

Week 1

AP Statistics

Homework bundle for this week

本周作业合集

exercises.lol

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1. Exploring One-Variable Data

Starter Quiz · 课前小测

AP Statistics

Name: _____ Score: _____

1. In a data set of 200 students' commute times, the individuals are...

- ☐ the commute times
- ☐ the 200 students
- ☐ the average
- ☐ the survey

2. Eye color is a _____ variable.

- ☐ quantitative
- ☐ categorical
- ☐ continuous
- ☐ discrete

3. Of 120 students, 30 prefer maths. What is the relative frequency (as a decimal)?

4. In a bar chart, the height of each bar represents...

- ☐ the category name
- ☐ the frequency or relative frequency
- ☐ the mean
- ☐ the range

5. A histogram's bars touch because the scale is continuous.

- ☐ True
- ☐ False

6. A distribution with a long tail stretching to the left is...

- ☐ skewed right
- ☐ skewed left
- ☐ symmetric
- ☐ uniform

7. Find the median of 4, 5, 6, 7, 100.

8. The length of the box in a boxplot represents the...

- ☐ range
- ☐ IQR
- ☐ mean
- ☐ standard deviation

9. A complete comparison of two groups addresses **which** features?

- ☐ shape
- ☐ center
- ☐ spread
- ☐ unusual features

Tick every correct option.

10. Bar chart bars are separated, unlike a histogram's touching bars.

- ☐ True
- ☐ False

1. Exploring One-Variable Data

Answers · 答案

AP Statistics

1. **the 200 students**

Individuals are the cases (students); commute time is the variable.

2. **categorical**

It records a label/group → categorical.

3. **0.25**

$30/120 = 0.25$.

4. **the frequency or relative frequency**

Bar height = count (or proportion) in that category.

5. **True**

Touching bars distinguish it from a categorical bar chart.

6. **skewed left**

Skew is named for the tail direction → left.

7. **6**

The middle value of the sorted data is 6.

8. **IQR**

Box from Q_1 to $Q_3 = \text{IQR}$.

9. **shape; center; spread; unusual features**

Shape, center, spread, and unusual features for both groups.

10. **True**

Categorical bars are separated; quantitative histogram bars touch.

AP STATISTICS · EXERCISE SHEET

1.1 Introducing Statistics: What Can We Learn from Data?

Name: _____ Class: _____ Date: _____ **Total: 9 marks**

KEY WORDS statistic 统计量 • data 数据 • individuals 个体 • statistical question 统计问题
• statistics 统计学

Objective

Build the skills to answer exam questions on **what we can learn from data**.

You must be able to:

- identify the **individuals** 个体 (cases) and the **variables** 变量 in a data set
- describe **statistics** 统计学 as the science of learning from data that vary
- pose a **statistical question** 统计问题 that anticipates variability

1 Worked examples

Study these first. Each one shows the method for a question type used later.

■ Individuals and variables

The **individuals** are the people or objects described; the **variables** are the characteristics recorded about them.

■ Statistics and statistical questions

Statistics is the science of learning from **data that vary**. A good **statistical question anticipates variability** — its answer is a distribution of values, not a single fixed number.

2 Practice

2.1 Define statistics. [1]

2.2 In a data set of students' heights, state the individuals and the variable. [2]

2.3 State one feature of a good statistical question. [1]

3 Exam-style questions

3.1 The people or objects described in a data set are the [1]

- **A** variables
- **B** individuals
- **C** statistics
- **D** graphs

3.2 A statistical question anticipates [1]

- **A** a single fixed answer
- **B** variability in the data
- **C** no data at all
- **D** a graph

3.3 A survey records each student's favourite sport.

(a) Name the individuals. [1]

(b) Name the variable. [1]

(c) State whether "What is my friend's height?" is a statistical question. [1]

Solutions

2.1 the science of learning from data that vary.

2.2 individuals: the students; variable: their height.

2.3 it anticipates variability (its answer is a distribution, not one value).

3.1 B.

3.2 B.

3.3 (a) the students. (b) favourite sport. (c) no — it has a single answer with no variability.

1.2 The Language of Variation: Variables

Name: _____ Class: _____ Date: _____ **Total: 10 marks**

KEY WORDS quantitative 数量 • categorical 分类 • variable 变量 • continuous 连续
• discrete 离散

Objective

Build the skills to answer exam questions on **the language of variation: variables**.

You must be able to:

- classify a **variable** as **categorical** 分类 (qualitative) or **quantitative** 数量 (numerical)
- distinguish a **discrete** 离散 quantitative variable from a **continuous** 连续 one
- explain that a variable's values **vary** from one individual to another

1 Worked examples

Study these first. Each one shows the method for a question type used later.

■ **Categorical vs quantitative**

- **Categorical** — puts individuals into groups (eye colour, favourite sport).
- **Quantitative** — a numerical measurement you can do arithmetic with (height, age).

■ **Discrete vs continuous**

- **Discrete** — countable values (number of siblings).
- **Continuous** — any value in a range (exact weight).

The whole point of statistics is that these values **vary** between individuals.

2 Practice

2.1 State the difference between a categorical and a quantitative variable. [2]

2.2 State the difference between discrete and continuous quantitative variables. [2]

2.3 Classify “eye colour” as categorical or quantitative. [1]

3 Exam-style questions

3.1 The number of siblings is a _____ variable. [1]

- **A** categorical
- **B** discrete quantitative
- **C** continuous quantitative
- **D** none of these

3.2 Height in centimetres is a _____ variable. [1]

- **A** categorical
- **B** discrete
- **C** continuous quantitative
- **D** binary

3.3 Classify each variable.

- (a) Favourite colour. [1]
- (b) The number of pets counted. [1]
- (c) A person's exact weight. [1]

Solutions

- 2.1 categorical sorts individuals into groups; quantitative is a numerical measurement.
- 2.2 discrete takes countable values; continuous can take any value in a range.
- 2.3 categorical.
- 3.1 B.
- 3.2 C.
- 3.3 (a) categorical. (b) discrete quantitative. (c) continuous quantitative.

1.3 Representing a Categorical Variable with Tables

Name: _____ Class: _____ Date: _____ Total: 9 marks

KEY WORDS relative frequency 相对频率 • two-way table 双向表 • proportion 比例
• frequency table 频数表 • relative frequencies 相对频数

Objective

Build the skills to answer exam questions on representing a categorical variable with tables.

You must be able to:

- summarise a categorical variable with a frequency table 频数表
- calculate relative frequencies 相对频数 (proportions or percentages)
- build a two-way table 双向表 for two categorical variables

1 Worked examples

Study these first. Each one shows the method for a question type used later.

Frequency and relative frequency

A frequency table lists the count in each category. A relative frequency is that count as a proportion of the total:

relative frequency = $\frac{\text{count in category}}{\text{total}}$.

Two-way tables

A two-way (contingency) table shows two categorical variables together — one across the rows, one across the columns — so you can study their relationship.

2 Practice

2.1 State what a frequency table shows. [1]

2.2 In a class of 25, 10 prefer tea. Find the relative frequency (proportion) who prefer tea. [2]

2.3 State what a two-way table displays. [1]

3 Exam-style questions

3.1 A relative frequency is a [1]

- **A** count
- **B** proportion or percentage
- **C** total
- **D** graph

3.2 A two-way table displays [1]

- **A** one categorical variable
- **B** two categorical variables together
- **C** a histogram
- **D** a mean

3.3 In a survey of 40 people, 12 answered "yes".

(a) Find the relative frequency of "yes". [2]

(b) State what a two-way table would add to this summary. [1]

Solutions

2.1 the count of individuals in each category.

2.2 $\frac{10}{25} = 0.40$ (40%).

2.3 two categorical variables together.

3.1 B.

3.2 B.

3.3 (a) $\frac{12}{40} = 0.30$ (30%). (b) a second categorical variable, to show the relationship between the two.

1.4 Representing a Categorical Variable with Graphs

Name: _____ Class: _____ Date: _____ Total: 8 marks

KEY WORDS bar chart 条形图 • pie chart 饼图 • segmented bar chart 分段条形图
• variable 变量 • categorical 分类

Objective

Build the skills to answer exam questions on **representing a categorical variable with graphs**.

You must be able to:

- display a categorical variable with a **bar chart** 条形图
- use a **pie chart** 饼图 to show each category’s share of the whole
- represent two categorical variables with a **side-by-side** or **segmented bar chart** 分段条形图

1 Worked examples

Study these first. Each one shows the method for a question type used later.

■ **Bar chart and pie chart**

A **bar chart** shows the distribution of a categorical variable using the frequency or relative frequency of each category. A **pie chart** shows each category as a **share of the whole**.

■ **Two categorical variables**

A **side-by-side** or **segmented (stacked) bar chart** compares two categorical variables — for example, drink choice broken down by age group.

2 Practice

2.1 Name a graph for a single categorical variable. [1]

2.2 State what a pie chart shows. [1]

2.3 Name a graph for displaying two categorical variables. [1]

3 Exam-style questions

3.1 A bar chart displays [1]

- A the shape of a quantitative variable
- B the distribution of a categorical variable
- C a scatterplot
- D a z-score

3.2 A pie chart shows each category as [1]

- A a count only
- B a share of the whole
- C a mean
- D an outlier

3.3 A café records customers’ drink choices.

(a) Name a suitable graph for this one variable. [1]

(b) State what a pie chart of the data would show. [1]

(c) Name a graph that compares drink choice by age group. [1]

Solutions

2.1 a bar chart.

2.2 each category as a share (proportion) of the whole.

2.3 a side-by-side or segmented (stacked) bar chart.

3.1 B.

3.2 B.

3.3 (a) a bar chart. (b) each drink as a share of the whole. (c) a segmented or side-by-side bar chart.

AP STATISTICS • EXERCISE SHEET

1.5 Representing a Quantitative Variable with Graphs

Name: _____ Class: _____ Date: _____ **Total: 9 marks**

KEY WORDS histogram 直方图 • bins 组距
• cumulative relative frequency graph 累积相对频数图 • dotplot 点图 • stem-and-leaf plot 茎叶图

Objective

Build the skills to answer exam questions on **representing a quantitative variable with graphs**.

You must be able to:

- display a quantitative variable with a **histogram** 直方图, **dotplot** 点图, or **stem-and-leaf plot** 茎叶图
- choose an appropriate number of **bins** 组距 for a histogram
- use a **cumulative relative frequency graph** 累积相对频数图 (ogive) to estimate percentiles

1 Worked examples

Study these first. Each one shows the method for a question type used later.

■ Graphs for quantitative data

A **histogram**, **dotplot**, or **stem-and-leaf plot** shows the distribution of a numerical variable.

■ Bins

A histogram groups values into **bins** (intervals). Too few bins hide the shape; too many make it look choppy — choose enough to reveal the shape.

■ Ogive

A **cumulative relative frequency graph** (ogive) rises to 1 and lets you read off percentiles.

2 Practice

2.1 Name two graphs for a quantitative variable. [2]

2.2 State what the bins of a histogram are. [1]

2.3 State what an ogive is used to estimate. [1]

3 Exam-style questions

3.1 A graph for a quantitative variable is a [1]

- A bar chart
- B pie chart
- C histogram
- D two-way table

3.2 A cumulative relative frequency graph (ogive) is used to estimate [1]

- A the mode
- B percentiles
- C categories
- D colours

3.3 Data on exam marks is to be displayed.

- (a) Name a suitable graph. [1]
- (b) State what using too few bins might hide. [1]
- (c) Name the graph used to read percentiles. [1]

Solutions

2.1 any two of: histogram, dotplot, stem-and-leaf plot.

2.2 the intervals into which the values are grouped.

2.3 percentiles.

3.1 C.

3.2 B.

3.3 (a) a histogram (or dotplot). (b) the shape of the data. (c) an ogive (cumulative relative frequency graph).

AP STATISTICS • EXERCISE SHEET

1.6 Describing the Distribution of a Quantitative Variable

Name: _____ Class: _____ Date: _____ **Total: 9 marks**

KEY WORDS shape 形状 • mean 均值 • skewed 偏斜 • outliers 离群值 • center 中心

Objective

Build the skills to answer exam questions on **describing the distribution of a quantitative variable**.

You must be able to:

- describe the **shape** 形状 as symmetric, skewed left, or skewed right
- identify the **center** 中心 and the **spread** 离散程度
- locate **clusters**, **gaps**, and possible **outliers** 离群值

1 Worked examples

Study these first. Each one shows the method for a question type used later.

■ Shape, center, spread

Always describe a distribution's **shape**, **center**, and **spread** — and note any distinctive features.

■ Shape

- **Symmetric** — roughly a mirror image.
- **Skewed right** — a long tail to the **right**.
- **Skewed left** — a long tail to the **left**.

■ Features

Look for **clusters** (groups), **gaps** (empty regions), and **outliers** (values far from the rest).

2 Practice

2.1 State the three features you should describe for any distribution. [2]

2.2 Describe what “skewed right” means. [1]

2.3 Define an outlier. [1]

3 Exam-style questions

3.1 A distribution with a long tail to the right is [1]

- A symmetric
- B skewed left
- C skewed right
- D uniform

3.2 When describing a distribution you should mention shape, center, and [1]

- A colour
- B spread
- C the mean only
- D the category

3.3 A histogram has its peak on the left and a long right tail.

(a) Name the shape. [1]

(b) Name one other feature you should describe. [1]

(c) Define an outlier. [1]

Solutions

2.1 shape, center, spread.

2.2 it has a long tail extending to the right (higher values).

2.3 a value that is far from the rest of the data.

3.1 C.

3.2 B.

3.3 (a) skewed right. (b) center or spread (or clusters/gaps/outliers). (c) a value far from the rest of the data.

Name:

AP Statistics: 2024

Begin your response to **QUESTION 6** on this page.

SECTION II, Part B

Suggested Time—25 minutes

1 Question

Directions: Show all your work. Indicate clearly the methods you use, because you will be scored on the correctness of your methods as well as on the accuracy and completeness of your results and explanations.

6. A company sells a certain type of whistle. The price of the whistle varies from store to store. Julio, a statistician at the company, wants to estimate the mean price, in dollars (\$), of this type of whistle at all stores that sell the whistle.

(a) (i) Identify the appropriate inference procedure for Julio to use.

(ii) Describe the parameter for the inference procedure you identified in part (a-i) in context.

Julio called the managers of 20 randomly selected stores that sell the whistle and recorded the price of the whistle at each store. Following is a dotplot of Julio's data.

Price of the Whistle at 20 Stores



GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 6** on this page.

The summary statistics for Julio's data are shown in the following table.

Summary Statistics for Julio's Data

Sample Size	Mean	Standard Deviation	Minimum	Q_1	Median	Q_3	Maximum
20	5.12	0.743	4.25	4.51	4.885	5.475	6.58

(b) Julio wants to examine some characteristics of the distribution of the sample of whistle prices.

(i) Describe the shape of the distribution of the sample of whistle prices. Justify your response using appropriate values from the summary statistics table.

(ii) Using the $1.5 \times \text{IQR}$ rule, determine whether there are any outliers in the sample of whistle prices. Justify your response.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 6** on this page.

It can often be difficult to determine whether the distribution of sample data is skewed by looking at a graph of the data and the summary statistics, particularly when the sample size is small. Thus, statisticians sometimes measure how skewed a data set is. One such measure is Pearson's coefficient of skewness, which is calculated using the following formula.

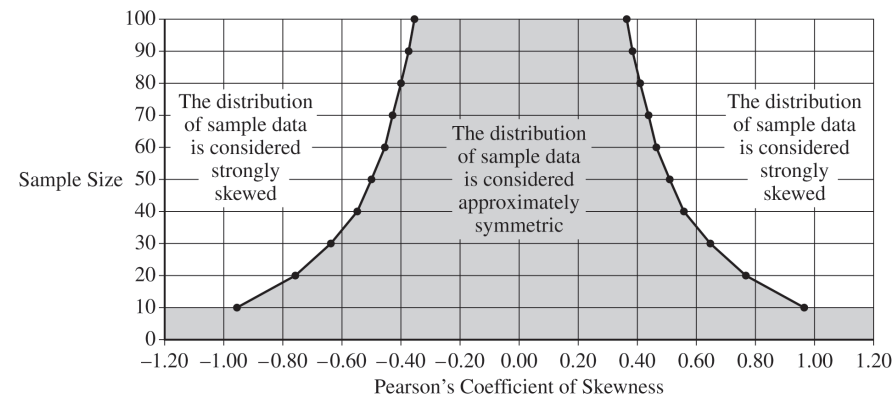
$$\text{Pearson's Coefficient of Skewness} = \frac{3(\bar{x} - m)}{s}$$

In the formula, $\bar{x}$ is the sample mean, m is the sample median, and s is the sample standard deviation.

(c) (i) Calculate Pearson's coefficient of skewness for Julio's sample of 20 whistle prices. Show your work.

The following graph shows conclusions that can be made about the shape of the distribution of sample data based on Pearson's coefficient of skewness and sample size.

Conclusion from Pearson's Coefficient of Skewness



(ii) Indicate the value of the Pearson's coefficient of skewness you calculated in part (c-i) for the appropriate sample size by marking it with an "X" on the preceding graph.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 6** on this page.

(d) Consider your work in part (c).

(i) What should you conclude about the shape of the distribution of the sample of whistle prices? Justify your response.

Julio's inference procedure in part (a-i) needs one of the following requirements to be satisfied to verify the normality condition.

- The sample size is greater than or equal to 30.
- If the sample size is less than 30, the distribution of the sample data is not strongly skewed and does not have outliers.

(ii) Using your response to (d-i) and the preceding requirements, is the normality condition satisfied for Julio's data? Explain your response.

GO ON TO THE NEXT PAGE.

STOP

END OF EXAM

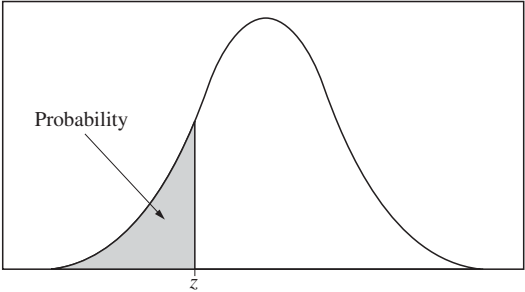


Table entry for z is the probability lying below z .

Table A Standard normal probabilities

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

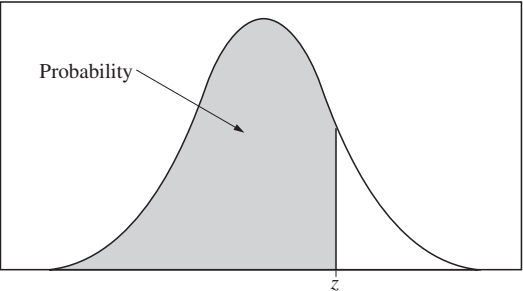


Table entry for z is the probability lying below z .

Table A (Continued)

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

Table entry for p and C is the point t^* with probability p lying above it and probability C lying between $-t^*$ and t^* .

