table entry for z is the probability lying below z.

Table A Standard normal probabilities

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

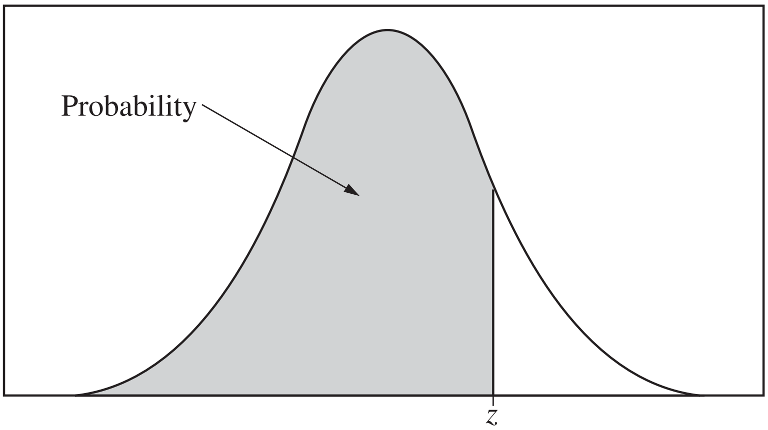


Table entry for z is the probability lying below z.

Table A (***Continued***)

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

Table entry for p and C is the point t^* with probability p lying above it and probability C lying between $-t^*$ and t^*.

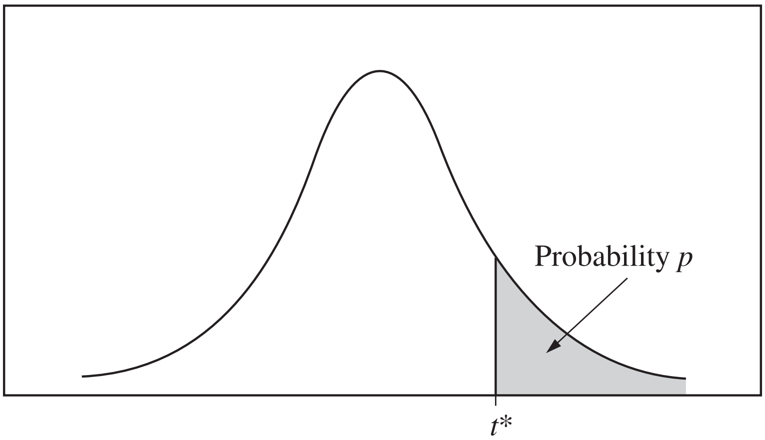


Table B ***t* distribution critical values**

	Tail probability p											
df	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.000	1.376	1.963	3.078	6.314	12.71	15.89	31.82	63.66	127.3	318.3	636.6
2	.816	1.061	1.386	1.886	2.920	4.303	4.849	6.965	9.925	14.09	22.33	31.60
3	.765	.978	1.250	1.638	2.353	3.182	3.482	4.541	5.841	7.453	10.21	12.92
4	.741	.941	1.190	1.533	2.132	2.776	2.999	3.747	4.604	5.598	7.173	8.610
5	.727	.920	1.156	1.476	2.015	2.571	2.757	3.365	4.032	4.773	5.893	6.869
6	.718	.906	1.134	1.440	1.943	2.447	2.612	3.143	3.707	4.317	5.208	5.959
7	.711	.896	1.119	1.415	1.895	2.365	2.517	2.998	3.499	4.029	4.785	5.408
8	.706	.889	1.108	1.397	1.860	2.306	2.449	2.896	3.355	3.833	4.501	5.041
9	.703	.883	1.100	1.383	1.833	2.262	2.398	2.821	3.250	3.690	4.297	4.781
10	.700	.879	1.093	1.372	1.812	2.228	2.359	2.764	3.169	3.581	4.144	4.587
11	.697	.876	1.088	1.363	1.796	2.201	2.328	2.718	3.106	3.497	4.025	4.437
12	.695	.873	1.083	1.356	1.782	2.179	2.303	2.681	3.055	3.428	3.930	4.318
13	.694	.870	1.079	1.350	1.771	2.160	2.282	2.650	3.012	3.372	3.852	4.221
14	.692	.868	1.076	1.345	1.761	2.145	2.264	2.624	2.977	3.326	3.787	4.140
15	.691	.866	1.074	1.341	1.753	2.131	2.249	2.602	2.947	3.286	3.733	4.073
16	.690	.865	1.071	1.337	1.746	2.120	2.235	2.583	2.921	3.252	3.686	4.015
17	.689	.863	1.069	1.333	1.740	2.110	2.224	2.567	2.898	3.222	3.646	3.965
18	.688	.862	1.067	1.330	1.734	2.101	2.214	2.552	2.878	3.197	3.611	3.922
19	.688	.861	1.066	1.328	1.729	2.093	2.205	2.539	2.861	3.174	3.579	3.883
20	.687	.860	1.064	1.325	1.725	2.086	2.197	2.528	2.845	3.153	3.552	3.850
21	.686	.859	1.063	1.323	1.721	2.080	2.189	2.518	2.831	3.135	3.527	3.819
22	.686	.858	1.061	1.321	1.717	2.074	2.183	2.508	2.819	3.119	3.505	3.792
23	.685	.858	1.060	1.319	1.714	2.069	2.177	2.500	2.807	3.104	3.485	3.768
24	.685	.857	1.059	1.318	1.711	2.064	2.172	2.492	2.797	3.091	3.467	3.745
25	.684	.856	1.058	1.316	1.708	2.060	2.167	2.485	2.787	3.078	3.450	3.725
26	.684	.856	1.058	1.315	1.706	2.056	2.162	2.479	2.779	3.067	3.435	3.707
27	.684	.855	1.057	1.314	1.703	2.052	2.158	2.473	2.771	3.057	3.421	3.690
28	.683	.855	1.056	1.313	1.701	2.048	2.154	2.467	2.763	3.047	3.408	3.674
29	.683	.854	1.055	1.311	1.699	2.045	2.150	2.462	2.756	3.038	3.396	3.659
30	.683	.854	1.055	1.310	1.697	2.042	2.147	2.457	2.750	3.030	3.385	3.646
40	.681	.851	1.050	1.303	1.684	2.021	2.123	2.423	2.704	2.971	3.307	3.551
50	.679	.849	1.047	1.299	1.676	2.009	2.109	2.403	2.678	2.937	3.261	3.496
60	.679	.848	1.045	1.296	1.671	2.000	2.099	2.390	2.660	2.915	3.232	3.460
80	.678	.846	1.043	1.292	1.664	1.990	2.088	2.374	2.639	2.887	3.195	3.416
100	.677	.845	1.042	1.290	1.660	1.984	2.081	2.364	2.626	2.871	3.174	3.390
1000	.675	.842	1.037	1.282	1.646	1.962	2.056	2.330	2.581	2.813	3.098	3.300
∞	.674	.841	1.036	1.282	1.645	1.960	2.054	2.326	2.576	2.807	3.091	3.291
	50%	60%	70%	80%	90%	95%	96%	98%	99%	99.5%	99.8%	99.9%
	Confidence level C											