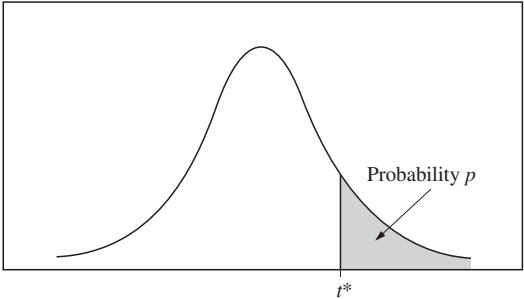


Table B t distribution critical values

df	Tail probability p											
	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.000	1.376	1.963	3.078	6.314	12.71	15.89	31.82	63.66	127.3	318.3	636.6
2	.816	1.061	1.386	1.886	2.920	4.303	4.849	6.965	9.925	14.09	22.33	31.60
3	.765	.978	1.250	1.638	2.353	3.182	3.482	4.541	5.841	7.453	10.21	12.92
4	.741	.941	1.190	1.533	2.132	2.776	2.999	3.747	4.604	5.598	7.173	8.610
5	.727	.920	1.156	1.476	2.015	2.571	2.757	3.365	4.032	4.773	5.893	6.869
6	.718	.906	1.134	1.440	1.943	2.447	2.612	3.143	3.707	4.317	5.208	5.959
7	.711	.896	1.119	1.415	1.895	2.365	2.517	2.998	3.499	4.029	4.785	5.408
8	.706	.889	1.108	1.397	1.860	2.306	2.449	2.896	3.355	3.833	4.501	5.041
9	.703	.883	1.100	1.383	1.833	2.262	2.398	2.821	3.250	3.690	4.297	4.781
10	.700	.879	1.093	1.372	1.812	2.228	2.359	2.764	3.169	3.581	4.144	4.587
11	.697	.876	1.088	1.363	1.796	2.201	2.328	2.718	3.106	3.497	4.025	4.437
12	.695	.873	1.083	1.356	1.782	2.179	2.303	2.681	3.055	3.428	3.930	4.318
13	.694	.870	1.079	1.350	1.771	2.160	2.282	2.650	3.012	3.372	3.852	4.221
14	.692	.868	1.076	1.345	1.761	2.145	2.264	2.624	2.977	3.326	3.787	4.140
15	.691	.866	1.074	1.341	1.753	2.131	2.249	2.602	2.947	3.286	3.733	4.073
16	.690	.865	1.071	1.337	1.746	2.120	2.235	2.583	2.921	3.252	3.686	4.015
17	.689	.863	1.069	1.333	1.740	2.110	2.224	2.567	2.898	3.222	3.646	3.965
18	.688	.862	1.067	1.330	1.734	2.101	2.214	2.552	2.878	3.197	3.611	3.922
19	.688	.861	1.066	1.328	1.729	2.093	2.205	2.539	2.861	3.174	3.579	3.883
20	.687	.860	1.064	1.325	1.725	2.086	2.197	2.528	2.845	3.153	3.552	3.850
21	.686	.859	1.063	1.323	1.721	2.080	2.189	2.518	2.831	3.135	3.527	3.819
22	.686	.858	1.061	1.321	1.717	2.074	2.183	2.508	2.819	3.119	3.505	3.792
23	.685	.858	1.060	1.319	1.714	2.069	2.177	2.500	2.807	3.104	3.485	3.768
24	.685	.857	1.059	1.318	1.711	2.064	2.172	2.492	2.797	3.091	3.467	3.745
25	.684	.856	1.058	1.316	1.708	2.060	2.167	2.485	2.787	3.078	3.450	3.725
26	.684	.856	1.058	1.315	1.706	2.056	2.162	2.479	2.779	3.067	3.435	3.707
27	.684	.855	1.057	1.314	1.703	2.052	2.158	2.473	2.771	3.057	3.421	3.690
28	.683	.855	1.056	1.313	1.701	2.048	2.154	2.467	2.763	3.047	3.408	3.674
29	.683	.854	1.055	1.311	1.699	2.045	2.150	2.462	2.756	3.038	3.396	3.659
30	.683	.854	1.055	1.310	1.697	2.042	2.147	2.457	2.750	3.030	3.385	3.646
40	.681	.851	1.050	1.303	1.684	2.021	2.123	2.423	2.704	2.971	3.307	3.551
50	.679	.849	1.047	1.299	1.676	2.009	2.109	2.403	2.678	2.937	3.261	3.496
60	.679	.848	1.045	1.296	1.671	2.000	2.099	2.390	2.660	2.915	3.232	3.460
80	.678	.846	1.043	1.292	1.664	1.990	2.088	2.374	2.639	2.887	3.195	3.416
100	.677	.845	1.042	1.290	1.660	1.984	2.081	2.364	2.626	2.871	3.174	3.390
1000	.675	.842	1.037	1.282	1.646	1.962	2.056	2.330	2.581	2.813	3.098	3.300
∞	.674	.841	1.036	1.282	1.645	1.960	2.054	2.326	2.576	2.807	3.091	3.291

50%	60%	70%	80%	90%	95%	96%	98%	99%	99.5%	99.8%	99.9%
Confidence level C											

Table entry for p is the point (χ^2) with probability p lying above it.

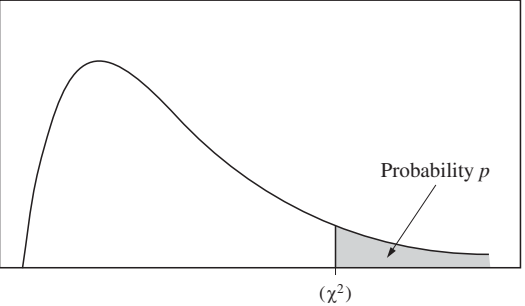


Table C χ^2 critical values

	Tail probability p											
df	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.32	1.64	2.07	2.71	3.84	5.02	5.41	6.63	7.88	9.14	10.83	12.12
2	2.77	3.22	3.79	4.61	5.99	7.38	7.82	9.21	10.60	11.98	13.82	15.20
3	4.11	4.64	5.32	6.25	7.81	9.35	9.84	11.34	12.84	14.32	16.27	17.73
4	5.39	5.99	6.74	7.78	9.49	11.14	11.67	13.28	14.86	16.42	18.47	20.00
5	6.63	7.29	8.12	9.24	11.07	12.83	13.39	15.09	16.75	18.39	20.51	22.11
6	7.84	8.56	9.45	10.64	12.59	14.45	15.03	16.81	18.55	20.25	22.46	24.10
7	9.04	9.80	10.75	12.02	14.07	16.01	16.62	18.48	20.28	22.04	24.32	26.02
8	10.22	11.03	12.03	13.36	15.51	17.53	18.17	20.09	21.95	23.77	26.12	27.87
9	11.39	12.24	13.29	14.68	16.92	19.02	19.68	21.67	23.59	25.46	27.88	29.67
10	12.55	13.44	14.53	15.99	18.31	20.48	21.16	23.21	25.19	27.11	29.59	31.42
11	13.70	14.63	15.77	17.28	19.68	21.92	22.62	24.72	26.76	28.73	31.26	33.14
12	14.85	15.81	16.99	18.55	21.03	23.34	24.05	26.22	28.30	30.32	32.91	34.82
13	15.98	16.98	18.20	19.81	22.36	24.74	25.47	27.69	29.82	31.88	34.53	36.48
14	17.12	18.15	19.41	21.06	23.68	26.12	26.87	29.14	31.32	33.43	36.12	38.11
15	18.25	19.31	20.60	22.31	25.00	27.49	28.26	30.58	32.80	34.95	37.70	39.72
16	19.37	20.47	21.79	23.54	26.30	28.85	29.63	32.00	34.27	36.46	39.25	41.31
17	20.49	21.61	22.98	24.77	27.59	30.19	31.00	33.41	35.72	37.95	40.79	42.88
18	21.60	22.76	24.16	25.99	28.87	31.53	32.35	34.81	37.16	39.42	42.31	44.43
19	22.72	23.90	25.33	27.20	30.14	32.85	33.69	36.19	38.58	40.88	43.82	45.97
20	23.83	25.04	26.50	28.41	31.41	34.17	35.02	37.57	40.00	42.34	45.31	47.50
21	24.93	26.17	27.66	29.62	32.67	35.48	36.34	38.93	41.40	43.78	46.80	49.01
22	26.04	27.30	28.82	30.81	33.92	36.78	37.66	40.29	42.80	45.20	48.27	50.51
23	27.14	28.43	29.98	32.01	35.17	38.08	38.97	41.64	44.18	46.62	49.73	52.00
24	28.24	29.55	31.13	33.20	36.42	39.36	40.27	42.98	45.56	48.03	51.18	53.48
25	29.34	30.68	32.28	34.38	37.65	40.65	41.57	44.31	46.93	49.44	52.62	54.95
26	30.43	31.79	33.43	35.56	38.89	41.92	42.86	45.64	48.29	50.83	54.05	56.41
27	31.53	32.91	34.57	36.74	40.11	43.19	44.14	46.96	49.64	52.22	55.48	57.86
28	32.62	34.03	35.71	37.92	41.34	44.46	45.42	48.28	50.99	53.59	56.89	59.30
29	33.71	35.14	36.85	39.09	42.56	45.72	46.69	49.59	52.34	54.97	58.30	60.73
30	34.80	36.25	37.99	40.26	43.77	46.98	47.96	50.89	53.67	56.33	59.70	62.16
40	45.62	47.27	49.24	51.81	55.76	59.34	60.44	63.69	66.77	69.70	73.40	76.09
50	56.33	58.16	60.35	63.17	67.50	71.42	72.61	76.15	79.49	82.66	86.66	89.56
60	66.98	68.97	71.34	74.40	79.08	83.30	84.58	88.38	91.95	95.34	99.61	102.7
80	88.13	90.41	93.11	96.58	101.9	106.6	108.1	112.3	116.3	120.1	124.8	128.3
100	109.1	111.7	114.7	118.5	124.3	129.6	131.1	135.8	140.2	144.3	149.4	153.2

Descriptive Statistics

$$\bar{x} = \frac{1}{n} \sum x_i = \frac{\sum x_i}{n}$$

$$s_x = \sqrt{\frac{1}{n-1} \sum (x_i - \bar{x})^2} = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n-1}}$$

$$\hat{y} = a + bx$$

$$\bar{y} = a + b\bar{x}$$

$$r = \frac{1}{n-1} \sum \left(\frac{x_i - \bar{x}}{s_x} \right) \left(\frac{y_i - \bar{y}}{s_y} \right)$$

$$b = r \frac{s_y}{s_x}$$

Descriptive Statistics

$$\bar{x} = \frac{1}{n} \sum x_i = \frac{\sum x_i}{n}$$

$$\hat{y} = a + bx$$

$$r = \frac{1}{n-1} \sum \left(\frac{x_i - \bar{x}}{s_x} \right) \left(\frac{y_i - \bar{y}}{s_y} \right)$$

$$s_x = \sqrt{\frac{1}{n-1} \sum (x_i - \bar{x})^2} = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n-1}}$$

$$\bar{y} = a + b\bar{x}$$

$$b = r \frac{s_y}{s_x}$$

STATISTICS

SECTION II

Part A

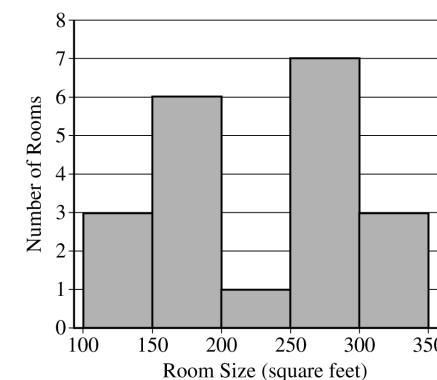
Questions 1-5

Spend about 1 hour and 5 minutes on this part of the exam.

Percent of Section II score—75

Directions: Show all your work. Indicate clearly the methods you use, because you will be scored on the correctness of your methods as well as on the accuracy and completeness of your results and explanations.

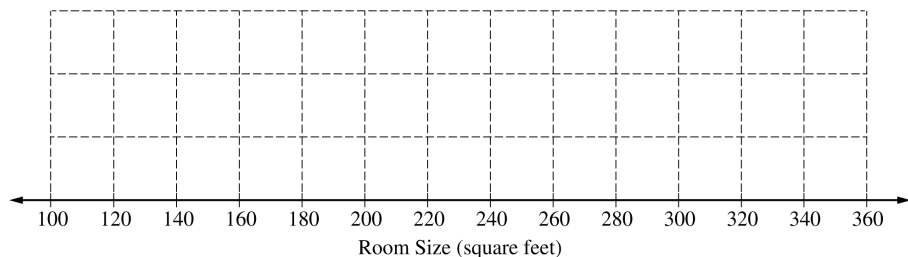
1. The sizes, in square feet, of the 20 rooms in a student residence hall at a certain university are summarized in the following histogram.



- (a) Based on the histogram, write a few sentences describing the distribution of room size in the residence hall.
- (b) Summary statistics for the sizes are given in the following table.

Mean	Standard Deviation	Min	Q1	Median	Q3	Max
231.4	68.12	134	174	253.5	292	315

Determine whether there are potential outliers in the data. Then use the following grid to sketch a boxplot of room size.



- (c) What characteristic of the shape of the distribution of room size is apparent from the histogram but not from the boxplot?

STATISTICS
SECTION II
Part A

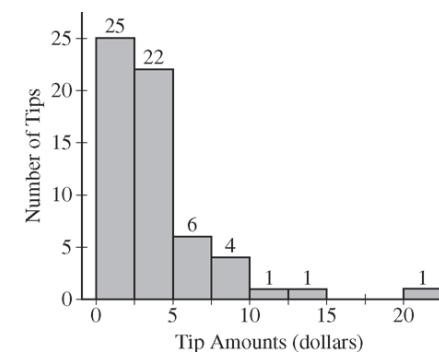
Questions 1-5

Spend about 65 minutes on this part of the exam.

Percent of Section II score—75

Directions: Show all your work. Indicate clearly the methods you use, because you will be scored on the correctness of your methods as well as on the accuracy and completeness of your results and explanations.

1. Robin works as a server in a small restaurant, where she can earn a tip (extra money) from each customer she serves. The histogram below shows the distribution of her 60 tip amounts for one day of work.



- (a) Write a few sentences to describe the distribution of tip amounts for the day shown.
- (b) One of the tip amounts was \$8. If the \$8 tip had been \$18, what effect would the increase have had on the following statistics? Justify your answers.

The mean:

The median:

1.7 Summary Statistics for a Quantitative Variable

Name: _____ Class: _____ Date: _____ **Total: 11 marks**

KEY WORDS mean 均值 • median 中位数 • standard deviation 标准差 • resistant 稳健
• interquartile range 四分位距

Objective

Build the skills to answer exam questions on **summary statistics for a quantitative variable**.

You must be able to:

- calculate the **mean** 均值 $\bar{x}$ and the **median** 中位数
- calculate the **range**, **interquartile range** 四分位距 $IQR = Q_3 - Q_1$, and **standard deviation** 标准差 s_x
- explain why the median and IQR are **resistant** 稳健 to outliers

1 Worked examples

Study these first. Each one shows the method for a question type used later.

■ Measures of center

- Mean** $\bar{x} = \frac{\sum x}{n}$.
- Median** — the middle value when the data are ordered.

■ Measures of spread

- Range** = max – min.
- IQR** = $Q_3 - Q_1$.
- Standard deviation** s_x — the typical distance from the mean.

■ Resistance

The **median** and **IQR** are **resistant** to outliers; the **mean** and **standard deviation** are **not**.

2 Practice

2.1 Find the mean of {3, 5, 7, 9}. [2]

2.2 Find the median of {2, 6, 4, 8, 10}. [2]

2.3 State which measures of center and spread resist outliers. [1]

3 Exam-style questions

3.1 The interquartile range equals [1]

- A max – min
- B $Q_3 - Q_1$
- C mean – median
- D $Q_1 - Q_3$

3.2 Which of these is resistant to outliers? [1]

- A the mean
- B the standard deviation
- C the median
- D the range

3.3 For the data {4, 8, 6, 10, 12}:

(a) find the mean. [2]

(b) find the median. [1]

(c) find the range. [1]

Solutions

2.1 $\bar{x} = \frac{3 + 5 + 7 + 9}{4} = \frac{24}{4} = 6.$

2.2 ordered: 2, 4, 6, 8, 10; the middle value is 6.

2.3 the median (center) and the IQR (spread).

3.1 B.

3.2 C.

3.3 (a) $\frac{4 + 8 + 6 + 10 + 12}{5} = \frac{40}{5} = 8.$ (b) ordered 4, 6, 8, 10, 12 $\rightarrow$ median 8. (c) $12 - 4 = 8.$

1. A goat farmer raises two breeds of goats, Breed H and Breed J. He wants to compare the weights of the two breeds and takes independent random samples of 14 goats from each breed. The weight, in pounds, of each goat is measured. The weights of the 14 Breed H goats are recorded.

Weights of Breed H Goats (pounds)

48
48
55
56
56
57
62
66
72
72
72
73
80
80

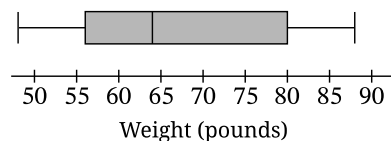
Part A

Use the data in the list to determine the five-number summary for the sample of Breed H goats.

Part B

The distribution of weight for Breed J goats is displayed in the boxplot.

Weights of Breed J Goats (pounds)



Use the five-number summary from part A and the boxplot of Breed J to compare the center and variability for the distribution of weight for the Breed H goats and the distribution of weight for the Breed J goats, in context.

Part C

A stem-and-leaf plot is another way to display the weights of goats. The distribution of weight for the Breed H goats is displayed in the given stem-and-leaf plot.

Weights of Breed H Goats (pounds)

4		8 8
5		5 6 6 7
6		2 6
7		2 2 2 3
8		0 0

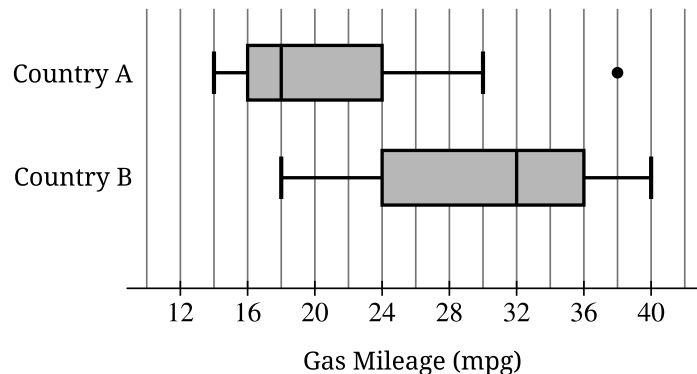
Key: 4|8 = 48 pounds

Consider the five-number summary from part A and the stem-and-leaf plot.

- If a boxplot were created from the five-number summary found in part A, what characteristic of the shape of the distribution of weight for the Breed H goats would be apparent from the stem-and-leaf plot but not from the boxplot?
- Explain why a boxplot would not display the characteristic of the shape of the distribution identified in part C (i).

1. The manager of an automotive company is interested in comparing the gas mileages for cars manufactured in Country A and cars manufactured in Country B. The manager selected a random sample of 100 cars manufactured in Country A and a random sample of 100 cars manufactured in Country B. The gas mileages for each sample, in miles per gallon (mpg), are summarized in the boxplots.

Boxplots of Gas Mileage for Each Country



- A. Compare the distributions of gas mileage for the sample of cars manufactured in Country A and the sample of cars manufactured in Country B.
- B. For the distribution of gas mileage for the sample of cars manufactured in Country A, would you expect the mean to be greater than 18 mpg, less than 18 mpg, or equal to 18 mpg? Justify your answer.
- C. The manager will create a new boxplot with the combined data from the sample of cars manufactured in Country A and the sample of cars manufactured in Country B.
- What is the range of the combined data set? Justify your answer.
 - What is a possible value of the median of the combined data set? Justify your answer by referencing the boxplots shown.

Begin your response to **QUESTION 6** on this page.

SECTION II, Part B

Suggested Time—25 minutes

1 Question

Directions: Show all your work. Indicate clearly the methods you use, because you will be scored on the correctness of your methods as well as on the accuracy and completeness of your results and explanations.

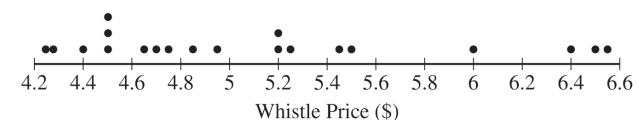
6. A company sells a certain type of whistle. The price of the whistle varies from store to store. Julio, a statistician at the company, wants to estimate the mean price, in dollars (\$), of this type of whistle at all stores that sell the whistle.

(a) (i) Identify the appropriate inference procedure for Julio to use.

(ii) Describe the parameter for the inference procedure you identified in part (a-i) in context.

Julio called the managers of 20 randomly selected stores that sell the whistle and recorded the price of the whistle at each store. Following is a dotplot of Julio's data.

Price of the Whistle at 20 Stores



GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 6** on this page.

The summary statistics for Julio's data are shown in the following table.

Summary Statistics for Julio's Data

Sample Size	Mean	Standard Deviation	Minimum	Q_1	Median	Q_3	Maximum
20	5.12	0.743	4.25	4.51	4.885	5.475	6.58

(b) Julio wants to examine some characteristics of the distribution of the sample of whistle prices.

(i) Describe the shape of the distribution of the sample of whistle prices. Justify your response using appropriate values from the summary statistics table.

(ii) Using the $1.5 \times \text{IQR}$ rule, determine whether there are any outliers in the sample of whistle prices. Justify your response.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 6** on this page.

It can often be difficult to determine whether the distribution of sample data is skewed by looking at a graph of the data and the summary statistics, particularly when the sample size is small. Thus, statisticians sometimes measure how skewed a data set is. One such measure is Pearson's coefficient of skewness, which is calculated using the following formula.

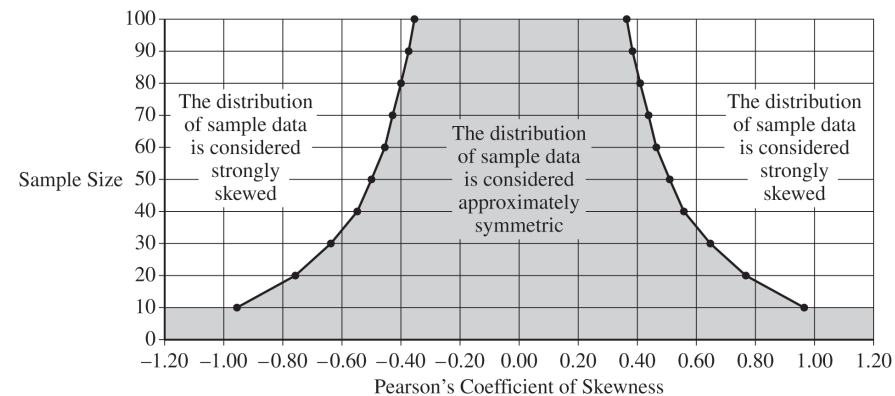
$$\text{Pearson's Coefficient of Skewness} = \frac{3(\bar{x} - m)}{s}$$

In the formula, $\bar{x}$ is the sample mean, m is the sample median, and s is the sample standard deviation.

(c) (i) Calculate Pearson's coefficient of skewness for Julio's sample of 20 whistle prices. Show your work.

The following graph shows conclusions that can be made about the shape of the distribution of sample data based on Pearson's coefficient of skewness and sample size.

Conclusion from Pearson's Coefficient of Skewness



(ii) Indicate the value of the Pearson's coefficient of skewness you calculated in part (c-i) for the appropriate sample size by marking it with an "X" on the preceding graph.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 6** on this page.

(d) Consider your work in part (c).

(i) What should you conclude about the shape of the distribution of the sample of whistle prices? Justify your response.

Julio's inference procedure in part (a-i) needs one of the following requirements to be satisfied to verify the normality condition.

- The sample size is greater than or equal to 30.
- If the sample size is less than 30, the distribution of the sample data is not strongly skewed and does not have outliers.

(ii) Using your response to (d-i) and the preceding requirements, is the normality condition satisfied for Julio's data? Explain your response.

GO ON TO THE NEXT PAGE.

STOP

END OF EXAM

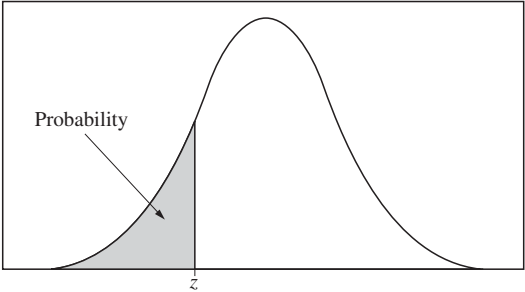


Table entry for z is the probability lying below z .

Table A Standard normal probabilities

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

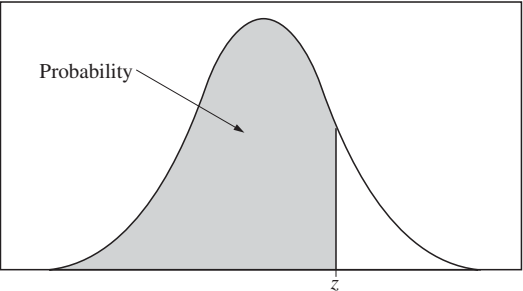


Table entry for z is the probability lying below z .

Table A (Continued)

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

Table entry for p and C is the point t^* with probability p lying above it and probability C lying between $-t^*$ and t^* .

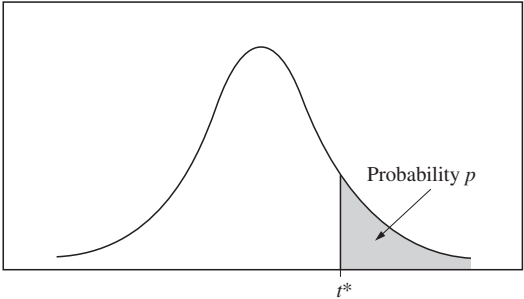


Table B t distribution critical values

df	Tail probability p											
	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.000	1.376	1.963	3.078	6.314	12.71	15.89	31.82	63.66	127.3	318.3	636.6
2	.816	1.061	1.386	1.886	2.920	4.303	4.849	6.965	9.925	14.09	22.33	31.60
3	.765	.978	1.250	1.638	2.353	3.182	3.482	4.541	5.841	7.453	10.21	12.92
4	.741	.941	1.190	1.533	2.132	2.776	2.999	3.747	4.604	5.598	7.173	8.610
5	.727	.920	1.156	1.476	2.015	2.571	2.757	3.365	4.032	4.773	5.893	6.869
6	.718	.906	1.134	1.440	1.943	2.447	2.612	3.143	3.707	4.317	5.208	5.959
7	.711	.896	1.119	1.415	1.895	2.365	2.517	2.998	3.499	4.029	4.785	5.408
8	.706	.889	1.108	1.397	1.860	2.306	2.449	2.896	3.355	3.833	4.501	5.041
9	.703	.883	1.100	1.383	1.833	2.262	2.398	2.821	3.250	3.690	4.297	4.781
10	.700	.879	1.093	1.372	1.812	2.228	2.359	2.764	3.169	3.581	4.144	4.587
11	.697	.876	1.088	1.363	1.796	2.201	2.328	2.718	3.106	3.497	4.025	4.437
12	.695	.873	1.083	1.356	1.782	2.179	2.303	2.681	3.055	3.428	3.930	4.318
13	.694	.870	1.079	1.350	1.771	2.160	2.282	2.650	3.012	3.372	3.852	4.221
14	.692	.868	1.076	1.345	1.761	2.145	2.264	2.624	2.977	3.326	3.787	4.140
15	.691	.866	1.074	1.341	1.753	2.131	2.249	2.602	2.947	3.286	3.733	4.073
16	.690	.865	1.071	1.337	1.746	2.120	2.235	2.583	2.921	3.252	3.686	4.015
17	.689	.863	1.069	1.333	1.740	2.110	2.224	2.567	2.898	3.222	3.646	3.965
18	.688	.862	1.067	1.330	1.734	2.101	2.214	2.552	2.878	3.197	3.611	3.922
19	.688	.861	1.066	1.328	1.729	2.093	2.205	2.539	2.861	3.174	3.579	3.883
20	.687	.860	1.064	1.325	1.725	2.086	2.197	2.528	2.845	3.153	3.552	3.850
21	.686	.859	1.063	1.323	1.721	2.080	2.189	2.518	2.831	3.135	3.527	3.819
22	.686	.858	1.061	1.321	1.717	2.074	2.183	2.508	2.819	3.119	3.505	3.792
23	.685	.858	1.060	1.319	1.714	2.069	2.177	2.500	2.807	3.104	3.485	3.768
24	.685	.857	1.059	1.318	1.711	2.064	2.172	2.492	2.797	3.091	3.467	3.745
25	.684	.856	1.058	1.316	1.708	2.060	2.167	2.485	2.787	3.078	3.450	3.725
26	.684	.856	1.058	1.315	1.706	2.056	2.162	2.479	2.779	3.067	3.435	3.707
27	.684	.855	1.057	1.314	1.703	2.052	2.158	2.473	2.771	3.057	3.421	3.690
28	.683	.855	1.056	1.313	1.701	2.048	2.154	2.467	2.763	3.047	3.408	3.674
29	.683	.854	1.055	1.311	1.699	2.045	2.150	2.462	2.756	3.038	3.396	3.659
30	.683	.854	1.055	1.310	1.697	2.042	2.147	2.457	2.750	3.030	3.385	3.646
40	.681	.851	1.050	1.303	1.684	2.021	2.123	2.423	2.704	2.971	3.307	3.551
50	.679	.849	1.047	1.299	1.676	2.009	2.109	2.403	2.678	2.937	3.261	3.496
60	.679	.848	1.045	1.296	1.671	2.000	2.099	2.390	2.660	2.915	3.232	3.460
80	.678	.846	1.043	1.292	1.664	1.990	2.088	2.374	2.639	2.887	3.195	3.416
100	.677	.845	1.042	1.290	1.660	1.984	2.081	2.364	2.626	2.871	3.174	3.390
1000	.675	.842	1.037	1.282	1.646	1.962	2.056	2.330	2.581	2.813	3.098	3.300
∞	.674	.841	1.036	1.282	1.645	1.960	2.054	2.326	2.576	2.807	3.091	3.291

50%	60%	70%	80%	90%	95%	96%	98%	99%	99.5%	99.8%	99.9%
Confidence level C											

Table entry for p is the point (χ^2) with probability p lying above it.

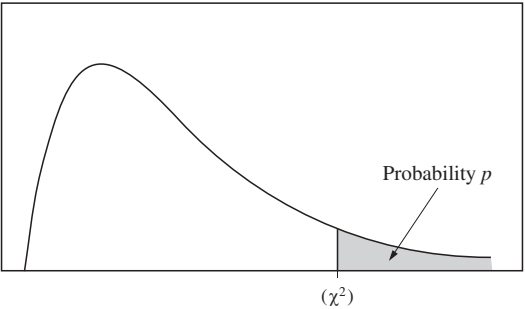


Table C χ^2 critical values

	Tail probability p											
df	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.32	1.64	2.07	2.71	3.84	5.02	5.41	6.63	7.88	9.14	10.83	12.12
2	2.77	3.22	3.79	4.61	5.99	7.38	7.82	9.21	10.60	11.98	13.82	15.20
3	4.11	4.64	5.32	6.25	7.81	9.35	9.84	11.34	12.84	14.32	16.27	17.73
4	5.39	5.99	6.74	7.78	9.49	11.14	11.67	13.28	14.86	16.42	18.47	20.00
5	6.63	7.29	8.12	9.24	11.07	12.83	13.39	15.09	16.75	18.39	20.51	22.11
6	7.84	8.56	9.45	10.64	12.59	14.45	15.03	16.81	18.55	20.25	22.46	24.10
7	9.04	9.80	10.75	12.02	14.07	16.01	16.62	18.48	20.28	22.04	24.32	26.02
8	10.22	11.03	12.03	13.36	15.51	17.53	18.17	20.09	21.95	23.77	26.12	27.87
9	11.39	12.24	13.29	14.68	16.92	19.02	19.68	21.67	23.59	25.46	27.88	29.67
10	12.55	13.44	14.53	15.99	18.31	20.48	21.16	23.21	25.19	27.11	29.59	31.42
11	13.70	14.63	15.77	17.28	19.68	21.92	22.62	24.72	26.76	28.73	31.26	33.14
12	14.85	15.81	16.99	18.55	21.03	23.34	24.05	26.22	28.30	30.32	32.91	34.82
13	15.98	16.98	18.20	19.81	22.36	24.74	25.47	27.69	29.82	31.88	34.53	36.48
14	17.12	18.15	19.41	21.06	23.68	26.12	26.87	29.14	31.32	33.43	36.12	38.11
15	18.25	19.31	20.60	22.31	25.00	27.49	28.26	30.58	32.80	34.95	37.70	39.72
16	19.37	20.47	21.79	23.54	26.30	28.85	29.63	32.00	34.27	36.46	39.25	41.31
17	20.49	21.61	22.98	24.77	27.59	30.19	31.00	33.41	35.72	37.95	40.79	42.88
18	21.60	22.76	24.16	25.99	28.87	31.53	32.35	34.81	37.16	39.42	42.31	44.43
19	22.72	23.90	25.33	27.20	30.14	32.85	33.69	36.19	38.58	40.88	43.82	45.97
20	23.83	25.04	26.50	28.41	31.41	34.17	35.02	37.57	40.00	42.34	45.31	47.50
21	24.93	26.17	27.66	29.62	32.67	35.48	36.34	38.93	41.40	43.78	46.80	49.01
22	26.04	27.30	28.82	30.81	33.92	36.78	37.66	40.29	42.80	45.20	48.27	50.51
23	27.14	28.43	29.98	32.01	35.17	38.08	38.97	41.64	44.18	46.62	49.73	52.00
24	28.24	29.55	31.13	33.20	36.42	39.36	40.27	42.98	45.56	48.03	51.18	53.48
25	29.34	30.68	32.28	34.38	37.65	40.65	41.57	44.31	46.93	49.44	52.62	54.95
26	30.43	31.79	33.43	35.56	38.89	41.92	42.86	45.64	48.29	50.83	54.05	56.41
27	31.53	32.91	34.57	36.74	40.11	43.19	44.14	46.96	49.64	52.22	55.48	57.86
28	32.62	34.03	35.71	37.92	41.34	44.46	45.42	48.28	50.99	53.59	56.89	59.30
29	33.71	35.14	36.85	39.09	42.56	45.72	46.69	49.59	52.34	54.97	58.30	60.73
30	34.80	36.25	37.99	40.26	43.77	46.98	47.96	50.89	53.67	56.33	59.70	62.16
40	45.62	47.27	49.24	51.81	55.76	59.34	60.44	63.69	66.77	69.70	73.40	76.09
50	56.33	58.16	60.35	63.17	67.50	71.42	72.61	76.15	79.49	82.66	86.66	89.56
60	66.98	68.97	71.34	74.40	79.08	83.30	84.58	88.38	91.95	95.34	99.61	102.7
80	88.13	90.41	93.11	96.58	101.9	106.6	108.1	112.3	116.3	120.1	124.8	128.3
100	109.1	111.7	114.7	118.5	124.3	129.6	131.1	135.8	140.2	144.3	149.4	153.2

Descriptive Statistics

$$\bar{x} = \frac{1}{n} \sum x_i = \frac{\sum x_i}{n}$$

$$s_x = \sqrt{\frac{1}{n-1} \sum (x_i - \bar{x})^2} = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n-1}}$$

$$\hat{y} = a + bx$$

$$\bar{y} = a + b\bar{x}$$

$$r = \frac{1}{n-1} \sum \left(\frac{x_i - \bar{x}}{s_x} \right) \left(\frac{y_i - \bar{y}}{s_y} \right)$$

$$b = r \frac{s_y}{s_x}$$

STATISTICS
SECTION II

Part A

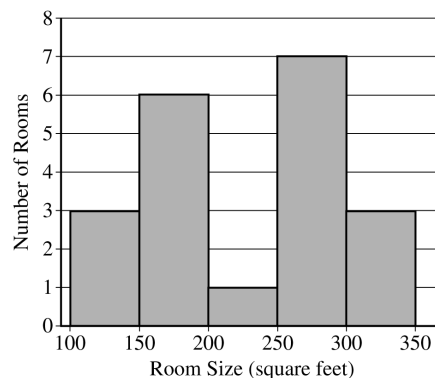
Questions 1-5

Spend about 1 hour and 5 minutes on this part of the exam.

Percent of Section II score—75

Directions: Show all your work. Indicate clearly the methods you use, because you will be scored on the correctness of your methods as well as on the accuracy and completeness of your results and explanations.

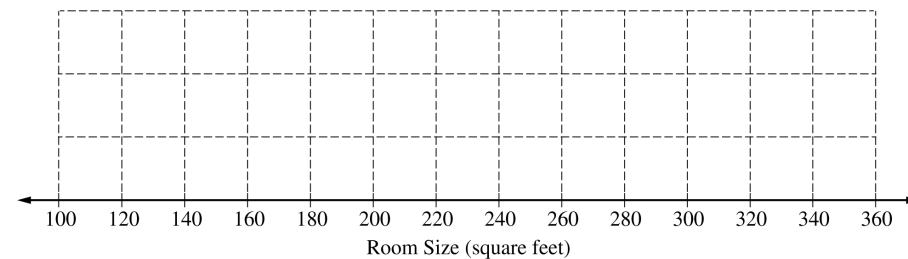
1. The sizes, in square feet, of the 20 rooms in a student residence hall at a certain university are summarized in the following histogram.



- (a) Based on the histogram, write a few sentences describing the distribution of room size in the residence hall.
- (b) Summary statistics for the sizes are given in the following table.

Mean	Standard Deviation	Min	Q1	Median	Q3	Max
231.4	68.12	134	174	253.5	292	315

Determine whether there are potential outliers in the data. Then use the following grid to sketch a boxplot of room size.



- (c) What characteristic of the shape of the distribution of room size is apparent from the histogram but not from the boxplot?

STATISTICS
SECTION II

Part B

Question 6

Spend about 25 minutes on this part of the exam.

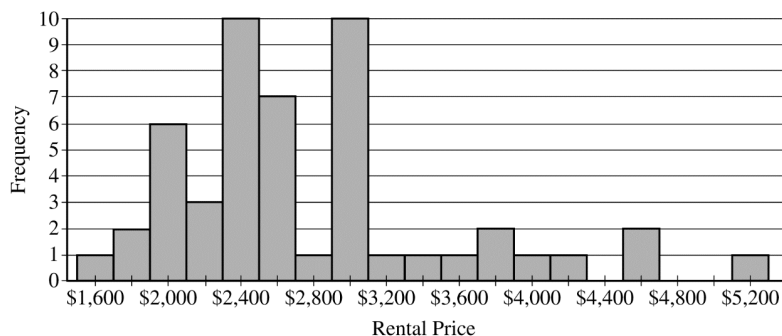
Percent of Section II score—25

Directions: Show all your work. Indicate clearly the methods you use, because you will be scored on the correctness of your methods as well as on the accuracy and completeness of your results and explanations.

6. Emma is moving to a large city and is investigating typical monthly rental prices of available one-bedroom apartments. She obtained a random sample of rental prices for 50 one-bedroom apartments taken from a Web site where people voluntarily list available apartments.

(a) Describe the population for which it is appropriate for Emma to generalize the results from her sample.

The distribution of the 50 rental prices of the available apartments is shown in the following histogram.



- (b) Emma wants to estimate the typical rental price of a one-bedroom apartment in the city. Based on the distribution shown, what is a disadvantage of using the mean rather than the median as an estimate of the typical rental price?
- (c) Instead of using the sample median as the point estimate for the population median, Emma wants to use an interval estimate. However, computing an interval estimate requires knowing the sampling distribution of the sample median for samples of size 50. Emma has one point, her sample median, in that sampling distribution. Using information about rental prices that are available on the Web site, describe how someone could develop a theoretical sampling distribution of the sample median for samples of size 50.

Because Emma does not have the resources to develop the theoretical sampling distribution, she estimates the sampling distribution of the sample median using a process called bootstrapping. In the bootstrapping process, a computer program performs the following steps.

- Take a random sample, with replacement, of size 50 from the original sample.
- Calculate and record the median of the sample.
- Repeat the process to obtain a total of 15,000 medians.

Emma ran the bootstrap process, and the following frequency table is the bootstrap distribution showing her results of generating 15,000 medians.

Bootstrap Distribution of Medians					
Median	Frequency	Median	Frequency	Median	Frequency
2,345	1	2,585	1	2,825	247
2,390	13	2,587.5	171	2,837.5	7
2,395	18	2,600	22	2,847.5	1
2,400	56	2,612.5	1,190	2,872.5	317
2,445	4	2,625	174	2,885	10
2,447.5	56	2,672.5	5	2,950	700
2,450	55	2,675	1,924	2,962.5	93
2,475	3	2,687.5	1,341	2,972.5	6
2,495	66	2,700	2,825	2,975	65
2,497.5	136	2,735	35	2,985	12
2,500	1,899	2,747.5	619	2,987.5	1
2,522.5	2	2,750	2	2,995	6
2,525	945	2,795	278	3,000	2
2,550	1,673	2,812.5	16	3,062.5	3

The bootstrap distribution provides an approximation of the sampling distribution of the sample median. A confidence interval for the median can be constructed using a percentage of the values in the middle of the bootstrap distribution.

- (d) Use the frequency table to find the following.
- Value of the 5th percentile:
 - Value of the 95th percentile:
- (e) Find the percentage of bootstrap medians in the table that are equal to or between the values found in part (d).
- (f) Use your values from parts (d) and (e) to construct and interpret a confidence interval for the median rental price.