Table entry for p is the point (χ^2) with probability p lying above it.

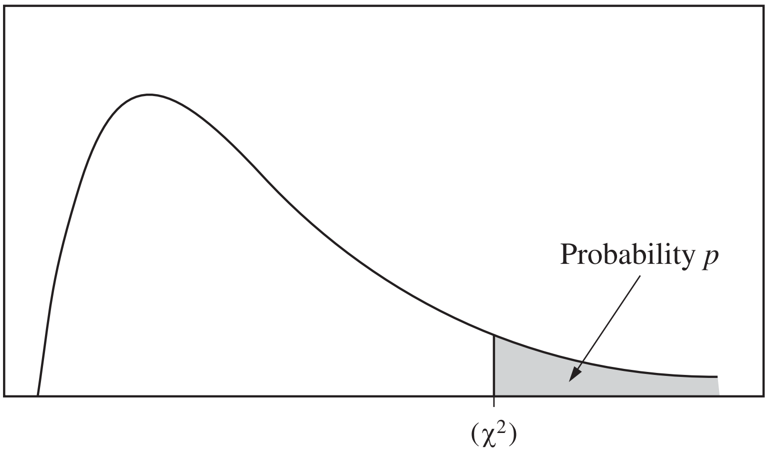


Table C χ^2 **critical values**

	Tail probability p											
df	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.32	1.64	2.07	2.71	3.84	5.02	5.41	6.63	7.88	9.14	10.83	12.12
2	2.77	3.22	3.79	4.61	5.99	7.38	7.82	9.21	10.60	11.98	13.82	15.20
3	4.11	4.64	5.32	6.25	7.81	9.35	9.84	11.34	12.84	14.32	16.27	17.73
4	5.39	5.99	6.74	7.78	9.49	11.14	11.67	13.28	14.86	16.42	18.47	20.00
5	6.63	7.29	8.12	9.24	11.07	12.83	13.39	15.09	16.75	18.39	20.51	22.11
6	7.84	8.56	9.45	10.64	12.59	14.45	15.03	16.81	18.55	20.25	22.46	24.10
7	9.04	9.80	10.75	12.02	14.07	16.01	16.62	18.48	20.28	22.04	24.32	26.02
8	10.22	11.03	12.03	13.36	15.51	17.53	18.17	20.09	21.95	23.77	26.12	27.87
9	11.39	12.24	13.29	14.68	16.92	19.02	19.68	21.67	23.59	25.46	27.88	29.67
10	12.55	13.44	14.53	15.99	18.31	20.48	21.16	23.21	25.19	27.11	29.59	31.42
11	13.70	14.63	15.77	17.28	19.68	21.92	22.62	24.72	26.76	28.73	31.26	33.14
12	14.85	15.81	16.99	18.55	21.03	23.34	24.05	26.22	28.30	30.32	32.91	34.82
13	15.98	16.98	18.20	19.81	22.36	24.74	25.47	27.69	29.82	31.88	34.53	36.48
14	17.12	18.15	19.41	21.06	23.68	26.12	26.87	29.14	31.32	33.43	36.12	38.11
15	18.25	19.31	20.60	22.31	25.00	27.49	28.26	30.58	32.80	34.95	37.70	39.72
16	19.37	20.47	21.79	23.54	26.30	28.85	29.63	32.00	34.27	36.46	39.25	41.31
17	20.49	21.61	22.98	24.77	27.59	30.19	31.00	33.41	35.72	37.95	40.79	42.88
18	21.60	22.76	24.16	25.99	28.87	31.53	32.35	34.81	37.16	39.42	42.31	44.43
19	22.72	23.90	25.33	27.20	30.14	32.85	33.69	36.19	38.58	40.88	43.82	45.97
20	23.83	25.04	26.50	28.41	31.41	34.17	35.02	37.57	40.00	42.34	45.31	47.50
21	24.93	26.17	27.66	29.62	32.67	35.48	36.34	38.93	41.40	43.78	46.80	49.01
22	26.04	27.30	28.82	30.81	33.92	36.78	37.66	40.29	42.80	45.20	48.27	50.51
23	27.14	28.43	29.98	32.01	35.17	38.08	38.97	41.64	44.18	46.62	49.73	52.00
24	28.24	29.55	31.13	33.20	36.42	39.36	40.27	42.98	45.56	48.03	51.18	53.48
25	29.34	30.68	32.28	34.38	37.65	40.65	41.57	44.31	46.93	49.44	52.62	54.95
26	30.43	31.79	33.43	35.56	38.89	41.92	42.86	45.64	48.29	50.83	54.05	56.41
27	31.53	32.91	34.57	36.74	40.11	43.19	44.14	46.96	49.64	52.22	55.48	57.86
28	32.62	34.03	35.71	37.92	41.34	44.46	45.42	48.28	50.99	53.59	56.89	59.30
29	33.71	35.14	36.85	39.09	42.56	45.72	46.69	49.59	52.34	54.97	58.30	60.73
30	34.80	36.25	37.99	40.26	43.77	46.98	47.96	50.89	53.67	56.33	59.70	62.16
40	45.62	47.27	49.24	51.81	55.76	59.34	60.44	63.69	66.77	69.70	73.40	76.09
50	56.33	58.16	60.35	63.17	67.50	71.42	72.61	76.15	79.49	82.66	86.66	89.56
60	66.98	68.97	71.34	74.40	79.08	83.30	84.58	88.38	91.95	95.34	99.61	102.7
80	88.13	90.41	93.11	96.58	101.9	106.6	108.1	112.3	116.3	120.1	124.8	128.3
100	109.1	111.7	114.7	118.5	124.3	129.6	131.1	135.8	140.2	144.3	149.4	153.2