STOP

END OF EXAM

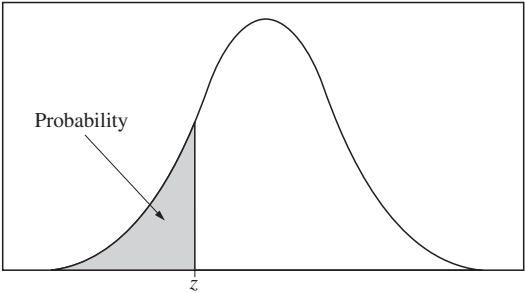


Table entry for z is the probability lying below z .

Table A Standard normal probabilities

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

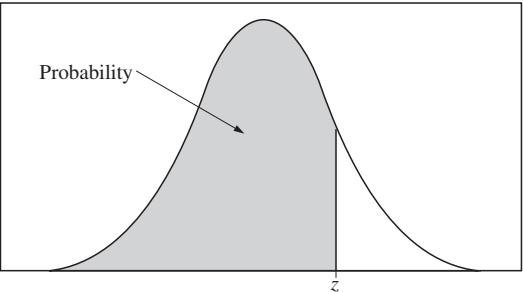


Table entry for z is the probability lying below z .

Table A (Continued)

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

Table entry for p and C is the point t^* with probability p lying above it and probability C lying between $-t^*$ and t^* .

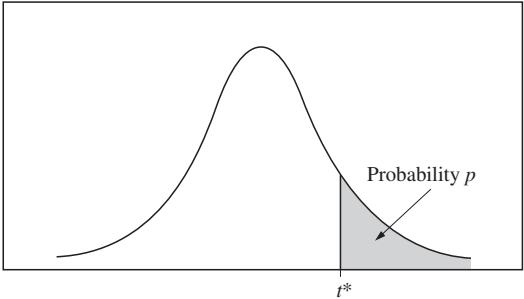


Table B t distribution critical values

df	Tail probability p											
	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.000	1.376	1.963	3.078	6.314	12.71	15.89	31.82	63.66	127.3	318.3	636.6
2	.816	1.061	1.386	1.886	2.920	4.303	4.849	6.965	9.925	14.09	22.33	31.60
3	.765	.978	1.250	1.638	2.353	3.182	3.482	4.541	5.841	7.453	10.21	12.92
4	.741	.941	1.190	1.533	2.132	2.776	2.999	3.747	4.604	5.598	7.173	8.610
5	.727	.920	1.156	1.476	2.015	2.571	2.757	3.365	4.032	4.773	5.893	6.869
6	.718	.906	1.134	1.440	1.943	2.447	2.612	3.143	3.707	4.317	5.208	5.959
7	.711	.896	1.119	1.415	1.895	2.365	2.517	2.998	3.499	4.029	4.785	5.408
8	.706	.889	1.108	1.397	1.860	2.306	2.449	2.896	3.355	3.833	4.501	5.041
9	.703	.883	1.100	1.383	1.833	2.262	2.398	2.821	3.250	3.690	4.297	4.781
10	.700	.879	1.093	1.372	1.812	2.228	2.359	2.764	3.169	3.581	4.144	4.587
11	.697	.876	1.088	1.363	1.796	2.201	2.328	2.718	3.106	3.497	4.025	4.437
12	.695	.873	1.083	1.356	1.782	2.179	2.303	2.681	3.055	3.428	3.930	4.318
13	.694	.870	1.079	1.350	1.771	2.160	2.282	2.650	3.012	3.372	3.852	4.221
14	.692	.868	1.076	1.345	1.761	2.145	2.264	2.624	2.977	3.326	3.787	4.140
15	.691	.866	1.074	1.341	1.753	2.131	2.249	2.602	2.947	3.286	3.733	4.073
16	.690	.865	1.071	1.337	1.746	2.120	2.235	2.583	2.921	3.252	3.686	4.015
17	.689	.863	1.069	1.333	1.740	2.110	2.224	2.567	2.898	3.222	3.646	3.965
18	.688	.862	1.067	1.330	1.734	2.101	2.214	2.552	2.878	3.197	3.611	3.922
19	.688	.861	1.066	1.328	1.729	2.093	2.205	2.539	2.861	3.174	3.579	3.883
20	.687	.860	1.064	1.325	1.725	2.086	2.197	2.528	2.845	3.153	3.552	3.850
21	.686	.859	1.063	1.323	1.721	2.080	2.189	2.518	2.831	3.135	3.527	3.819
22	.686	.858	1.061	1.321	1.717	2.074	2.183	2.508	2.819	3.119	3.505	3.792
23	.685	.858	1.060	1.319	1.714	2.069	2.177	2.500	2.807	3.104	3.485	3.768
24	.685	.857	1.059	1.318	1.711	2.064	2.172	2.492	2.797	3.091	3.467	3.745
25	.684	.856	1.058	1.316	1.708	2.060	2.167	2.485	2.787	3.078	3.450	3.725
26	.684	.856	1.058	1.315	1.706	2.056	2.162	2.479	2.779	3.067	3.435	3.707
27	.684	.855	1.057	1.314	1.703	2.052	2.158	2.473	2.771	3.057	3.421	3.690
28	.683	.855	1.056	1.313	1.701	2.048	2.154	2.467	2.763	3.047	3.408	3.674
29	.683	.854	1.055	1.311	1.699	2.045	2.150	2.462	2.756	3.038	3.396	3.659
30	.683	.854	1.055	1.310	1.697	2.042	2.147	2.457	2.750	3.030	3.385	3.646
40	.681	.851	1.050	1.303	1.684	2.021	2.123	2.423	2.704	2.971	3.307	3.551
50	.679	.849	1.047	1.299	1.676	2.009	2.109	2.403	2.678	2.937	3.261	3.496
60	.679	.848	1.045	1.296	1.671	2.000	2.099	2.390	2.660	2.915	3.232	3.460
80	.678	.846	1.043	1.292	1.664	1.990	2.088	2.374	2.639	2.887	3.195	3.416
100	.677	.845	1.042	1.290	1.660	1.984	2.081	2.364	2.626	2.871	3.174	3.390
1000	.675	.842	1.037	1.282	1.646	1.962	2.056	2.330	2.581	2.813	3.098	3.300
∞	.674	.841	1.036	1.282	1.645	1.960	2.054	2.326	2.576	2.807	3.091	3.291
50% 60% 70% 80% 90% 95% 96% 98% 99% 99.5% 99.8% 99.9%												
Confidence level C												

Table entry for p is the point (χ^2) with probability p lying above it.

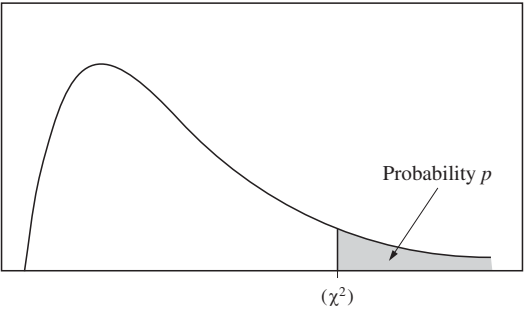
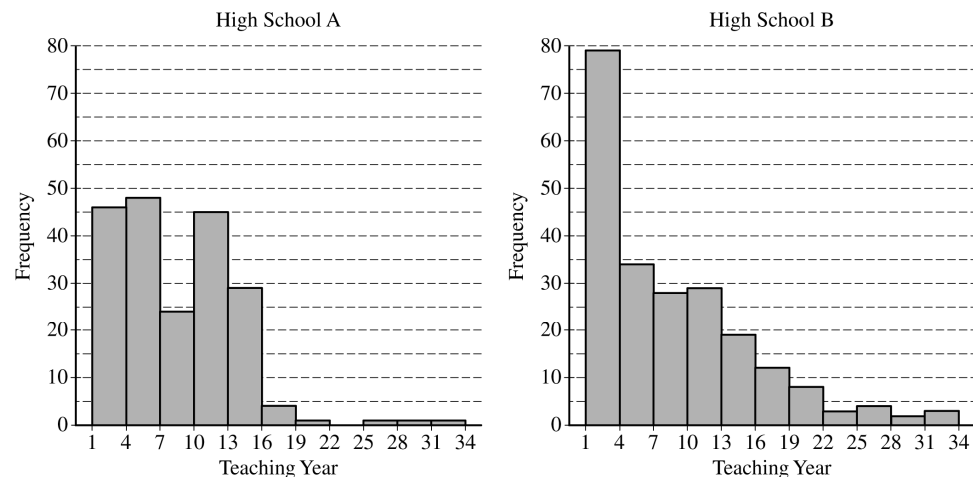


Table C χ^2 critical values

	Tail probability p											
df	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.32	1.64	2.07	2.71	3.84	5.02	5.41	6.63	7.88	9.14	10.83	12.12
2	2.77	3.22	3.79	4.61	5.99	7.38	7.82	9.21	10.60	11.98	13.82	15.20
3	4.11	4.64	5.32	6.25	7.81	9.35	9.84	11.34	12.84	14.32	16.27	17.73
4	5.39	5.99	6.74	7.78	9.49	11.14	11.67	13.28	14.86	16.42	18.47	20.00
5	6.63	7.29	8.12	9.24	11.07	12.83	13.39	15.09	16.75	18.39	20.51	22.11
6	7.84	8.56	9.45	10.64	12.59	14.45	15.03	16.81	18.55	20.25	22.46	24.10
7	9.04	9.80	10.75	12.02	14.07	16.01	16.62	18.48	20.28	22.04	24.32	26.02
8	10.22	11.03	12.03	13.36	15.51	17.53	18.17	20.09	21.95	23.77	26.12	27.87
9	11.39	12.24	13.29	14.68	16.92	19.02	19.68	21.67	23.59	25.46	27.88	29.67
10	12.55	13.44	14.53	15.99	18.31	20.48	21.16	23.21	25.19	27.11	29.59	31.42
11	13.70	14.63	15.77	17.28	19.68	21.92	22.62	24.72	26.76	28.73	31.26	33.14
12	14.85	15.81	16.99	18.55	21.03	23.34	24.05	26.22	28.30	30.32	32.91	34.82
13	15.98	16.98	18.20	19.81	22.36	24.74	25.47	27.69	29.82	31.88	34.53	36.48
14	17.12	18.15	19.41	21.06	23.68	26.12	26.87	29.14	31.32	33.43	36.12	38.11
15	18.25	19.31	20.60	22.31	25.00	27.49	28.26	30.58	32.80	34.95	37.70	39.72
16	19.37	20.47	21.79	23.54	26.30	28.85	29.63	32.00	34.27	36.46	39.25	41.31
17	20.49	21.61	22.98	24.77	27.59	30.19	31.00	33.41	35.72	37.95	40.79	42.88
18	21.60	22.76	24.16	25.99	28.87	31.53	32.35	34.81	37.16	39.42	42.31	44.43
19	22.72	23.90	25.33	27.20	30.14	32.85	33.69	36.19	38.58	40.88	43.82	45.97
20	23.83	25.04	26.50	28.41	31.41	34.17	35.02	37.57	40.00	42.34	45.31	47.50
21	24.93	26.17	27.66	29.62	32.67	35.48	36.34	38.93	41.40	43.78	46.80	49.01
22	26.04	27.30	28.82	30.81	33.92	36.78	37.66	40.29	42.80	45.20	48.27	50.51
23	27.14	28.43	29.98	32.01	35.17	38.08	38.97	41.64	44.18	46.62	49.73	52.00
24	28.24	29.55	31.13	33.20	36.42	39.36	40.27	42.98	45.56	48.03	51.18	53.48
25	29.34	30.68	32.28	34.38	37.65	40.65	41.57	44.31	46.93	49.44	52.62	54.95
26	30.43	31.79	33.43	35.56	38.89	41.92	42.86	45.64	48.29	50.83	54.05	56.41
27	31.53	32.91	34.57	36.74	40.11	43.19	44.14	46.96	49.64	52.22	55.48	57.86
28	32.62	34.03	35.71	37.92	41.34	44.46	45.42	48.28	50.99	53.59	56.89	59.30
29	33.71	35.14	36.85	39.09	42.56	45.72	46.69	49.59	52.34	54.97	58.30	60.73
30	34.80	36.25	37.99	40.26	43.77	46.98	47.96	50.89	53.67	56.33	59.70	62.16
40	45.62	47.27	49.24	51.81	55.76	59.34	60.44	63.69	66.77	69.70	73.40	76.09
50	56.33	58.16	60.35	63.17	67.50	71.42	72.61	76.15	79.49	82.66	86.66	89.56
60	66.98	68.97	71.34	74.40	79.08	83.30	84.58	88.38	91.95	95.34	99.61	102.7
80	88.13	90.41	93.11	96.58	101.9	106.6	108.1	112.3	116.3	120.1	124.8	128.3
100	109.1	111.7	114.7	118.5	124.3	129.6	131.1	135.8	140.2	144.3	149.4	153.2

5. The following histograms summarize the teaching year for the teachers at two high schools, A and B.



Teaching year is recorded as an integer, with first-year teachers recorded as 1, second-year teachers recorded as 2, and so on. Both sets of data have a mean teaching year of 8.2, with data recorded from 200 teachers at High School A and 221 teachers at High School B. On the histograms, each interval represents possible integer values from the left endpoint up to but not including the right endpoint.

- The median teaching year for one high school is 6, and the median teaching year for the other high school is 7. Identify which high school has each median and justify your answer.
- An additional 18 teachers were not included with the data recorded from the 200 teachers at High School A. The mean teaching year of the 18 teachers is 2.5. What is the mean teaching year for all 218 teachers at High School A?
- The standard deviation of the teaching year for the 221 teachers at High School B is 7.2. If one teacher is selected at random from High School B, what is the probability that the teaching year for the selected teacher will be within 1 standard deviation of the mean of 8.2? Justify your answer.

STATISTICS
SECTION II
Part A

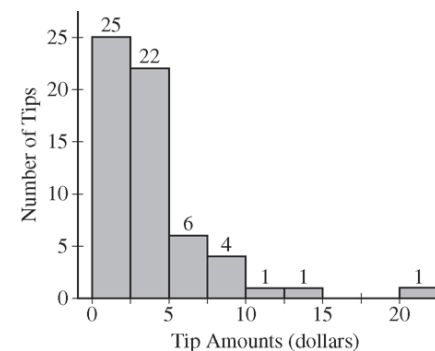
Questions 1-5

Spend about 65 minutes on this part of the exam.

Percent of Section II score—75

Directions: Show all your work. Indicate clearly the methods you use, because you will be scored on the correctness of your methods as well as on the accuracy and completeness of your results and explanations.

- Robin works as a server in a small restaurant, where she can earn a tip (extra money) from each customer she serves. The histogram below shows the distribution of her 60 tip amounts for one day of work.



- Write a few sentences to describe the distribution of tip amounts for the day shown.
- One of the tip amounts was \$8. If the \$8 tip had been \$18, what effect would the increase have had on the following statistics? Justify your answers.

The mean:

The median:

4. As part of its twenty-fifth reunion celebration, the class of 1988 (students who graduated in 1988) at a state university held a reception on campus. In an informal survey, the director of alumni development asked 50 of the attendees about their incomes. The director computed the mean income of the 50 attendees to be \$189,952. In a news release, the director announced, “The members of our class of 1988 enjoyed resounding success. Last year’s mean income of its members was \$189,952!”

(a) What would be a statistical advantage of using the median of the reported incomes, rather than the mean, as the estimate of the typical income?

(b) The director felt the members who attended the reception may be different from the class as a whole. A more detailed survey of the class was planned to find a better estimate of the income as well as other facts about the alumni. The staff developed two methods based on the available funds to carry out the survey.

Method 1: Send out an e-mail to all 6,826 members of the class asking them to complete an online form. The staff estimates that at least 600 members will respond.

Method 2: Select a simple random sample of members of the class and contact the selected members directly by phone. Follow up to ensure that all responses are obtained. Because method 2 will require more time than method 1, the staff estimates that only 100 members of the class could be contacted using method 2.

Which of the two methods would you select for estimating the average yearly income of all 6,826 members of the class of 1988 ? Explain your reasoning by comparing the two methods and the effect of each method on the estimate.

1.8 Graphical Representations of Summary Statistics

Name: _____ Class: _____ Date: _____ **Total: 9 marks**

KEY WORDS boxplot 箱线图 • five-number summary 五数概括 • percentiles 百分位数
• median 中位数 • outliers 离群值

Objective

Build the skills to answer exam questions on **graphical representations of summary statistics**.

You must be able to:

- construct a **boxplot** 箱线图 from the **five-number summary** 五数概括
- mark **outliers** with the $1.5 \times \text{IQR}$ rule (a modified boxplot)
- estimate **percentiles** 百分位数 and describe position

1 Worked examples

Study these first. Each one shows the method for a question type used later.

■ The five-number summary and boxplot

The **five-number summary** is minimum, Q_1 , median, Q_3 , maximum. A **boxplot** draws a box from Q_1 to Q_3 (with the median inside) and whiskers to the extremes.

■ The 1.5 IQR rule

A value is an **outlier** if it is below $Q_1 - 1.5 \times \text{IQR}$ or above $Q_3 + 1.5 \times \text{IQR}$.

■ Percentiles

Read position (percentiles) from a boxplot or ogive — e.g. Q_3 is the 75th percentile.

2 Practice

2.1 Name the five values in a five-number summary. [2]

2.2 State the rule for identifying an outlier. [1]

2.3 For a data set with $Q_1 = 10$ and $Q_3 = 20$, find the IQR. [1]

3 Exam-style questions

3.1 A boxplot is built from the [1]

- **A** mean and standard deviation
- **B** five-number summary
- **C** histogram
- **D** z -scores

3.2 A value is an outlier if it is more than _____ $\times$ IQR beyond a quartile. [1]

- **A** 1
- **B** 1.5
- **C** 2
- **D** 3

3.3 A data set has $Q_1 = 15$ and $Q_3 = 27$.

(a) Find the IQR. [1]

(b) Find the upper outlier boundary, $Q_3 + 1.5 \times \text{IQR}$. [2]

Solutions

2.1 minimum, Q_1 , median, Q_3 , maximum.

2.2 a value below $Q_1 - 1.5 \times \text{IQR}$ or above $Q_3 + 1.5 \times \text{IQR}$.

2.3 $20 - 10 = 10$.

3.1 B.

3.2 B.

3.3 (a) $27 - 15 = 12$. (b) $27 + 1.5 \times 12 = 27 + 18 = 45$.

1. A goat farmer raises two breeds of goats, Breed H and Breed J. He wants to compare the weights of the two breeds and takes independent random samples of 14 goats from each breed. The weight, in pounds, of each goat is measured. The weights of the 14 Breed H goats are recorded.

Weights of Breed H Goats (pounds)

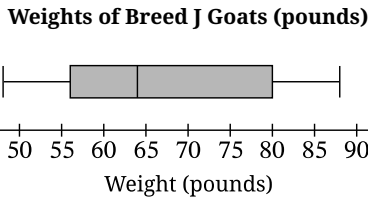
- 48
- 48
- 55
- 56
- 56
- 57
- 62
- 66
- 72
- 72
- 72
- 73
- 80
- 80

Part A

Use the data in the list to determine the five-number summary for the sample of Breed H goats.

Part B

The distribution of weight for Breed J goats is displayed in the boxplot.



Use the five-number summary from part A and the boxplot of Breed J to compare the center and variability for the distribution of weight for the Breed H goats and the distribution of weight for the Breed J goats, in context.



Part C

A stem-and-leaf plot is another way to display the weights of goats. The distribution of weight for the Breed H goats is displayed in the given stem-and-leaf plot.

Weights of Breed H Goats (pounds)

4	8 8
5	5 6 6 7
6	2 6
7	2 2 2 3
8	0 0

Key: 4|8 = 48 pounds

Consider the five-number summary from part A and the stem-and-leaf plot.

- i. If a boxplot were created from the five-number summary found in part A, what characteristic of the shape of the distribution of weight for the Breed H goats would be apparent from the stem-and-leaf plot but not from the boxplot?
- ii. Explain why a boxplot would not display the characteristic of the shape of the distribution identified in part C (i).

Descriptive Statistics

$$\bar{x} = \frac{1}{n} \sum x_i = \frac{\sum x_i}{n}$$

$$\hat{y} = a + bx$$

$$r = \frac{1}{n-1} \sum \left(\frac{x_i - \bar{x}}{s_x} \right) \left(\frac{y_i - \bar{y}}{s_y} \right)$$

$$s_x = \sqrt{\frac{1}{n-1} \sum (x_i - \bar{x})^2} = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n-1}}$$

$$\bar{y} = a + b\bar{x}$$

$$b = r \frac{s_y}{s_x}$$



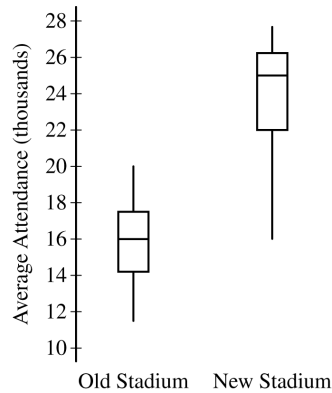
Part B

Question 6

Spend about 25 minutes on this part of the exam.

Directions: Show all your work. Indicate clearly the methods you use, because you will be scored on the correctness of your methods as well as on the accuracy and completeness of your results and explanations.

5. Attendance at games for a certain baseball team is being investigated by the team owner. The following boxplots summarize the attendance, measured as average number of attendees per game, for 47 years of the team's existence. The boxplots include the 30 years of games played in the old stadium and the 17 years played in the new stadium.

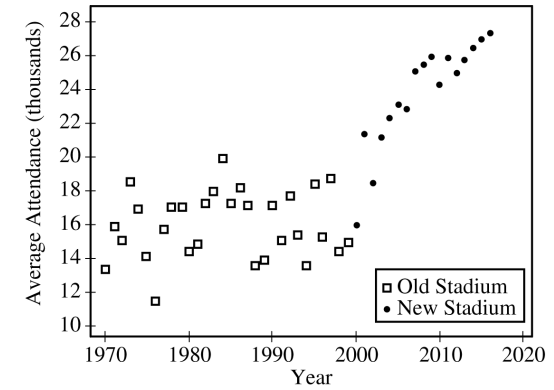


- (a) Compare the distributions of average attendance between the old and new stadiums.

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The following scatterplot shows average attendance versus year.

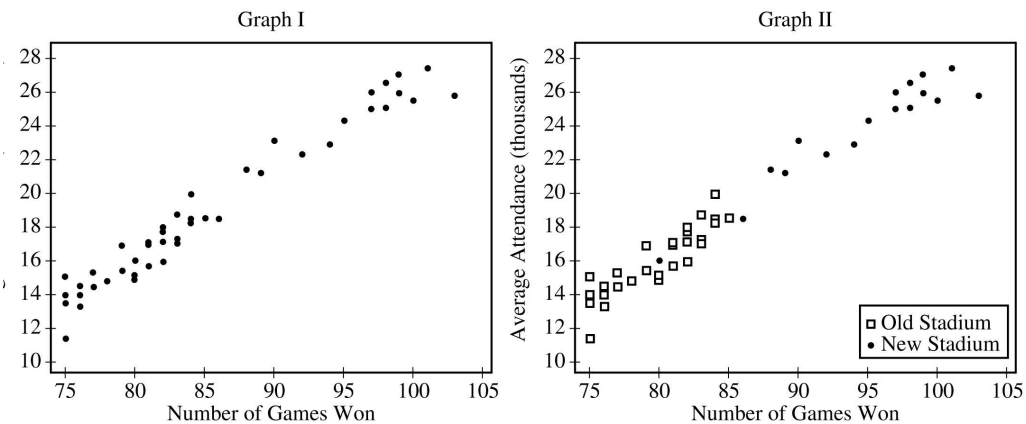


- (b) Compare the trends in average attendance over time between the old and new stadium.

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(c) Consider the following scatterplots.



(i) Graph I shows the average attendance versus number of games won for each year. Describe the relationship between the variables.

(ii) Graph II shows the same information as Graph I, but also indicates the old and new stadiums. Does Graph II suggest that the rate at which attendance changes as number of games won increases is different in the new stadium compared to the old stadium? Explain your reasoning.

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(d) Consider the three variables: number of games won, year, and stadium. Based on the graphs, explain how one of those variables could be a confounding variable in the relationship between average attendance and the other variables.

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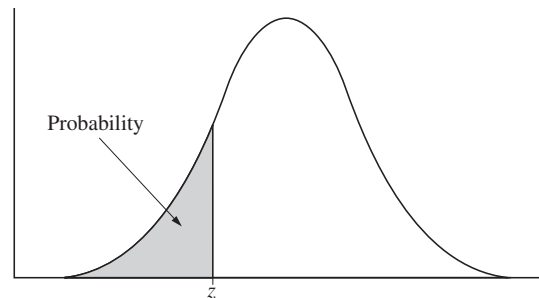
AP Statistics: 2021

STOP**END OF EXAM**

Use a pencil or pen with black or dark blue ink only. Do NOT write your name. Do NOT write outside the box.

Name:

AP Statistics: 2021

Table entry for z is the probability lying below z .**Table A** Standard normal probabilities

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
-3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
-3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
-3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
-3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
-2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
-2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
-2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
-2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
-2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
-2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
-2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
-2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
-2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
-2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
-1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
-1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
-1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
-1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
-1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
-1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
-1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
-1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
-1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
-1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
-0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
-0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
-0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
-0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
-0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
-0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
-0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
-0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
-0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
-0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641



Name:

AP Statistics: 2021

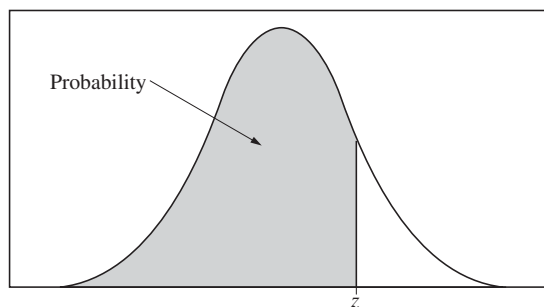


Table entry for z is the probability lying below z .

Table A (Continued)

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

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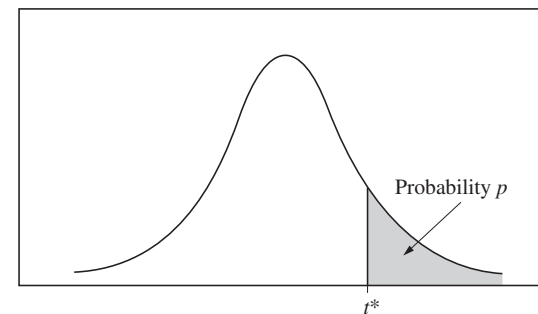


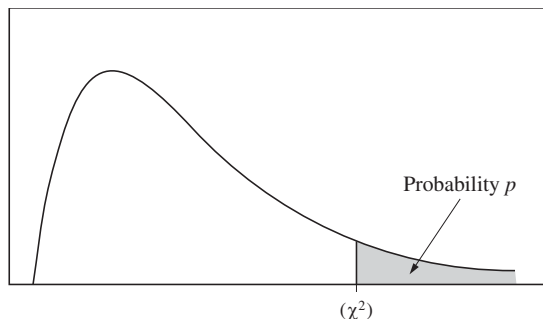
Table entry for p and C is the point t^* with probability p lying above it and probability C lying between $-t^*$ and t^* .

Table B t distribution critical values

df	Tail probability p									
	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025
1	1.000	1.376	1.963	3.078	6.314	12.71	15.89	31.82	63.66	127.3
2	.816	1.061	1.386	1.886	2.920	4.303	4.849	6.965	9.925	14.09
3	.765	.978	1.250	1.638	2.353	3.182	3.482	4.541	5.841	7.453
4	.741	.941	1.190	1.533	2.132	2.776	2.999	3.747	4.604	5.598
5	.727	.920	1.156	1.476	2.015	2.571	2.757	3.365	4.032	4.773
6	.718	.906	1.134	1.440	1.943	2.447	2.612	3.143	3.707	4.317
7	.711	.896	1.119	1.415	1.895	2.365	2.517	2.998	3.499	4.029
8	.706	.889	1.108	1.397	1.860	2.306	2.449	2.896	3.355	3.833
9	.703	.883	1.100	1.383	1.833	2.262	2.398	2.821	3.250	3.690
10	.700	.879	1.093	1.372	1.812	2.228	2.359	2.764	3.169	3.581
11	.697	.876	1.088	1.363	1.796	2.201	2.328	2.718	3.106	3.497
12	.695	.873	1.083	1.356	1.782	2.179	2.303	2.681	3.055	3.428
13	.694	.870	1.079	1.350	1.771	2.160	2.282	2.650	3.012	3.372
14	.692	.868	1.076	1.345	1.761	2.145	2.264	2.624	2.977	3.326
15	.691	.866	1.074	1.341	1.753	2.131	2.249	2.602	2.947	3.286
16	.690	.865	1.071	1.337	1.746	2.120	2.235	2.583	2.921	3.252
17	.689	.863	1.069	1.333	1.740	2.110	2.224	2.567	2.898	3.222
18	.688	.862	1.067	1.330	1.734	2.101	2.214	2.552	2.878	3.197
19	.688	.861	1.066	1.328	1.729	2.093	2.205	2.539	2.861	3.174
20	.687	.860	1.064	1.325	1.725	2.086	2.197	2.528	2.845	3.153
21	.686	.859	1.063	1.323	1.721	2.080	2.189	2.518	2.831	3.135
22	.686	.858	1.061	1.321	1.717	2.074	2.183	2.508	2.819	3.119
23	.685	.858	1.060	1.319	1.714	2.069	2.177	2.500	2.807	3.104
24	.685	.857	1.059	1.318	1.711	2.064	2.172	2.492	2.797	3.091
25	.684	.856	1.058	1.316	1.708	2.060	2.167	2.485	2.787	3.078
26	.684	.856	1.058	1.315	1.706	2.056	2.162	2.479	2.779	3.067
27	.684	.855	1.057	1.314	1.703	2.052	2.158	2.473	2.771	3.057
28	.683	.855	1.056	1.313	1.701	2.048	2.154	2.467	2.763	3.047
29	.683	.854	1.055	1.311	1.699	2.045	2.150	2.462	2.756	3.038
30	.683	.854	1.055	1.310	1.697	2.042	2.147	2.457	2.750	3.030
40	.681	.851	1.050	1.303	1.684	2.021	2.123	2.423	2.704	2.971
50	.679	.849	1.047	1.299	1.676	2.009	2.109	2.403	2.678	2.937
60	.679	.848	1.045	1.296	1.671	2.000	2.099	2.390	2.660	2.915
80	.678	.846	1.043	1.292	1.664	1.990	2.088	2.374	2.639	2.887
100	.677	.845	1.042	1.290	1.660	1.984	2.081	2.364	2.626	2.871
1000	.675	.842	1.037	1.282	1.646	1.962	2.056	2.330	2.581	2.813
∞	.674	.841	1.036	1.282	1.645	1.960	2.054	2.326	2.576	2.807
	50%	60%	70%	80%	90%	95%	96%	98%	99%	99.5%
										99.8%
										99.9%

Confidence level C

Table entry for p is the point (χ^2) with probability p lying above it.

Table C χ^2 critical values

df	Tail probability p											
	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.32	1.64	2.07	2.71	3.84	5.02	5.41	6.63	7.88	9.14	10.83	12.12
2	2.77	3.22	3.79	4.61	5.99	7.38	7.82	9.21	10.60	11.98	13.82	15.20
3	4.11	4.64	5.32	6.25	7.81	9.35	9.84	11.34	12.84	14.32	16.27	17.73
4	5.39	5.99	6.74	7.78	9.49	11.14	11.67	13.28	14.86	16.42	18.47	20.00
5	6.63	7.29	8.12	9.24	11.07	12.83	13.39	15.09	16.75	18.39	20.51	22.11
6	7.84	8.56	9.45	10.64	12.59	14.45	15.03	16.81	18.55	20.25	22.46	24.10
7	9.04	9.80	10.75	12.02	14.07	16.01	16.62	18.48	20.28	22.04	24.32	26.02
8	10.22	11.03	12.03	13.36	15.51	17.53	18.17	20.09	21.95	23.77	26.12	27.87
9	11.39	12.24	13.29	14.68	16.92	19.02	19.68	21.67	23.59	25.46	27.88	29.67
10	12.55	13.44	14.53	15.99	18.31	20.48	21.16	23.21	25.19	27.11	29.59	31.42
11	13.70	14.63	15.77	17.28	19.68	21.92	22.62	24.72	26.76	28.73	31.26	33.14
12	14.85	15.81	16.99	18.55	21.03	23.34	24.05	26.22	28.30	30.32	32.91	34.82
13	15.98	16.98	18.20	19.81	22.36	24.74	25.47	27.69	29.82	31.88	34.53	36.48
14	17.12	18.15	19.41	21.06	23.68	26.12	26.87	29.14	31.32	33.43	36.12	38.11
15	18.25	19.31	20.60	22.31	25.00	27.49	28.26	30.58	32.80	34.95	37.70	39.72
16	19.37	20.47	21.79	23.54	26.30	28.85	29.63	32.00	34.27	36.46	39.25	41.31
17	20.49	21.61	22.98	24.77	27.59	30.19	31.00	33.41	35.72	37.95	40.79	42.88
18	21.60	22.76	24.16	25.99	28.87	31.53	32.35	34.81	37.16	39.42	42.31	44.43
19	22.72	23.90	25.33	27.20	30.14	32.85	33.69	36.19	38.58	40.88	43.82	45.97
20	23.83	25.04	26.50	28.41	31.41	34.17	35.02	37.57	40.00	42.34	45.31	47.50
21	24.93	26.17	27.66	29.62	32.67	35.48	36.34	38.93	41.40	43.78	46.80	49.01
22	26.04	27.30	28.82	30.81	33.92	36.78	37.66	40.29	42.80	45.20	48.27	50.51
23	27.14	28.43	29.98	32.01	35.17	38.08	38.97	41.64	44.18	46.62	49.73	52.00
24	28.24	29.55	31.13	33.20	36.42	39.36	40.27	42.98	45.56	48.03	51.18	53.48
25	29.34	30.68	32.28	34.38	37.65	40.65	41.57	44.31	46.93	49.44	52.62	54.95
26	30.43	31.79	33.43	35.56	38.89	41.92	42.86	45.64	48.29	50.83	54.05	56.41
27	31.53	32.91	34.57	36.74	40.11	43.19	44.14	46.96	49.64	52.22	55.48	57.86
28	32.62	34.03	35.71	37.92	41.34	44.46	45.42	48.28	50.99	53.59	56.89	59.30
29	33.71	35.14	36.85	39.09	42.56	45.72	46.69	49.59	52.34	54.97	58.30	60.73
30	34.80	36.25	37.99	40.26	43.77	46.98	47.96	50.89	53.67	56.33	59.70	62.16
40	45.62	47.27	49.24	51.81	55.76	59.34	60.44	63.69	66.77	69.70	73.40	76.09
50	56.33	58.16	60.35	63.17	67.50	71.42	72.61	76.15	79.49	82.66	86.66	89.56
60	66.98	68.97	71.34	74.40	79.08	83.30	84.58	88.38	91.95	95.34	99.61	102.7
80	88.13	90.41	93.11	96.58	101.9	106.6	108.1	112.3	116.3	120.1	124.8	128.3
100	109.1	111.7	114.7	118.5	124.3	129.6	131.1	135.8	140.2	144.3	149.4	153.2

STATISTICS SECTION II Part A

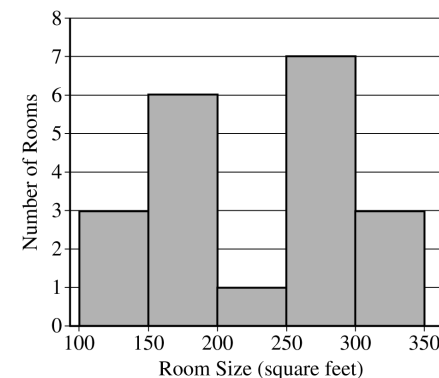
Questions 1-5

Spend about 1 hour and 5 minutes on this part of the exam.

Percent of Section II score—75

Directions: Show all your work. Indicate clearly the methods you use, because you will be scored on the correctness of your methods as well as on the accuracy and completeness of your results and explanations.

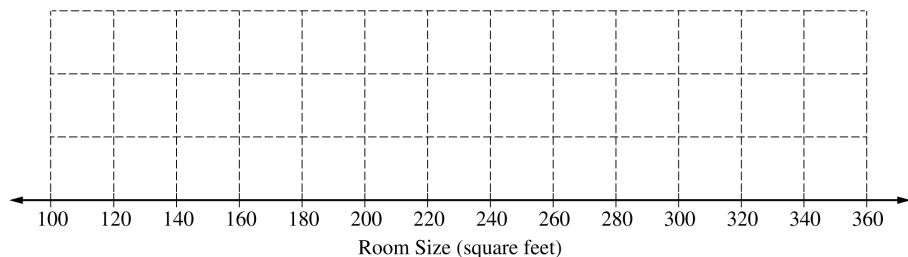
1. The sizes, in square feet, of the 20 rooms in a student residence hall at a certain university are summarized in the following histogram.



- (a) Based on the histogram, write a few sentences describing the distribution of room size in the residence hall.
- (b) Summary statistics for the sizes are given in the following table.

Mean	Standard Deviation	Min	Q1	Median	Q3	Max
231.4	68.12	134	174	253.5	292	315

Determine whether there are potential outliers in the data. Then use the following grid to sketch a boxplot of room size.



- (c) What characteristic of the shape of the distribution of room size is apparent from the histogram but not from the boxplot?

STATISTICS
SECTION II
Part A

Questions 1-5

Spend about 65 minutes on this part of the exam.

Percent of Section II score—75

Directions: Show all your work. Indicate clearly the methods you use, because you will be scored on the correctness of your methods as well as on the accuracy and completeness of your results and explanations.

1. Two large corporations, A and B, hire many new college graduates as accountants at entry-level positions. In 2009 the starting salary for an entry-level accountant position was \$36,000 a year at both corporations. At each corporation, data were collected from 30 employees who were hired in 2009 as entry-level accountants and were still employed at the corporation five years later. The yearly salaries of the 60 employees in 2014 are summarized in the boxplots below.



- (a) Write a few sentences comparing the distributions of the yearly salaries at the two corporations.
- (b) Suppose both corporations offered you a job for \$36,000 a year as an entry-level accountant.
- Based on the boxplots, give one reason why you might choose to accept the job at corporation A.
 - Based on the boxplots, give one reason why you might choose to accept the job at corporation B.

1.9 Comparing Distributions of a Quantitative Variable

Name: _____ Class: _____ Date: _____ **Total: 9 marks**

KEY WORDS in context 结合背景 • parallel boxplots 并列箱线图 • median 中位数
• boxplot 箱线图 • distribution 分布

Objective

Build the skills to answer exam questions on **comparing distributions of a quantitative variable**.

You must be able to:

- compare two or more distributions using **parallel boxplots** 并列箱线图
- compare the **shape**, **center**, and **spread** of the groups explicitly
- use comparative language, always **in context** 结合背景

1 Worked examples

Study these first. Each one shows the method for a question type used later.

■ Comparing groups

Use **parallel boxplots** (or back-to-back displays) to compare distributions. Compare **shape**, **center**, and **spread** — and state each comparison **explicitly**.

■ Comparative language in context

Say things like “Class A’s median score is **higher than** Class B’s” or “Group X is **more variable than** Group Y”, always referring to the real-world context so the comparison means something.

2 Practice

2.1 Name a graph useful for comparing two distributions. [1]

2.2 State the three features to compare between the groups. [2]

2.3 State why comparisons should be made “in context”. [1]

3 Exam-style questions

3.1 To compare two distributions, you can use [1]

- **A** a single pie chart
- **B** parallel boxplots
- **C** one dotplot
- **D** a *z*-score

3.2 When comparing groups, you should compare shape, center, and [1]

- **A** colour
- **B** spread
- **C** the total
- **D** the category

3.3 Two classes’ test scores are shown with parallel boxplots.

- (a) Name one feature to compare. [1]
(b) Give an example of comparative language. [1]
(c) State why context matters in the comparison. [1]

Solutions

- 2.1 parallel boxplots.
2.2 shape, center, spread.
2.3 so the comparison is meaningful and refers to the real situation.
3.1 B.
3.2 B.
3.3 (a) shape, center, or spread (any one). (b) e.g. “Class A’s median is higher than Class B’s”. (c) so the comparison relates to the actual data being studied.