7.6 Confidence Intervals for the Difference of Two Means

Name: ____________ Class: ________ Date: __________ **Total: 8 marks**

Objective

Build the skills to answer exam questions on **a confidence interval for the difference of two means**.

You must be able to:

- use the point estimate $\bar{x}_1 - \bar{x}_2$
- build the interval $(\bar{x}_1 - \bar{x}_2) \pm t^* \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$
- use the interval to judge whether the two means differ

1 Worked examples

Study these first. Each one shows the method for a question type used later.

■ **The two-mean interval**

$$(\bar{x}_1 - \bar{x}_2) \pm t^* \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}.$$

■ **Judging a difference**

If the interval **contains** 0, the two means may not differ. If it is entirely **positive** ($\mu_1 > \mu_2$) or **negative** ($\mu_1 < \mu_2$), there is a significant difference.

2 Practice

2.1 Write the point estimate for the difference of two means. [1]

2.2 State how to judge whether the means differ using the interval. [1]

2.3 For $\bar{x}_1 = 52$ and $\bar{x}_2 = 48$, find the point estimate of the difference. [1]

3 Exam-style questions

3.1 A confidence interval for $\mu_1 - \mu_2$ is centred on [1]

- **A** $\bar{x}_1 \cdot \bar{x}_2$
- **B** $\bar{x}_1 - \bar{x}_2$
- **C** 0
- **D** $\bar{x}_1 + \bar{x}_2$

3.2 If a 95% CI for $\mu_1 - \mu_2$ contains 0, the two means [1]

- **A** definitely differ
- **B** may not differ
- **C** are exactly equal
- **D** are both zero

3.3 A 95% CI for $\mu_1 - \mu_2$ is $(1.5, 4.5)$.

(a) State which mean is larger. [1]

(b) State whether 0 is in the interval. [1]

(c) State whether there is a significant difference. [1]

Solutions

2.1 $\bar{x}_1 - \bar{x}_2$.

2.2 if the interval contains 0, there is no significant difference; if not, there is.

2.3 $52 - 48 = 4$.

3.1 **B**.

3.2 **B**.

3.3 (a) μ_1 (the interval is positive). (b) no. (c) yes.

7.7 Justifying a Claim About the Difference of Two Means Based on a Confidence Interval

Name: ____________ Class: ________ Date: __________ **Total: 8 marks**

Objective

Build the skills to answer exam questions on **justifying a claim about a difference of means from a confidence interval**.

You must be able to:

- use a CI for $\mu_1 - \mu_2$ to judge a claim of no difference
- recognise an interval containing 0 as "no significant difference"

1 Worked examples

Study these first. Each one shows the method for a question type used later.

■ Judging a difference

A confidence interval for $\mu_1 - \mu_2$ gives the plausible differences:

- if it **contains** 0, "no difference" ($\mu_1 = \mu_2$) is plausible;
- if it is entirely positive or negative, there is a significant difference.

■ Reading a worked interval

A 95% interval for $\mu_1 - \mu_2$ comes out as $(1.2,\ 5.4)$. It is entirely **positive** and excludes 0, so group 1's mean is significantly higher. An interval of $(-1.1,\ 3.0)$ would contain 0 – no significant difference.

2 Practice

2.1 State how a CI for $\mu_1 - \mu_2$ judges "no difference". [1]

2.2 A 95% CI for $\mu_1 - \mu_2$ is $(-1, 3)$. State whether "no difference" is plausible. [1]

2.3 State what a fully positive interval implies. [1]

3 Exam-style questions

3.1 "No difference" ($\mu_1 = \mu_2$) is plausible if the CI for $\mu_1 - \mu_2$ contains [1]

- **A** 1
- **B** 0
- **C** $\bar{x}_1$
- **D** the mean

3.2 A 95% CI for $\mu_1 - \mu_2$ is $(2, 6)$. The claim $\mu_1 = \mu_2$ is [1]

- **A** supported
- **B** not supported
- **C** certainly true
- **D** unknown

3.3 A 95% CI for the difference in mean scores is $(-2, 5)$.

(a) State whether 0 is in the interval. [1]

(b) State whether the means differ significantly. [1]

(c) State the plausible conclusion. [1]

Solutions

2.1 if the interval contains 0, ”no difference” is plausible.

2.2 yes, plausible (0 is inside).

2.3 that $\mu_1 > \mu_2$ (a real difference).

3.1 **B**.

3.2 **B**.

3.3 (a) yes. (b) no. (c) there is no significant difference between the means.

7.8 Setting Up a Test for the Difference of Two Population Means

Name: ______________ Class: __________ Date: ____________ **Total: 10 marks**

Objective

Build the skills to answer exam questions on **setting up a test for a difference of two means**.

You must be able to:

- state the hypotheses $H_0 : \mu_1 = \mu_2$ and H_a
- write the test statistic $t = \dfrac{\bar{x}_1 - \bar{x}_2}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}}$

1 Worked examples

Study these first. Each one shows the method for a question type used later.

Hypotheses

- **Null** $H_0 : \mu_1 = \mu_2$ (no difference).
- **Alternative** $H_a : \mu_1 \neq \mu_2$ (or one-sided).

Test statistic

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}}.$$

The standard error again adds the two **variances** (s^2/n).

2 Practice

2.1 State the null hypothesis for a two-sample mean test. [1]

2.2 Write the test-statistic formula for a difference of means. [1]

2.3 For $\bar{x}_1 = 50$, $s_1 = 6$, $n_1 = 36$, $\bar{x}_2 = 45$, $s_2 = 8$, $n_2 = 64$, find the standard error $\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$. [2]

3 Exam-style questions

3.1 The null hypothesis for a two-sample mean test is [1]

- **A** $\mu_1 \neq \mu_2$
- **B** $\mu_1 = \mu_2$
- **C** $\mu_1 = 0$
- **D** $\bar{x}_1 = \bar{x}_2$

3.2 The test statistic for a difference of means uses the standard error [1]

- **A** $\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$
- **B** $s_1 + s_2$
- **C** $s_1 \cdot s_2$
- **D** $\sqrt{np}$

3.3 $\bar{x}_1 = 70$, $s_1 = 10$, $n_1 = 25$, $\bar{x}_2 = 65$, $s_2 = 10$, $n_2 = 25$.

(a) Find the standard error. [2]

(b) Compute the test statistic t. [2]

Solutions

2.1 $H_0 : \mu_1 = \mu_2$.

2.2 $t = \dfrac{\bar{x}_1 - \bar{x}_2}{\sqrt{s_1^2/n_1 + s_2^2/n_2}}$.

2.3 $\sqrt{\dfrac{36}{36} + \dfrac{64}{64}} = \sqrt{1+1} = \sqrt{2} \approx 1.41$.

3.1 **B**.

3.2 **A**.

3.3 (a) $\sqrt{\dfrac{100}{25} + \dfrac{100}{25}} = \sqrt{8} \approx 2.83$. (b) $t = \dfrac{70 - 65}{2.83} \approx 1.77$.

4. A farmer wants to know whether there is a difference in the mean number of oranges that grow on trees fertilized with the current fertilizer, Brand C, or a new fertilizer, Brand N. The farmer randomly assigns 58 trees to be treated with Brand C and 58 trees to be treated with Brand N. The number of oranges from each tree is recorded. The summary statistics by fertilizer brand are shown in the table.

Summary Statistics for the Number of Oranges by Fertilizer Brand

	n	Mean	Standard Deviation
Brand C	58	141	15
Brand N	58	148	19

At the $\alpha = 0.05$ significance level, do the data provide convincing statistical evidence that the mean number of oranges on trees fertilized with Brand C is different from the mean number of oranges on trees fertilized with Brand N for all trees similar to those in the study?

AP Statistics: **2025**

6. Stefan, a psychologist, conducted a study to investigate the effect of time of day on reading comprehension in children. One hundred children volunteered, with their parents' consent, to participate in the study. Fifty of the children were randomly assigned to read a story at 9 a.m. and then answer 25 questions about it. The remaining 50 children were assigned to read the same story at 3 p.m. and answer the same 25 questions. The reading comprehension for each child was measured by a reading score, which was determined by the number of questions that were answered correctly about the story. Stefan is interested in comparing the mean reading scores for the two times of day. Table 1 shows the results of Stefan's study.