1. A goat farmer raises two breeds of goats, Breed H and Breed J. He wants to compare the weights of the two breeds and takes independent random samples of 14 goats from each breed. The weight, in pounds, of each goat is measured. The weights of the 14 Breed H goats are recorded.

Weights of Breed H Goats (pounds)

- 48
48
55
56
56
57
62
66
72
72
72
73
80
80

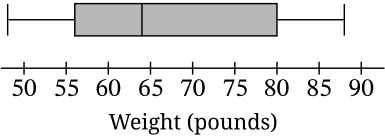
Part A

Use the data in the list to determine the five-number summary for the sample of Breed H goats.

Part B

The distribution of weight for Breed J goats is displayed in the boxplot.

Weights of Breed J Goats (pounds)



Use the five-number summary from part A and the boxplot of Breed J to compare the center and variability for the distribution of weight for the Breed H goats and the distribution of weight for the Breed J goats, in context.

Part C

A stem-and-leaf plot is another way to display the weights of goats. The distribution of weight for the Breed H goats is displayed in the given stem-and-leaf plot.

Weights of Breed H Goats (pounds)

```

4 | 8 8
5 | 5 6 6 7
6 | 2 6
7 | 2 2 2 3
8 | 0 0

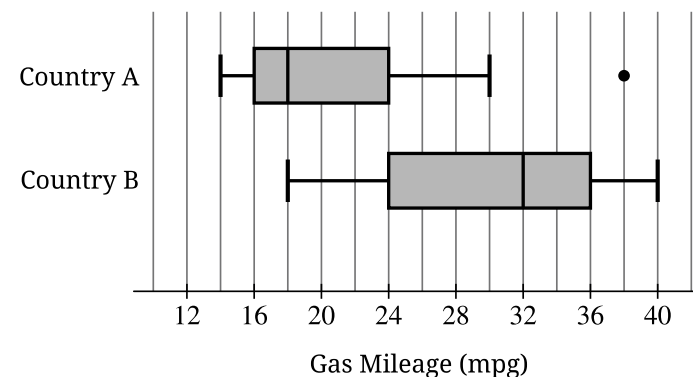
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Key: 4|8 = 48 pounds

Consider the five-number summary from part A and the stem-and-leaf plot.

- If a boxplot were created from the five-number summary found in part A, what characteristic of the shape of the distribution of weight for the Breed H goats would be apparent from the stem-and-leaf plot but not from the boxplot?
- Explain why a boxplot would not display the characteristic of the shape of the distribution identified in part C (i).

- The manager of an automotive company is interested in comparing the gas mileages for cars manufactured in Country A and cars manufactured in Country B. The manager selected a random sample of 100 cars manufactured in Country A and a random sample of 100 cars manufactured in Country B. The gas mileages for each sample, in miles per gallon (mpg), are summarized in the boxplots.

Boxplots of Gas Mileage for Each Country

- Compare the distributions of gas mileage for the sample of cars manufactured in Country A and the sample of cars manufactured in Country B.
- For the distribution of gas mileage for the sample of cars manufactured in Country A, would you expect the mean to be greater than 18 mpg, less than 18 mpg, or equal to 18 mpg? Justify your answer.
- The manager will create a new boxplot with the combined data from the sample of cars manufactured in Country A and the sample of cars manufactured in Country B.
 - What is the range of the combined data set? Justify your answer.
 - What is a possible value of the median of the combined data set? Justify your answer by referencing the boxplots shown.

Descriptive Statistics

$$\bar{x} = \frac{1}{n} \sum x_i = \frac{\sum x_i}{n}$$

$$\hat{y} = a + bx$$

$$r = \frac{1}{n-1} \sum \left(\frac{x_i - \bar{x}}{s_x} \right) \left(\frac{y_i - \bar{y}}{s_y} \right)$$

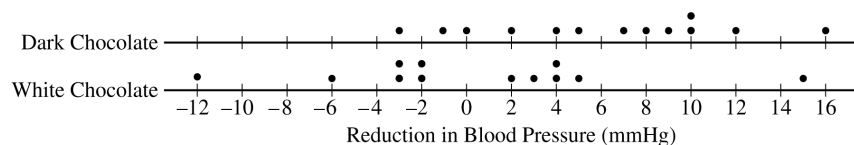
$$s_x = \sqrt{\frac{1}{n-1} \sum (x_i - \bar{x})^2} = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n-1}}$$

$$\bar{y} = a + b\bar{x}$$

$$b = r \frac{s_y}{s_x}$$

Studies have shown that foods rich in compounds known as flavonoids help lower blood pressure. Researchers conducted a study to investigate whether there was a greater reduction in blood pressure for people who consumed dark chocolate, which contains flavonoids, than people who consumed white chocolate, which does not contain flavonoids. Twenty-five healthy adults agreed to participate in the study and add 3.5 ounces of chocolate to their daily diets. Of the 25 participants, 13 were randomly assigned to the dark chocolate group and the rest were assigned to the white chocolate group. All participants had their blood pressure recorded, in millimeters of mercury (mmHg), before adding chocolate to their daily diets and again 30 days after adding chocolate to their daily diets.

The reduction in blood pressure (before minus after) for each of the participants in the two groups is shown in the dotplots below.



- (a) Determine and compare the medians of the reduction in blood pressure for the two groups.

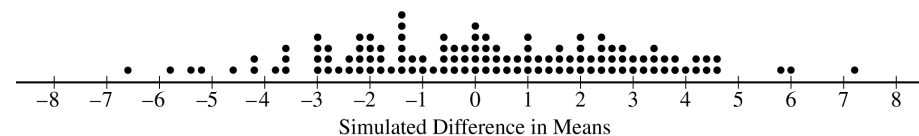
The researchers found the mean reduction in blood pressure for those who consumed dark chocolate is $\bar{x}_{dark} = 6.08$ mmHg and the mean reduction in blood pressure for those who consumed white chocolate is $\bar{x}_{white} = 0.42$ mmHg.

- (b) One researcher indicated that because the difference in sample means of 5.66 mmHg is greater than 0 there is convincing statistical evidence to conclude that the population mean blood pressure reduction for those who consume dark chocolate is greater than for those who consume white chocolate. Why might the researcher's conclusion, based only on the difference in sample means of 5.66 mmHg, not necessarily be true?

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 5** on this page.

A simulation was conducted to investigate whether there is a greater reduction of blood pressure for those who consume dark chocolate than for those who consume white chocolate. The simulation was conducted under the assumption that no difference exists. The results of 120 trials of the simulation are shown in the following dotplot.



- (c) Use the results of the simulation to determine whether the results from the 25 participants in the study provide convincing statistical evidence, at a 5 percent level of significance, that adding dark chocolate to a daily diet will result in a greater reduction in blood pressure, on average, than adding white chocolate to a daily diet. Justify your answer.

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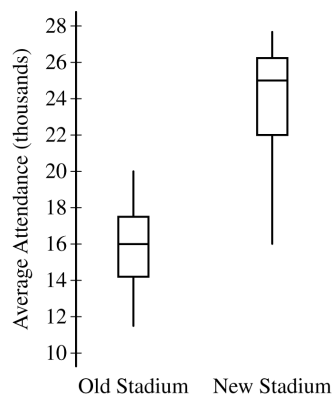
Part B

Question 6

Spend about 25 minutes on this part of the exam.

Directions: Show all your work. Indicate clearly the methods you use, because you will be scored on the correctness of your methods as well as on the accuracy and completeness of your results and explanations.

5. Attendance at games for a certain baseball team is being investigated by the team owner. The following boxplots summarize the attendance, measured as average number of attendees per game, for 47 years of the team's existence. The boxplots include the 30 years of games played in the old stadium and the 17 years played in the new stadium.



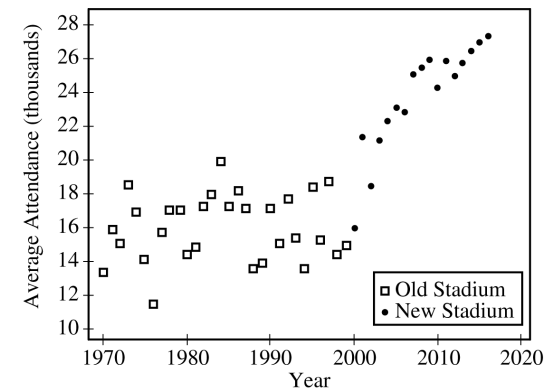
- (a) Compare the distributions of average attendance between the old and new stadiums.

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The following scatterplot shows average attendance versus year.



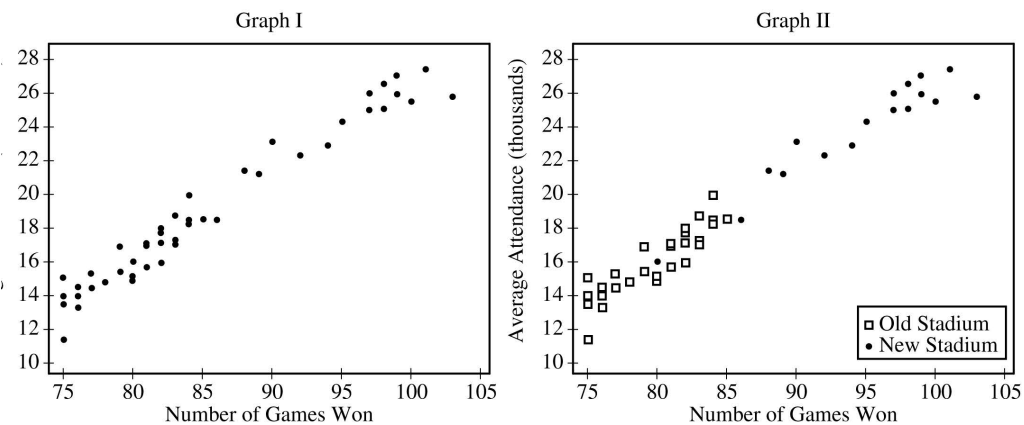
- (b) Compare the trends in average attendance over time between the old and new stadium.

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(c) Consider the following scatterplots.



(i) Graph I shows the average attendance versus number of games won for each year. Describe the relationship between the variables.

(ii) Graph II shows the same information as Graph I, but also indicates the old and new stadiums. Does Graph II suggest that the rate at which attendance changes as number of games won increases is different in the new stadium compared to the old stadium? Explain your reasoning.

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(d) Consider the three variables: number of games won, year, and stadium. Based on the graphs, explain how one of those variables could be a confounding variable in the relationship between average attendance and the other variables.

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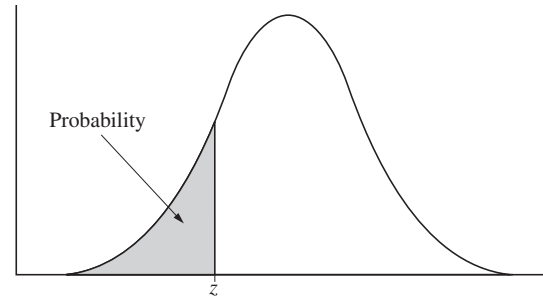
AP Statistics: 2021

STOP**END OF EXAM**

Use a pencil or pen with black or dark blue ink only. Do NOT write your name. Do NOT write outside the box.

Name:

AP Statistics: 2021

Table entry for z is the probability lying below z .**Table A** Standard normal probabilities

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

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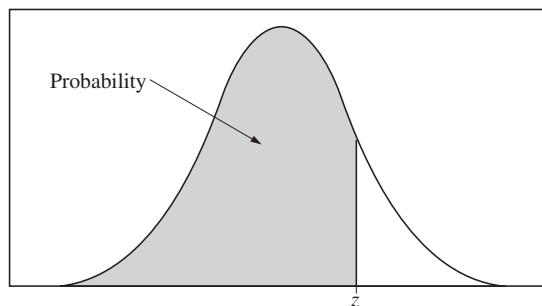


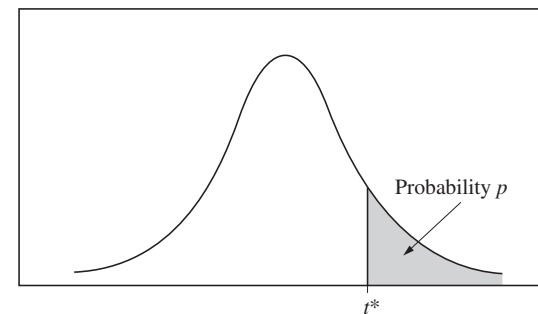
Table A (Continued)

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

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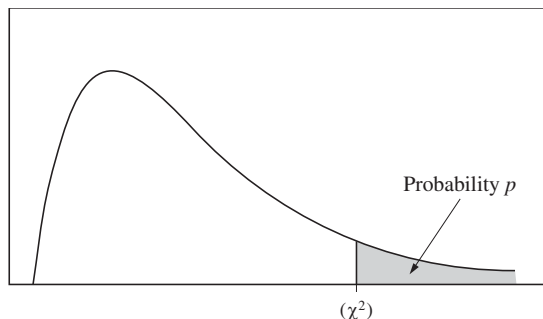
AP Statistics: 2021

Table entry for p and C is the point t^* with probability p lying above it and probability C lying between $-t^*$ and t^* .

Table B t distribution critical values

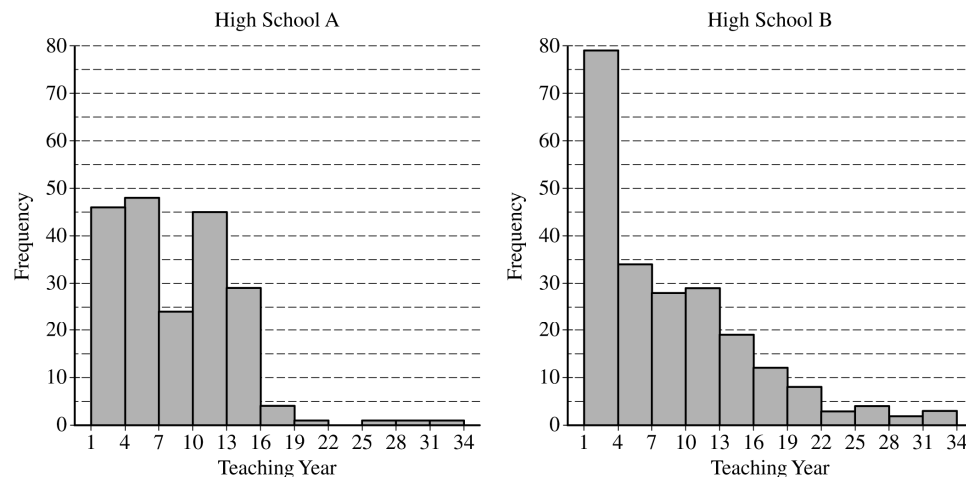
df	Tail probability p									
	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025
1	1.000	1.376	1.963	3.078	6.314	12.71	15.89	31.82	63.66	127.3
2	.816	1.061	1.386	1.886	2.920	4.303	4.849	6.965	9.925	14.09
3	.765	.978	1.250	1.638	2.353	3.182	3.482	4.541	5.841	7.453
4	.741	.941	1.190	1.533	2.132	2.776	2.999	3.747	4.604	5.598
5	.727	.920	1.156	1.476	2.015	2.571	2.757	3.365	4.032	4.773
6	.718	.906	1.134	1.440	1.943	2.447	2.612	3.143	3.707	4.317
7	.711	.896	1.119	1.415	1.895	2.365	2.517	2.998	3.499	4.029
8	.706	.889	1.108	1.397	1.860	2.306	2.449	2.896	3.355	3.833
9	.703	.883	1.100	1.383	1.833	2.262	2.398	2.821	3.250	3.690
10	.700	.879	1.093	1.372	1.812	2.228	2.359	2.764	3.169	3.581
11	.697	.876	1.088	1.363	1.796	2.201	2.328	2.718	3.106	3.497
12	.695	.873	1.083	1.356	1.782	2.179	2.303	2.681	3.055	3.428
13	.694	.870	1.079	1.350	1.771	2.160	2.282	2.650	3.012	3.372
14	.692	.868	1.076	1.345	1.761	2.145	2.264	2.624	2.977	3.326
15	.691	.866	1.074	1.341	1.753	2.131	2.249	2.602	2.947	3.286
16	.690	.865	1.071	1.337	1.746	2.120	2.235	2.583	2.921	3.252
17	.689	.863	1.069	1.333	1.740	2.110	2.224	2.567	2.898	3.222
18	.688	.862	1.067	1.330	1.734	2.101	2.214	2.552	2.878	3.197
19	.688	.861	1.066	1.328	1.729	2.093	2.205	2.539	2.861	3.174
20	.687	.860	1.064	1.325	1.725	2.086	2.197	2.528	2.845	3.153
21	.686	.859	1.063	1.323	1.721	2.080	2.189	2.518	2.831	3.135
22	.686	.858	1.061	1.321	1.717	2.074	2.183	2.508	2.819	3.119
23	.685	.858	1.060	1.319	1.714	2.069	2.177	2.500	2.807	3.104
24	.685	.857	1.059	1.318	1.711	2.064	2.172	2.492	2.797	3.091
25	.684	.856	1.058	1.316	1.708	2.060	2.167	2.485	2.787	3.078
26	.684	.856	1.058	1.315	1.706	2.056	2.162	2.479	2.779	3.067
27	.684	.855	1.057	1.314	1.703	2.052	2.158	2.473	2.771	3.057
28	.683	.855	1.056	1.313	1.701	2.048	2.154	2.467	2.763	3.047
29	.683	.854	1.055	1.311	1.699	2.045	2.150	2.462	2.756	3.038
30	.683	.854	1.055	1.310	1.697	2.042	2.147	2.457	2.750	3.030
40	.681	.851	1.050	1.303	1.684	2.021	2.123	2.423	2.704	2.971
50	.679	.849	1.047	1.299	1.676	2.009	2.109	2.403	2.678	2.937
60	.679	.848	1.045	1.296	1.671	2.000	2.099	2.390	2.660	2.915
80	.678	.846	1.043	1.292	1.664	1.990	2.088	2.374	2.639	2.887
100	.677	.845	1.042	1.290	1.660	1.984	2.081	2.364	2.626	2.871
1000	.675	.842	1.037	1.282	1.646	1.962	2.056	2.330	2.581	2.813
∞	.674	.841	1.036	1.282	1.645	1.960	2.054	2.326	2.576	2.807
Confidence level C										
50% 60% 70% 80% 90% 95% 96% 98% 99% 99.5% 99.8% 99.9%										

Table entry for p is the point (χ^2) with probability p lying above it.

Table C χ^2 critical values

df	Tail probability p										
	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001
1	1.32	1.64	2.07	2.71	3.84	5.02	5.41	6.63	7.88	9.14	10.83
2	2.77	3.22	3.79	4.61	5.99	7.38	7.82	9.21	10.60	11.98	13.82
3	4.11	4.64	5.32	6.25	7.81	9.35	9.84	11.34	12.84	14.32	16.27
4	5.39	5.99	6.74	7.78	9.49	11.14	11.67	13.28	14.86	16.42	18.47
5	6.63	7.29	8.12	9.24	11.07	12.83	13.39	15.09	16.75	18.39	20.51
6	7.84	8.56	9.45	10.64	12.59	14.45	15.03	16.81	18.55	20.25	22.46
7	9.04	9.80	10.75	12.02	14.07	16.01	16.62	18.48	20.28	22.04	24.32
8	10.22	11.03	12.03	13.36	15.51	17.53	18.17	20.09	21.95	23.77	26.12
9	11.39	12.24	13.29	14.68	16.92	19.02	19.68	21.67	23.59	25.46	27.88
10	12.55	13.44	14.53	15.99	18.31	20.48	21.16	23.21	25.19	27.11	29.59
11	13.70	14.63	15.77	17.28	19.68	21.92	22.62	24.72	26.76	28.73	31.26
12	14.85	15.81	16.99	18.55	21.03	23.34	24.05	26.22	28.30	30.32	32.91
13	15.98	16.98	18.20	19.81	22.36	24.74	25.47	27.69	29.82	31.88	34.53
14	17.12	18.15	19.41	21.06	23.68	26.12	26.87	29.14	31.32	33.43	36.12
15	18.25	19.31	20.60	22.31	25.00	27.49	28.26	30.58	32.80	34.95	37.70
16	19.37	20.47	21.79	23.54	26.30	28.85	29.63	32.00	34.27	36.46	39.25
17	20.49	21.61	22.98	24.77	27.59	30.19	31.00	33.41	35.72	37.95	40.79
18	21.60	22.76	24.16	25.99	28.87	31.53	32.35	34.81	37.16	39.42	42.31
19	22.72	23.90	25.33	27.20	30.14	32.85	33.69	36.19	38.58	40.88	43.82
20	23.83	25.04	26.50	28.41	31.41	34.17	35.02	37.57	40.00	42.34	45.31
21	24.93	26.17	27.66	29.62	32.67	35.48	36.34	38.93	41.40	43.78	46.80
22	26.04	27.30	28.82	30.81	33.92	36.78	37.66	40.29	42.80	45.20	48.27
23	27.14	28.43	29.98	32.01	35.17	38.08	38.97	41.64	44.18	46.62	49.73
24	28.24	29.55	31.13	33.20	36.42	39.36	40.27	42.98	45.56	48.03	51.18
25	29.34	30.68	32.28	34.38	37.65	40.65	41.57	44.31	46.93	49.44	52.62
26	30.43	31.79	33.43	35.56	38.89	41.92	42.86	45.64	48.29	50.83	54.05
27	31.53	32.91	34.57	36.74	40.11	43.19	44.14	46.96	49.64	52.22	55.48
28	32.62	34.03	35.71	37.92	41.34	44.46	45.42	48.28	50.99	53.59	56.89
29	33.71	35.14	36.85	39.09	42.56	45.72	46.69	49.59	52.34	54.97	58.30
30	34.80	36.25	37.99	40.26	43.77	46.98	47.96	50.89	53.67	56.33	59.70
40	45.62	47.27	49.24	51.81	55.76	59.34	60.44	63.69	66.77	69.70	73.40
50	56.33	58.16	60.35	63.17	67.50	71.42	72.61	76.15	79.49	82.66	86.66
60	66.98	68.97	71.34	74.40	79.08	83.30	84.58	88.38	91.95	95.34	99.61
80	88.13	90.41	93.11	96.58	101.9	106.6	108.1	112.3	116.3	120.1	124.8
100	109.1	111.7	114.7	118.5	124.3	129.6	131.1	135.8	140.2	144.3	149.4

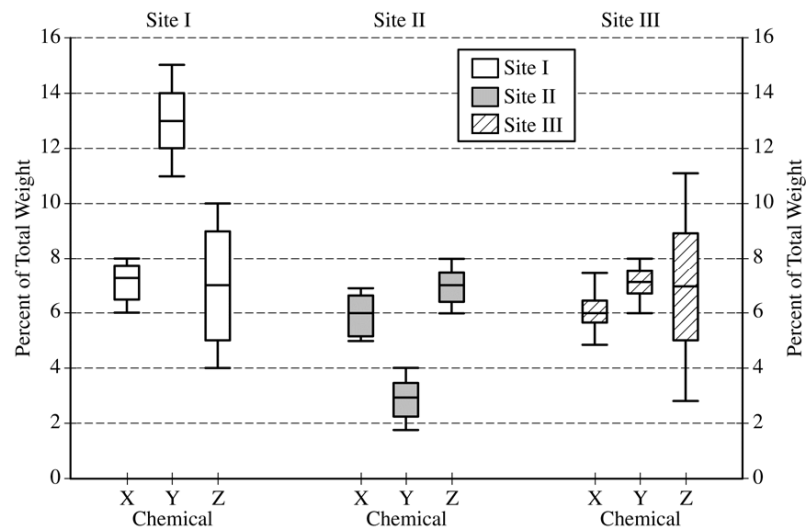
5. The following histograms summarize the teaching year for the teachers at two high schools, A and B.



Teaching year is recorded as an integer, with first-year teachers recorded as 1, second-year teachers recorded as 2, and so on. Both sets of data have a mean teaching year of 8.2, with data recorded from 200 teachers at High School A and 221 teachers at High School B. On the histograms, each interval represents possible integer values from the left endpoint up to but not including the right endpoint.

- The median teaching year for one high school is 6, and the median teaching year for the other high school is 7. Identify which high school has each median and justify your answer.
- An additional 18 teachers were not included with the data recorded from the 200 teachers at High School A. The mean teaching year of the 18 teachers is 2.5. What is the mean teaching year for all 218 teachers at High School A?
- The standard deviation of the teaching year for the 221 teachers at High School B is 7.2. If one teacher is selected at random from High School B, what is the probability that the teaching year for the selected teacher will be within 1 standard deviation of the mean of 8.2? Justify your answer.

4. The chemicals in clay used to make pottery can differ depending on the geographical region where the clay originated. Sometimes, archaeologists use a chemical analysis of clay to help identify where a piece of pottery originated. Such an analysis measures the amount of a chemical in the clay as a percent of the total weight of the piece of pottery. The boxplots below summarize analyses done for three chemicals—X, Y, and Z—on pieces of pottery that originated at one of three sites: I, II, or III.



- (a) For chemical Z, describe how the percents found in the pieces of pottery are similar and how they differ among the three sites.
- (b) Consider a piece of pottery known to have originated at one of the three sites, but the actual site is not known.
- Suppose an analysis of the clay reveals that the sum of the percents of the three chemicals X, Y, and Z is 20.5%. Based on the boxplots, which site—I, II, or III—is the most likely site where the piece of pottery originated? Justify your choice.
 - Suppose only one chemical could be analyzed in the piece of pottery. Which chemical—X, Y, or Z—would be the most useful in identifying the site where the piece of pottery originated? Justify your choice.

STATISTICS

SECTION II

Part B

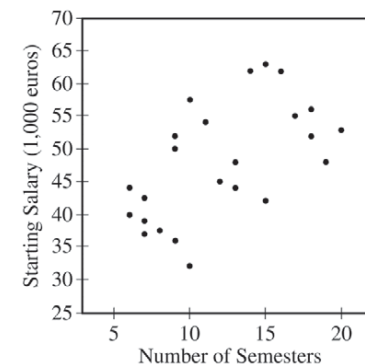
Question 6

Spend about 25 minutes on this part of the exam.

Percent of Section II score—25

Directions: Show all your work. Indicate clearly the methods you use, because you will be scored on the correctness of your methods as well as on the accuracy and completeness of your results and explanations.

6. A newspaper in Germany reported that the more semesters needed to complete an academic program at the university, the greater the starting salary in the first year of a job. The report was based on a study that used a random sample of 24 people who had recently completed an academic program. Information was collected on the number of semesters each person in the sample needed to complete the program and the starting salary, in thousands of euros, for the first year of a job. The data are shown in the scatterplot below.



- (a) Does the scatterplot support the newspaper report about number of semesters and starting salary? Justify your answer.

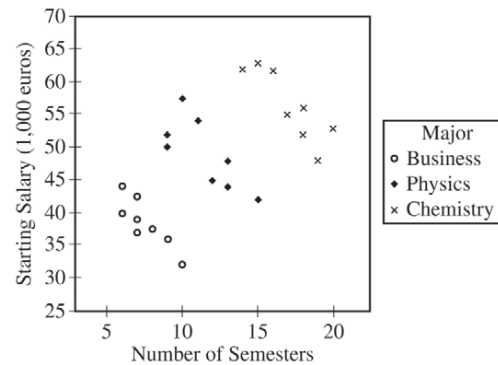
The table below shows computer output from a linear regression analysis on the data.

Predictor	Coef	SE Coef	T	P
Constant	34.018	4.455	7.64	0.000
Semesters	1.1594	0.3482	3.33	0.003

$S = 7.37702$ $R\text{-Sq} = 33.5\%$ $R\text{-Sq(adj)} = 30.5\%$

- (b) Identify the slope of the least-squares regression line, and interpret the slope in context.

An independent researcher received the data from the newspaper and conducted a new analysis by separating the data into three groups based on the major of each person. A revised scatterplot identifying the major of each person is shown below.



- (c) Based on the people in the sample, describe the association between starting salary and number of semesters for the business majors.
- (d) Based on the people in the sample, compare the median starting salaries for the three majors.
- (e) Based on the analysis conducted by the independent researcher, how could the newspaper report be modified to give a better description of the relationship between the number of semesters and the starting salary for the people in the sample?

STOP

END OF EXAM

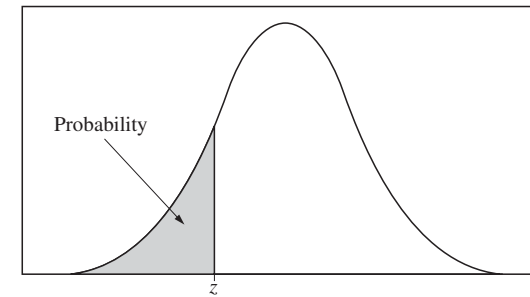


Table entry for z is the probability lying below z .

Table A Standard normal probabilities

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
-3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
-3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
-3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
-3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
-2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
-2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
-2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
-2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
-2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
-2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
-2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
-2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
-2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
-2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
-1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
-1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
-1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
-1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
-1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
-1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
-1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
-1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
-1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
-1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
-0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
-0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
-0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
-0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
-0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
-0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
-0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
-0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
-0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
-0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

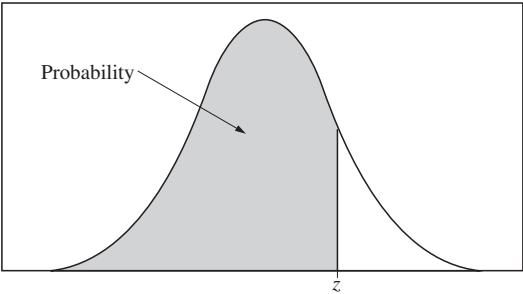


Table entry for z is the probability lying below z .

Table A (Continued)

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

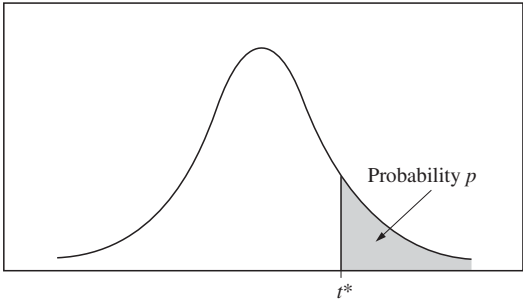


Table entry for p and C is the point t^* with probability p lying above it and probability C lying between $-t^*$ and t^* .

Table B t distribution critical values

df	Tail probability p									
	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025
1	1.000	1.376	1.963	3.078	6.314	12.71	15.89	31.82	63.66	127.3
2	.816	1.061	1.386	1.886	2.920	4.303	4.849	6.965	9.925	14.09
3	.765	.978	1.250	1.638	2.353	3.182	3.482	4.541	5.841	7.453
4	.741	.941	1.190	1.533	2.132	2.776	2.999	3.747	4.604	5.598
5	.727	.920	1.156	1.476	2.015	2.571	2.757	3.365	4.032	4.773
6	.718	.906	1.134	1.440	1.943	2.447	2.612	3.143	3.707	4.317
7	.711	.896	1.119	1.415	1.895	2.365	2.517	2.998	3.499	4.029
8	.706	.889	1.108	1.397	1.860	2.306	2.449	2.896	3.355	3.833
9	.703	.883	1.100	1.383	1.833	2.262	2.398	2.821	3.250	3.690
10	.700	.879	1.093	1.372	1.812	2.228	2.359	2.764	3.169	3.581
11	.697	.876	1.088	1.363	1.796	2.201	2.328	2.718	3.106	3.497
12	.695	.873	1.083	1.356	1.782	2.179	2.303	2.681	3.055	3.428
13	.694	.870	1.079	1.350	1.771	2.160	2.282	2.650	3.012	3.372
14	.692	.868	1.076	1.345	1.761	2.145	2.264	2.624	2.977	3.326
15	.691	.866	1.074	1.341	1.753	2.131	2.249	2.602	2.947	3.286
16	.690	.865	1.071	1.337	1.746	2.120	2.235	2.583	2.921	3.252
17	.689	.863	1.069	1.333	1.740	2.110	2.224	2.567	2.898	3.222
18	.688	.862	1.067	1.330	1.734	2.101	2.214	2.552	2.878	3.197
19	.688	.861	1.066	1.328	1.729	2.093	2.205	2.539	2.861	3.174
20	.687	.860	1.064	1.325	1.725	2.086	2.197	2.528	2.845	3.153
21	.686	.859	1.063	1.323	1.721	2.080	2.189	2.518	2.831	3.135
22	.686	.858	1.061	1.321	1.717	2.074	2.183	2.508	2.819	3.119
23	.685	.858	1.060	1.319	1.714	2.069	2.177	2.500	2.807	3.104
24	.685	.857	1.059	1.318	1.711	2.064	2.172	2.492	2.797	3.091
25	.684	.856	1.058	1.316	1.708	2.060	2.167	2.485	2.787	3.078
26	.684	.856	1.058	1.315	1.706	2.056	2.162	2.479	2.779	3.067
27	.684	.855	1.057	1.314	1.703	2.052	2.158	2.473	2.771	3.057
28	.683	.855	1.056	1.313	1.701	2.048	2.154	2.467	2.763	3.047
29	.683	.854	1.055	1.311	1.699	2.045	2.150	2.462	2.756	3.038
30	.683	.854	1.055	1.310	1.697	2.042	2.147	2.457	2.750	3.030
40	.681	.851	1.050	1.303	1.684	2.021	2.123	2.423	2.704	2.971
50	.679	.849	1.047	1.299	1.676	2.009	2.109	2.403	2.678	2.937
60	.679	.848	1.045	1.296	1.671	2.000	2.099	2.390	2.660	2.915
80	.678	.846	1.043	1.292	1.664	1.990	2.088	2.374	2.639	2.887
100	.677	.845	1.042	1.290	1.660	1.984	2.081	2.364	2.626	2.871
1000	.675	.842	1.037	1.282	1.646	1.962	2.056	2.330	2.581	2.813
∞	.674	.841	1.036	1.282	1.645	1.960	2.054	2.326	2.576	2.807

50% 60% 70% 80% 90% 95% 96% 98% 99% 99.5% 99.8% 99.9%

Confidence level C

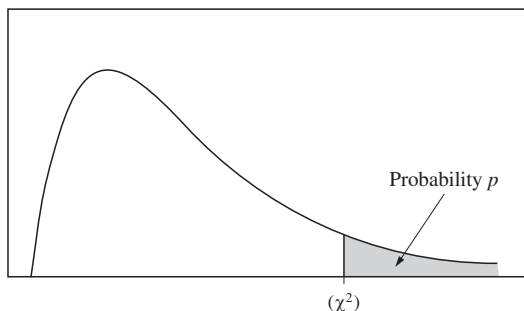


Table entry for p is the point (χ^2) with probability p lying above it.

Table C χ^2 critical values

df	Tail probability p											
	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.32	1.64	2.07	2.71	3.84	5.02	5.41	6.63	7.88	9.14	10.83	12.12
2	2.77	3.22	3.79	4.61	5.99	7.38	7.82	9.21	10.60	11.98	13.82	15.20
3	4.11	4.64	5.32	6.25	7.81	9.35	9.84	11.34	12.84	14.32	16.27	17.73
4	5.39	5.99	6.74	7.78	9.49	11.14	11.67	13.28	14.86	16.42	18.47	20.00
5	6.63	7.29	8.12	9.24	11.07	12.83	13.39	15.09	16.75	18.39	20.51	22.11
6	7.84	8.56	9.45	10.64	12.59	14.45	15.03	16.81	18.55	20.25	22.46	24.10
7	9.04	9.80	10.75	12.02	14.07	16.01	16.62	18.48	20.28	22.04	24.32	26.02
8	10.22	11.03	12.03	13.36	15.51	17.53	18.17	20.09	21.95	23.77	26.12	27.87
9	11.39	12.24	13.29	14.68	16.92	19.02	19.68	21.67	23.59	25.46	27.88	29.67
10	12.55	13.44	14.53	15.99	18.31	20.48	21.16	23.21	25.19	27.11	29.59	31.42
11	13.70	14.63	15.77	17.28	19.68	21.92	22.62	24.72	26.76	28.73	31.26	33.14
12	14.85	15.81	16.99	18.55	21.03	23.34	24.05	26.22	28.30	30.32	32.91	34.82
13	15.98	16.98	18.20	19.81	22.36	24.74	25.47	27.69	29.82	31.88	34.53	36.48
14	17.12	18.15	19.41	21.06	23.68	26.12	26.87	29.14	31.32	33.43	36.12	38.11
15	18.25	19.31	20.60	22.31	25.00	27.49	28.26	30.58	32.80	34.95	37.70	39.72
16	19.37	20.47	21.79	23.54	26.30	28.85	29.63	32.00	34.27	36.46	39.25	41.31
17	20.49	21.61	22.98	24.77	27.59	30.19	31.00	33.41	35.72	37.95	40.79	42.88
18	21.60	22.76	24.16	25.99	28.87	31.53	32.35	34.81	37.16	39.42	42.31	44.43
19	22.72	23.90	25.33	27.20	30.14	32.85	33.69	36.19	38.58	40.88	43.82	45.97
20	23.83	25.04	26.50	28.41	31.41	34.17	35.02	37.57	40.00	42.34	45.31	47.50
21	24.93	26.17	27.66	29.62	32.67	35.48	36.34	38.93	41.40	43.78	46.80	49.01
22	26.04	27.30	28.82	30.81	33.92	36.78	37.66	40.29	42.80	45.20	48.27	50.51
23	27.14	28.43	29.98	32.01	35.17	38.08	38.97	41.64	44.18	46.62	49.73	52.00
24	28.24	29.55	31.13	33.20	36.42	39.36	40.27	42.98	45.56	48.03	51.18	53.48
25	29.34	30.68	32.28	34.38	37.65	40.65	41.57	44.31	46.93	49.44	52.62	54.95
26	30.43	31.79	33.43	35.56	38.89	41.92	42.86	45.64	48.29	50.83	54.05	56.41
27	31.53	32.91	34.57	36.74	40.11	43.19	44.14	46.96	49.64	52.22	55.48	57.86
28	32.62	34.03	35.71	37.92	41.34	44.46	45.42	48.28	50.99	53.59	56.89	59.30
29	33.71	35.14	36.85	39.09	42.56	45.72	46.69	49.59	52.34	54.97	58.30	60.73
30	34.80	36.25	37.99	40.26	43.77	46.98	47.96	50.89	53.67	56.33	59.70	62.16
40	45.62	47.27	49.24	51.81	55.76	59.34	60.44	63.69	66.77	69.70	73.40	76.09
50	56.33	58.16	60.35	63.17	67.50	71.42	72.61	76.15	79.49	82.66	86.66	89.56
60	66.98	68.97	71.34	74.40	79.08	83.30	84.58	88.38	91.95	95.34	99.61	102.7
80	88.13	90.41	93.11	96.58	101.9	106.6	108.1	112.3	116.3	120.1	124.8	128.3
100	109.1	111.7	114.7	118.5	124.3	129.6	131.1	135.8	140.2	144.3	149.4	153.2

STATISTICS

SECTION II

Part A

Questions 1-5

Spend about 65 minutes on this part of the exam.

Percent of Section II score—75

Directions: Show all your work. Indicate clearly the methods you use, because you will be scored on the correctness of your methods as well as on the accuracy and completeness of your results and explanations.

- Two large corporations, A and B, hire many new college graduates as accountants at entry-level positions. In 2009 the starting salary for an entry-level accountant position was \$36,000 a year at both corporations. At each corporation, data were collected from 30 employees who were hired in 2009 as entry-level accountants and were still employed at the corporation five years later. The yearly salaries of the 60 employees in 2014 are summarized in the boxplots below.



- Write a few sentences comparing the distributions of the yearly salaries at the two corporations.
- Suppose both corporations offered you a job for \$36,000 a year as an entry-level accountant.
 - Based on the boxplots, give one reason why you might choose to accept the job at corporation A.
 - Based on the boxplots, give one reason why you might choose to accept the job at corporation B.

1.10 The Normal Distribution

Name: _____ Class: _____ Date: _____ **Total: 10 marks**

KEY WORDS normal distribution 正态分布 • -score 分数 • empirical rule 经验法则 • mean 均值
• standard deviation 标准差

Objective

Build the skills to answer exam questions on **the normal distribution**.

You must be able to:

- describe the **normal distribution** 正态分布 as a symmetric, bell-shaped model with mean μ and standard deviation σ
- apply the **empirical rule** 经验法则 (68-95-99.7)
- calculate a **z -score** z 分数 and use the standard normal distribution

1 Worked examples

Study these first. Each one shows the method for a question type used later.

■ The normal distribution

Symmetric and bell-shaped, fully described by its **mean** μ (center) and **standard deviation** σ (spread).

■ The empirical rule

For a normal distribution, about **68%** of values lie within 1σ , **95%** within 2σ , and **99.7%** within 3σ of the mean.

■ The z -score

$$z = \frac{x - \mu}{\sigma}$$

gives how many standard deviations x is from the mean. With $\mu = 100$, $\sigma = 15$, $x = 130$: $z = 2$.

2 Practice

2.1 State the three percentages of the empirical rule. [2]

2.2 A normal distribution has $\mu = 50$, $\sigma = 5$. Find the z -score of $x = 60$. [2]

2.3 State the two parameters that define a normal distribution. [1]

3 Exam-style questions

3.1 The empirical rule says about 95% of data lie within [1]

- A** 1
- B** 2
- C** 3
- D** 4

standard deviations of the mean.