Table 1: Summary Statistics of Reading Scores

	n	Mean	Standard Deviation
9 a.m.	50	15.2	4.12
3 p.m.	50	17.9	4.43

Stefan found the conditions for inference were met and conducted a two-sample *t*-test for the difference in two population means. Let μ_{AM} represent the mean reading score for all children, similar to those in the study, who would read the story at 9 a.m. Let μ_{PM} represent the mean reading score for all children, similar to those in the study, who would read the story at 3 p.m. Stefan's hypotheses are as shown.

$$\text{H}_0\text{:}\mu_{\text{AM}} = \mu_{\text{PM}}$$
$$\text{H}_a\text{:}\mu_{\text{AM}} \neq \mu_{\text{PM}}$$

A. The *p*-value for Stefan's hypothesis test was 0.002. State an appropriate conclusion, at the 5 percent significance level, for Stefan's test in the context of the investigation. Justify your answer.

B. Explain why it was appropriate for Stefan to conduct a two-sample *t*-test for the difference in two population means instead of a paired for the population mean difference.

C. Researchers are usually interested in the practical importance of their results as well as the statistical significance of the hypothesis test. The practical importance of the results indicates whether the observed results are meaningful in real life. For example, in an investigation of the heights of two groups of students, a difference in the two group means of 3.8 inches is much more meaningful, or has more practical importance, than a difference in the two group means of only 0.2 inches.

One indicator of practical importance is effect size. A common method for measuring effect size for the difference in two group means is Cohen's *d* coefficient. Cohen's *d* coefficient compares the absolute value of the difference in the means of the two groups to the pooled variability of the observed data values from the two groups.

Cohen's *d* coefficient can be calculated using $d = \frac{|\bar{x}_1 - \bar{x}_2|}{s_p}$, where s_p represents the pooled standard deviation, $\bar{x}_1$ represents the sample mean for the first group, and $\bar{x}_2$ represents the sample mean for the second group. When the sizes of the groups are equal, s_p is calculated as $s_p = \sqrt{\frac{s_1^2 + s_2^2}{2}}$, where s_1 represents the sample standard deviation for the first group and s_2 represents the sample standard deviation for the second group.

Consider the summary statistics from Stefan's study in Table 1.

i. Calculate Cohen's *d* coefficient for Stefan's study. Show your work.

ii. Higher values of Cohen's *d* indicate greater practical importance and lower values of Cohen's *d* indicate less practical importance. Typically, we use the intervals listed in Table 2 to help interpret practical importance.

Table 2: Guidelines for Interpreting Cohen's *d* Coefficient

Cohen's *d* Coefficient	Practical Importance
$0 \le d \le 0.20$	Not very meaningful in real life
$0.20 < d < 0.80$	Somewhat meaningful in real life
$d \ge 0.80$	Very meaningful in real life

Based on your answer to part C (i) and the information in Tables 1 and 2, describe the practical importance of Stefan's results, in context.

D. Suppose the results of Stefan's study, summarized in Table 1, instead had a standard deviation for the 9 a.m. reading scores, s_1, and a standard deviation for the 3 p.m. reading scores, s_2, that were both greater than 4.43. Assume the group sample sizes and the means are not changed.

i. Would the Cohen's *d* coefficient in this new situation be smaller than, larger than, or the same as the Cohen's *d* coefficient calculated in part C (i)? Explain your answer.

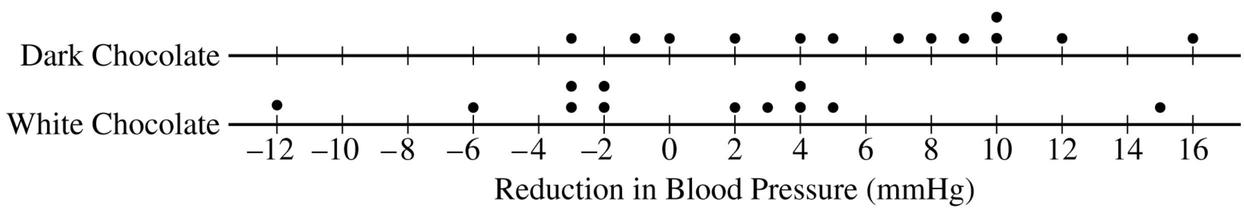
ii. Does the Cohen's *d* coefficient described in part D (i) indicate that Stefan's observed difference in the means in the new situation would have more practical importance than, less practical importance than, or the same practical importance as what was originally determined in part C (ii)? Explain your answer.

STOP

END OF EXAM

. Studies have shown that foods rich in compounds known as flavonoids help lower blood pressure. Researchers conducted a study to investigate whether there was a greater reduction in blood pressure for people who consumed dark chocolate, which contains flavonoids, than people who consumed white chocolate, which does not contain flavonoids. Twenty-five healthy adults agreed to participate in the study and add 3.5 ounces of chocolate to their daily diets. Of the 25 participants, 13 were randomly assigned to the dark chocolate group and the rest were assigned to the white chocolate group. All participants had their blood pressure recorded, in millimeters of mercury (mmHg), before adding chocolate to their daily diets and again 30 days after adding chocolate to their daily diets.

The reduction in blood pressure (before minus after) for each of the participants in the two groups is shown in the dotplots below.



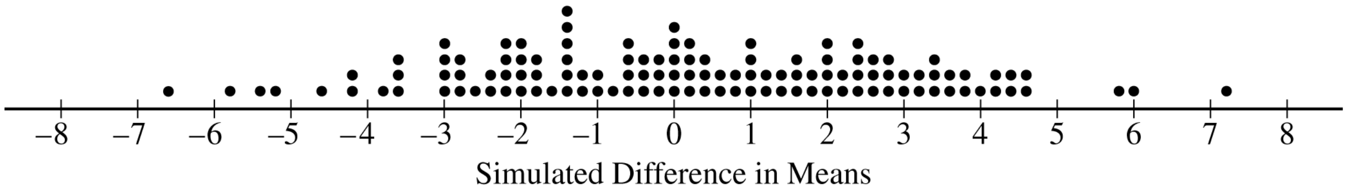
(a) Determine and compare the medians of the reduction in blood pressure for the two groups.

The researchers found the mean reduction in blood pressure for those who consumed dark chocolate is $\bar{x}_{dark} = 6.08$ mmHg and the mean reduction in blood pressure for those who consumed white chocolate is $\bar{x}_{white} = 0.42$ mmHg.

(b) One researcher indicated that because the difference in sample means of 5.66 mmHg is greater than 0 there is convincing statistical evidence to conclude that the population mean blood pressure reduction for those who consume dark chocolate is greater than for those who consume white chocolate. Why might the researcher's conclusion, based only on the difference in sample means of 5.66 mmHg, not necessarily be true?

GO ON TO THE NEXT PAGE.

A simulation was conducted to investigate whether there is a greater reduction of blood pressure for those who consume dark chocolate than for those who consume white chocolate. The simulation was conducted under the assumption that no difference exists. The results of 120 trials of the simulation are shown in the following dotplot.



(c) Use the results of the simulation to determine whether the results from the 25 participants in the study provide convincing statistical evidence, at a 5 percent level of significance, that adding dark chocolate to a daily diet will result in a greater reduction in blood pressure, on average, than adding white chocolate to a daily diet. Justify your answer.

GO ON TO THE NEXT PAGE.

4. The anterior cruciate ligament (ACL) is one of the ligaments that help stabilize the knee. Surgery is often recommended if the ACL is completely torn, and recovery time from the surgery can be lengthy. A medical center developed a new surgical procedure designed to reduce the average recovery time from the surgery. To test the effectiveness of the new procedure, a study was conducted in which 210 patients needing surgery to repair a torn ACL were randomly assigned to receive either the standard procedure or the new procedure.

   (a) Based on the design of the study, would a statistically significant result allow the medical center to conclude that the new procedure causes a reduction in recovery time compared to the standard procedure, for patients similar to those in the study? Explain your answer.

   (b) Summary statistics on the recovery times from the surgery are shown in the table.

Type of Procedure	Sample Size	Mean Recovery Time (days)	Standard Deviation Recovery Time (days)
Standard	110	217	34
New	100	186	29

   Do the data provide convincing statistical evidence that those who receive the new procedure will have less recovery time from the surgery, on average, than those who receive the standard procedure, for patients similar to those in the study?

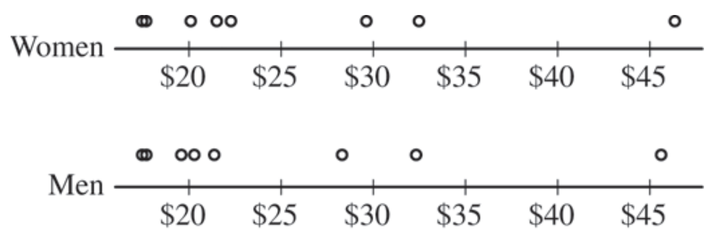
5. A researcher conducted a study to investigate whether local car dealers tend to charge women more than men for the same car model. Using information from the county tax collector's records, the researcher randomly selected one man and one woman from among everyone who had purchased the same model of an identically equipped car from the same dealer. The process was repeated for a total of 8 randomly selected car models.

The purchase prices and the differences (woman – man) are shown in the table below. Summary statistics are also shown.

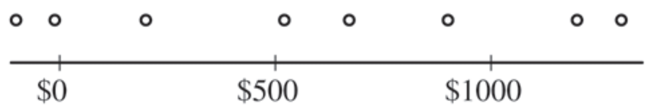
Car model	1	2	3	4	5	6	7	8
Women	\$20,100	\$17,400	\$22,300	\$32,500	\$17,710	\$21,500	\$29,600	\$46,300
Men	\$19,580	\$17,500	\$21,400	\$32,300	\$17,720	\$20,300	\$28,300	\$45,630
Difference	\$520	–\$100	\$900	\$200	–\$10	\$1,200	\$1,300	\$670

	Mean	Standard Deviation
Women	\$25,926.25	\$9,846.61
Men	\$25,341.25	\$9,728.60
Difference	\$585.00	\$530.71

Dotplots of the data and the differences are shown below.



Purchase Price (in thousands of dollars)



Difference in Purchase Price
(woman – man, in dollars)

Do the data provide convincing evidence that, on average, women pay more than men in the county for the same car model?

AP STATISTICS • EXERCISE SHEET

7.9 Carrying Out a Test for the Difference of Two Population Means

Name: ________ Class: ________ Date: ________ **Total: 9 marks**

KEY WORDS distribution 分布

Objective

Build the skills to answer exam questions on **carrying out a test for a difference of two means**.

You must be able to:

- compute the two-sample t statistic and find the **p-value**
- compare the p-value to α and reach a decision
- state a conclusion in context

1 Worked examples

Study these first. Each one shows the method for a question type used later.

■ The procedure

1. State $H_0 : \mu_1 = \mu_2$ and H_a; check conditions.
2. Compute $t = \dfrac{\bar{x}_1 - \bar{x}_2}{\sqrt{s_1^2/n_1 + s_2^2/n_2}}$.
3. Find the **p-value** from the t-distribution.
4. Compare to α and decide (reject or fail to reject).
5. State the conclusion **in context**.

A **large** $|t|$ gives a small p-value — stronger evidence of a difference.

■ A worked test

$\bar{x}_1 = 20, s_1 = 3, n_1 = 25$ and $\bar{x}_2 = 18, s_2 = 4, n_2 = 25$. Then

$$t = \frac{20 - 18}{\sqrt{3^2/25 + 4^2/25}} = \frac{2}{\sqrt{1}} = 2.0.$$

This gives a p-value near 0.05 – borderline evidence of a difference in means.

2 Practice

2.1 State the steps to carry out a two-sample t-test. [2]

2.2 State what a large $|t|$ suggests. [1]

2.3 A two-sample t-test gives a p-value of 0.06 at $\alpha = 0.05$. State the decision. [1]

3 Exam-style questions

3.1 A two-sample t-test compares [1]

- **A** two proportions
- **B** two means
- **C** one mean to a fixed value
- **D** categories

3.2 A two-sample t-test fails to reject H_0 when the p-value is [1]

- **A** below α
- **B** above α
- **C** exactly zero
- **D** negative

3.3 A two-sample t-test gives a p-value of 0.01 at $\alpha = 0.05$.

(a) State the decision. [1]

(b) State whether the means differ significantly. [1]

(c) State the conclusion in context. [1]

Solutions

2.1 state hypotheses and check conditions; compute t; find the p-value; compare to α and conclude.

2.2 a difference between the two means (a small p-value).

2.3 fail to reject H_0 ($0.06 > 0.05$).

3.1 **B**.

3.2 **B**.

3.3 (a) reject H_0. (b) yes. (c) there is convincing evidence that the two means differ.

4. A farmer wants to know whether there is a difference in the mean number of oranges that grow on trees fertilized with the current fertilizer, Brand C, or a new fertilizer, Brand N. The farmer randomly assigns 58 trees to be treated with Brand C and 58 trees to be treated with Brand N. The number of oranges from each tree is recorded. The summary statistics by fertilizer brand are shown in the table.

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AP Statistics: **2025**

6. Stefan, a psychologist, conducted a study to investigate the effect of time of day on reading comprehension in children. One hundred children volunteered, with their parents' consent, to participate in the study. Fifty of the children were randomly assigned to read a story at 9 a.m. and then answer 25 questions about it. The remaining 50 children were assigned to read the same story at 3 p.m. and answer the same 25 questions. The reading comprehension for each child was measured by a reading score, which was determined by the number of questions that were answered correctly about the story. Stefan is interested in comparing the mean reading scores for the two times of day. Table 1 shows the results of Stefan's study.

Table 1: Summary Statistics of Reading Scores

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$$\text{H}_0: \mu_{\text{AM}} = \mu_{\text{PM}}$$
$$\text{H}_a: \mu_{\text{AM}} \neq \mu_{\text{PM}}$$

A. The p-value for Stefan's hypothesis test was 0.002. State an appropriate conclusion, at the 5 percent significance level, for Stefan's test in the context of the investigation. Justify your answer.

B. Explain why it was appropriate for Stefan to conduct a two-sample t-test for the difference in two population means instead of a paired for the population mean difference.

C. Researchers are usually interested in the practical importance of their results as well as the statistical significance of the hypothesis test. The practical importance of the results indicates whether the observed results are meaningful in real life. For example, in an investigation of the heights of two groups of students, a difference in the two group means of 3.8 inches is much more meaningful, or has more practical importance, than a difference in the two group means of only 0.2 inches.

One indicator of practical importance is effect size. A common method for measuring effect size for the difference in two group means is Cohen's *d* coefficient. Cohen's *d* coefficient compares the absolute value of the difference in the means of the two groups to the pooled variability of the observed data values from the two groups.

Cohen's *d* coefficient can be calculated using $d = \frac{|\bar{x}_1 - \bar{x}_2|}{s_p}$, where s_p represents the pooled standard deviation, $\bar{x}_1$ represents the sample mean for the first group, and $\bar{x}_2$ represents the sample mean for the second group. When the sizes of the groups are equal, s_p is calculated as $s_p = \sqrt{\frac{s_1^2 + s_2^2}{2}}$, where s_1 represents the sample standard deviation for the first group and s_2 represents the sample standard deviation for the second group.

Consider the summary statistics from Stefan's study in Table 1.

i. Calculate Cohen's *d* coefficient for Stefan's study. Show your work.

ii. Higher values of Cohen's *d* indicate greater practical importance and lower values of Cohen's *d* indicate less practical importance. Typically, we use the intervals listed in Table 2 to help interpret practical importance.

Table 2: Guidelines for Interpreting Cohen's *d* Coefficient

Cohen's *d* Coefficient	Practical Importance
$0 \le d \le 0.20$	Not very meaningful in real life
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Based on your answer to part C (i) and the information in Tables 1 and 2, describe the practical importance of Stefan's results, in context.

D. Suppose the results of Stefan's study, summarized in Table 1, instead had a standard deviation for the 9 a.m. reading scores, s_1, and a standard deviation for the 3 p.m. reading scores, s_2, that were both greater than 4.43. Assume the group sample sizes and the means are not changed.

i. Would the Cohen's *d* coefficient in this new situation be smaller than, larger than, or the same as the Cohen's *d* coefficient calculated in part C (i)? Explain your answer.

ii. Does the Cohen's *d* coefficient described in part D (i) indicate that Stefan's observed difference in the means in the new situation would have more practical importance than, less practical importance than, or the same practical importance as what was originally determined in part C (ii)? Explain your answer.

STOP

END OF EXAM

4. The anterior cruciate ligament (ACL) is one of the ligaments that help stabilize the knee. Surgery is often recommended if the ACL is completely torn, and recovery time from the surgery can be lengthy. A medical center developed a new surgical procedure designed to reduce the average recovery time from the surgery. To test the effectiveness of the new procedure, a study was conducted in which 210 patients needing surgery to repair a torn ACL were randomly assigned to receive either the standard procedure or the new procedure.

(a) Based on the design of the study, would a statistically significant result allow the medical center to conclude that the new procedure causes a reduction in recovery time compared to the standard procedure, for patients similar to those in the study? Explain your answer.

(b) Summary statistics on the recovery times from the surgery are shown in the table.

Type of Procedure	Sample Size	Mean Recovery Time (days)	Standard Deviation Recovery Time (days)
Standard	110	217	34
New	100	186	29

Do the data provide convincing statistical evidence that those who receive the new procedure will have less recovery time from the surgery, on average, than those who receive the standard procedure, for patients similar to those in the study?

5. A researcher conducted a study to investigate whether local car dealers tend to charge women more than men for the same car model. Using information from the county tax collector's records, the researcher randomly selected one man and one woman from among everyone who had purchased the same model of an identically equipped car from the same dealer. The process was repeated for a total of 8 randomly selected car models.

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	Mean	Standard Deviation
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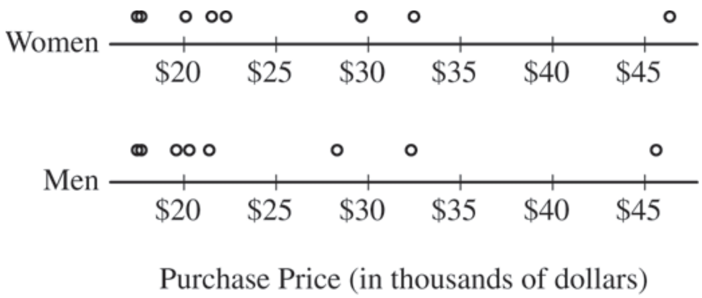
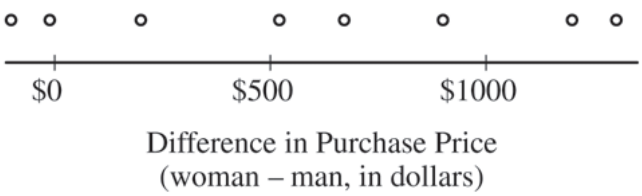
Dotplots of the data and the differences are shown below.





Do the data provide convincing evidence that, on average, women pay more than men in the county for the same car model?

AP STATISTICS • EXERCISE SHEET

7.10 Skills Focus: Selecting, Implementing, and Communicating Inference Procedures

Name: ____________ Class: ________ Date: __________ **Total: 9 marks**

Objective

Build the skills to answer exam questions on **selecting an inference procedure**.

You must be able to:

- decide between a procedure for a **proportion** (categorical) and a **mean** (quantitative)
- decide between a **one-sample** and a **two-sample** procedure
- decide between a **confidence interval** and a **hypothesis test**

1 Worked examples

Study these first. Each one shows the method for a question type used later.

Choosing a procedure

- **Proportion vs mean** — categorical data (counts/percentages) → **proportion**; quantitative data (measurements) → **mean**.
- **One vs two sample** — comparing to a fixed value → **one-sample**; comparing two groups → **two-sample**.
- **Interval vs test** — *estimate* a value → **confidence interval**; *test a claim* → **hypothesis test**.

Worked choices

"Estimate the mean weight of a bag of rice" – quantitative, one sample, estimate → **one-sample t interval**. "Test whether two brands differ in mean weight" – quantitative, two samples, test a claim → **two-sample t test**.

2 Practice

2.1 State how to decide between a proportion and a mean procedure. [1]

2.2 State how to decide between a one-sample and a two-sample procedure. [1]

2.3 State the difference between a confidence interval and a hypothesis test. [2]

3 Exam-style questions

3.1 To estimate a categorical variable's population value, use a procedure for a [1]

- **A** mean
- **B** proportion
- **C** slope
- **D** variance

3.2 Comparing the average heights of two groups uses a [1]

- **A** one-proportion test
- **B** two-sample t procedure
- **C** chi-square test
- **D** one-mean interval only

3.3 A study compares the proportion of defective items from two factories.

(a) Name the type of data. [1]

(b) State whether it is a one- or two-sample situation. [1]

(c) Name a suitable procedure. [1]

Solutions

2.1 categorical data uses a proportion procedure; quantitative data uses a mean procedure.

2.2 comparing to a fixed value is one-sample; comparing two groups is two-sample.

2.3 a confidence interval estimates a parameter; a hypothesis test evaluates a specific claim.

3.1 **B**.

3.2 **B**.

3.3 (a) categorical (proportions). (b) two-sample. (c) a two-proportion test (or confidence interval).