3.2 A z -score measures [1]

- A** the mean
- B** how many standard deviations a value is from the mean
- C** the median
- D** the range

3.3 IQ scores are normal with $\mu = 100$ and $\sigma = 15$.

(a) Find the z -score for $x = 115$. [2]

(b) State the percentage of scores within 1 standard deviation of the mean. [1]

Solutions

2.1 about 68% within 1σ , 95% within 2σ , 99.7% within 3σ .

2.2 $z = \frac{60 - 50}{5} = 2$.

2.3 the mean μ and the standard deviation σ .

3.1 B.

3.2 B.

3.3 (a) $z = \frac{115 - 100}{15} = 1$. (b) about 68%.

3. A certain sports team has its team song performed by a different musician at the beginning of each of its games. The time it takes for the team song to be performed by different musicians can be modeled using a normal distribution with mean $\mu = 109$ seconds and standard deviation $\sigma = 16$ seconds. All performances of the team song are independent.

Part A

What is the probability that a randomly selected performance of the team song will take longer than 120 seconds? Show your work.

Part B

Ten performances of the team song will be randomly selected. Let the random variable X represent the number of games at which the performance of the team song takes longer than 120 seconds. Find $P(X \geq 3)$. Show your work.

Part C

Ben will attend all of the games for this sports team. Let the random variable Y represent the number of games Ben will attend until a performance of the team song takes longer than 120 seconds.

- Calculate the mean of Y . Show your work.
- Calculate the standard deviation of Y . Show your work.

Part D

Interpret the standard deviation calculated in part C (ii) in context.

STATISTICS**SECTION II, Part B****Suggested Time—25 minutes****1 Question**

Directions: Show all your work. Indicate clearly the methods you use, because you will be scored on the correctness of your methods as well as on the accuracy and completeness of your results and explanations.

- A jewelry company uses a machine to apply a coating of gold on a certain style of necklace. The amount of gold applied to a necklace is approximately normally distributed. When the machine is working properly, the amount of gold applied to a necklace has a mean of 300 milligrams (mg) and standard deviation of 5 mg.
- (a) A necklace is randomly selected from the necklaces produced by the machine. Assuming that the machine is working properly, calculate the probability that the amount of gold applied to the necklace is between 296mg and 304mg.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 6** on this page.

The jewelry company wants to make sure the machine is working properly. Each day, Cleo, a statistician at the jewelry company, will take a random sample of the necklaces produced that day. Each selected necklace will be melted down and the amount of the gold applied to that necklace will be determined. Because a necklace must be destroyed to determine the amount of gold that was applied, Cleo will use random samples of size $n = 2$ necklaces.

Cleo starts by considering the mean amount of gold being applied to the necklaces. After Cleo takes a random sample of $n = 2$ necklaces, she computes the sample mean amount of gold applied to the two necklaces.

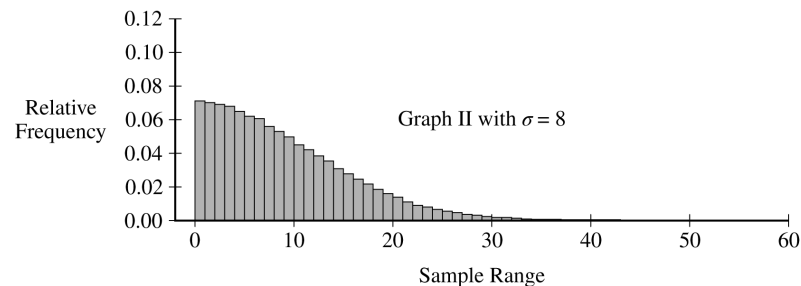
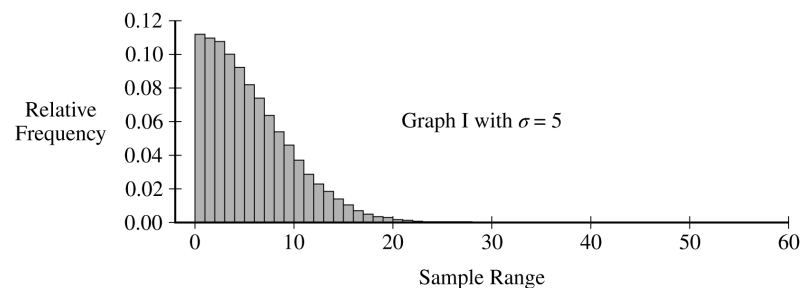
- (b) Suppose the machine is working properly with a population mean amount of gold being applied of 300mg and a population standard deviation of 5mg.
- (i) Calculate the probability that the sample mean amount of gold applied to a random sample of $n = 2$ necklaces will be greater than 303mg.

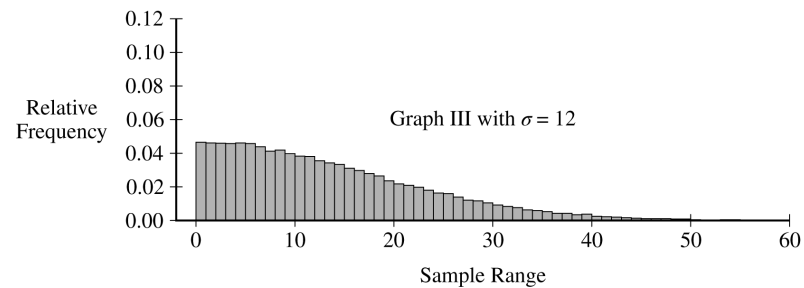
- (ii) Suppose Cleo took a random sample of $n = 2$ necklaces that resulted in a sample mean amount of gold applied of 303mg. Would that result indicate that the population mean amount of gold being applied by the machine is different from 300mg? Justify your answer without performing an inference procedure.

GO ON TO THE NEXT PAGE.Continue your response to **QUESTION 6** on this page.

Now, Cleo will consider the variation in the amount of gold the machine applies to the necklaces. Because of the small sample size, $n = 2$, Cleo will use the sample range of the data for the two randomly selected necklaces, rather than the sample standard deviation.

Cleo will investigate the behavior of the range for samples of size $n = 2$. She will simulate the sampling distribution of the range of the amount of gold applied to two randomly sampled necklaces. Cleo generates 100,000 random samples of size $n = 2$ independent values from a normal distribution with mean $\mu = 300$ and standard deviation $\sigma = 5$. The range is calculated for the two observations in each sample. The simulated sampling distribution of the range is shown in Graph I. This process is repeated using $\sigma = 8$, as shown in Graph II, and again using $\sigma = 12$, as shown in Graph III.

**GO ON TO THE NEXT PAGE.**

Continue your response to **QUESTION 6** on this page.

(c) Use the information in the graphs to complete the following.

- (i) Describe the sampling distribution of the sample range for random samples of size $n = 2$ from a normal distribution with standard deviation $\sigma = 5$, as shown in Graph I.

- (ii) Describe how the sampling distribution of the sample range for samples of size $n = 2$ changes as the value of the population standard deviation σ increases.

GO ON TO THE NEXT PAGE.Continue your response to **QUESTION 6** on this page.

Recall that Cleo needs to consider both the mean and standard deviation of the amount of gold applied to necklaces to determine whether the machine is working properly. Suppose that one month later, Cleo is again checking the machine to make sure it is working properly. Cleo takes a random sample of 2 necklaces and calculates the sample mean amount of gold applied as 303mg and the sample range as 10mg.

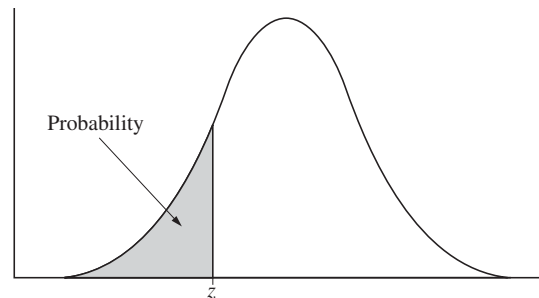
- (d) Recall that the machine is working properly if the amount of gold applied to the necklaces has a mean of 300mg and standard deviation of 5mg.
- (i) Consider Cleo's range of 10mg from the sample of size $n = 2$. If the machine is working properly with a standard deviation of 5mg, is a sample range of 10mg unusual? Justify your answer.

- (ii) Do Cleo's sample mean of 303mg and range of 10mg indicate that the machine is not working properly? Explain your answer.

GO ON TO THE NEXT PAGE.

STOP**END OF EXAM**

Table entry for z is the probability lying below z .

**Table A** Standard normal probabilities

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
-3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
-3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
-3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
-3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
-2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
-2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
-2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
-2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
-2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
-2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
-2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
-2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
-2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
-2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
-1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
-1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
-1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
-1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
-1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
-1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
-1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
-1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
-1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
-1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
-0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
-0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
-0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
-0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
-0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
-0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
-0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
-0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
-0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
-0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

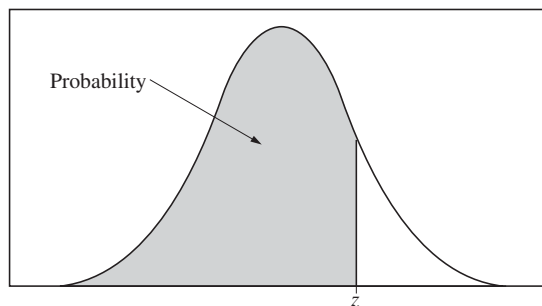
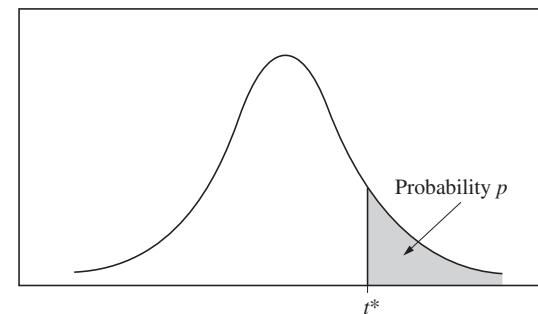
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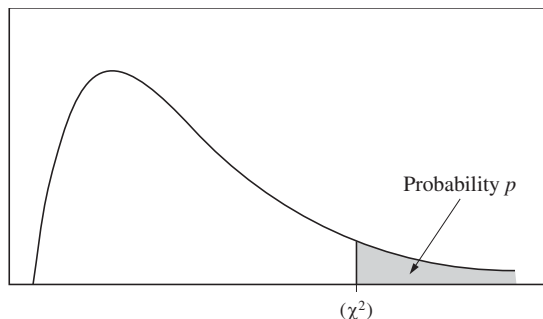
Table A (Continued)

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

Name: Table B t distribution critical values

df	Tail probability p											
	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.000	1.376	1.963	3.078	6.314	12.71	15.89	31.82	63.66	127.3	318.3	636.6
2	.816	1.061	1.386	1.886	2.920	4.303	4.849	6.965	9.925	14.09	22.33	31.60
3	.765	.978	1.250	1.638	2.353	3.182	3.482	4.541	5.841	7.453	10.21	12.92
4	.741	.941	1.190	1.533	2.132	2.776	2.999	3.747	4.604	5.598	7.173	8.610
5	.727	.920	1.156	1.476	2.015	2.571	2.757	3.365	4.032	4.773	5.893	6.869
6	.718	.906	1.134	1.440	1.943	2.447	2.612	3.143	3.707	4.317	5.208	5.959
7	.711	.896	1.119	1.415	1.895	2.365	2.517	2.998	3.499	4.029	4.785	5.408
8	.706	.889	1.108	1.397	1.860	2.306	2.449	2.896	3.355	3.833	4.501	5.041
9	.703	.883	1.100	1.383	1.833	2.262	2.398	2.821	3.250	3.690	4.297	4.781
10	.700	.879	1.093	1.372	1.812	2.228	2.359	2.764	3.169	3.581	4.144	4.587
11	.697	.876	1.088	1.363	1.796	2.201	2.328	2.718	3.106	3.497	4.025	4.437
12	.695	.873	1.083	1.356	1.782	2.179	2.303	2.681	3.055	3.428	3.930	4.318
13	.694	.870	1.079	1.350	1.771	2.160	2.282	2.650	3.012	3.372	3.852	4.221
14	.692	.868	1.076	1.345	1.761	2.145	2.264	2.624	2.977	3.326	3.787	4.140
15	.691	.866	1.074	1.341	1.753	2.131	2.249	2.602	2.947	3.286	3.733	4.073
16	.690	.865	1.071	1.337	1.746	2.120	2.235	2.583	2.921	3.252	3.686	4.015
17	.689	.863	1.069	1.333	1.740	2.110	2.224	2.567	2.898	3.222	3.646	3.965
18	.688	.862	1.067	1.330	1.734	2.101	2.214	2.552	2.878	3.197	3.611	3.922
19	.688	.861	1.066	1.328	1.729	2.093	2.205	2.539	2.861	3.174	3.579	3.883
20	.687	.860	1.064	1.325	1.725	2.086	2.197	2.528	2.845	3.153	3.552	3.850
21	.686	.859	1.063	1.323	1.721	2.080	2.189	2.518	2.831	3.135	3.527	3.819
22	.686	.858	1.061	1.321	1.717	2.074	2.183	2.508	2.819	3.119	3.505	3.792
23	.685	.858	1.060	1.319	1.714	2.069	2.177	2.500	2.807	3.104	3.485	3.768
24	.685	.857	1.059	1.318	1.711	2.064	2.172	2.492	2.797	3.091	3.467	3.745
25	.684	.856	1.058	1.316	1.708	2.060	2.167	2.485	2.787	3.078	3.450	3.725
26	.684	.856	1.058	1.315	1.706	2.056	2.162	2.479	2.779	3.067	3.435	3.707
27	.684	.855	1.057	1.314	1.703	2.052	2.158	2.473	2.771	3.057	3.421	3.690
28	.683	.855	1.056	1.313	1.701	2.048	2.154	2.467	2.763	3.047	3.408	3.674
29	.683	.854	1.055	1.311	1.699	2.045	2.150	2.462	2.756	3.038	3.396	3.659
30	.683	.854	1.055	1.310	1.697	2.042	2.147	2.457	2.750	3.030	3.385	3.646
40	.681	.851	1.050	1.303	1.684	2.021	2.123	2.423	2.704	2.971	3.307	3.551
50	.679	.849	1.047	1.299	1.676	2.009	2.109	2.403	2.678	2.937	3.261	3.496
60	.679	.848	1.045	1.296	1.671	2.000	2.099	2.390	2.660	2.915	3.232	3.460
80	.678	.846	1.043	1.292	1.664	1.990	2.088	2.374	2.639	2.887	3.195	3.416
100	.677	.845	1.042	1.290	1.660	1.984	2.081	2.364	2.626	2.871	3.174	3.390
1000	.675	.842	1.037	1.282	1.646	1.962	2.056	2.330	2.581	2.813	3.098	3.300
∞	.674	.841	1.036	1.282	1.645	1.960	2.054	2.326	2.576	2.807	3.091	3.291
50% 60% 70% 80% 90% 95% 96% 98% 99% 99.5% 99.8% 99.9%												
Confidence level C												

Table entry for p is the point (χ^2) with probability p lying above it.

Table C χ^2 critical values

df	Tail probability p										
	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001
1	1.32	1.64	2.07	2.71	3.84	5.02	5.41	6.63	7.88	9.14	10.83
2	2.77	3.22	3.79	4.61	5.99	7.38	7.82	9.21	10.60	11.98	13.82
3	4.11	4.64	5.32	6.25	7.81	9.35	9.84	11.34	12.84	14.32	16.27
4	5.39	5.99	6.74	7.78	9.49	11.14	11.67	13.28	14.86	16.42	18.47
5	6.63	7.29	8.12	9.24	11.07	12.83	13.39	15.09	16.75	18.39	20.51
6	7.84	8.56	9.45	10.64	12.59	14.45	15.03	16.81	18.55	20.25	22.46
7	9.04	9.80	10.75	12.02	14.07	16.01	16.62	18.48	20.28	22.04	24.32
8	10.22	11.03	12.03	13.36	15.51	17.53	18.17	20.09	21.95	23.77	26.12
9	11.39	12.24	13.29	14.68	16.92	19.02	19.68	21.67	23.59	25.46	27.88
10	12.55	13.44	14.53	15.99	18.31	20.48	21.16	23.21	25.19	27.11	29.59
11	13.70	14.63	15.77	17.28	19.68	21.92	22.62	24.72	26.76	28.73	31.26
12	14.85	15.81	16.99	18.55	21.03	23.34	24.05	26.22	28.30	30.32	32.91
13	15.98	16.98	18.20	19.81	22.36	24.74	25.47	27.69	29.82	31.88	34.53
14	17.12	18.15	19.41	21.06	23.68	26.12	26.87	29.14	31.32	33.43	36.12
15	18.25	19.31	20.60	22.31	25.00	27.49	28.26	30.58	32.80	34.95	37.70
16	19.37	20.47	21.79	23.54	26.30	28.85	29.63	32.00	34.27	36.46	39.25
17	20.49	21.61	22.98	24.77	27.59	30.19	31.00	33.41	35.72	37.95	40.79
18	21.60	22.76	24.16	25.99	28.87	31.53	32.35	34.81	37.16	39.42	42.31
19	22.72	23.90	25.33	27.20	30.14	32.85	33.69	36.19	38.58	40.88	43.82
20	23.83	25.04	26.50	28.41	31.41	34.17	35.02	37.57	40.00	42.34	45.31
21	24.93	26.17	27.66	29.62	32.67	35.48	36.34	38.93	41.40	43.78	46.80
22	26.04	27.30	28.82	30.81	33.92	36.78	37.66	40.29	42.80	45.20	48.27
23	27.14	28.43	29.98	32.01	35.17	38.08	38.97	41.64	44.18	46.62	49.73
24	28.24	29.55	31.13	33.20	36.42	39.36	40.27	42.98	45.56	48.03	51.18
25	29.34	30.68	32.28	34.38	37.65	40.65	41.57	44.31	46.93	49.44	52.62
26	30.43	31.79	33.43	35.56	38.89	41.92	42.86	45.64	48.29	50.83	54.05
27	31.53	32.91	34.57	36.74	40.11	43.19	44.14	46.96	49.64	52.22	55.48
28	32.62	34.03	35.71	37.92	41.34	44.46	45.42	48.28	50.99	53.59	56.89
29	33.71	35.14	36.85	39.09	42.56	45.72	46.69	49.59	52.34	54.97	58.30
30	34.80	36.25	37.99	40.26	43.77	46.98	47.96	50.89	53.67	56.33	59.70
40	45.62	47.27	49.24	51.81	55.76	59.34	60.44	63.69	66.77	69.70	73.40
50	56.33	58.16	60.35	63.17	67.50	71.42	72.61	76.15	79.49	82.66	86.66
60	66.98	68.97	71.34	74.40	79.08	83.30	84.58	88.38	91.95	95.34	99.61
80	88.13	90.41	93.11	96.58	101.9	106.6	108.1	112.3	116.3	120.1	124.8
100	109.1	111.7	114.7	118.5	124.3	129.6	131.1	135.8	140.2	144.3	149.4

A machine at a manufacturing company is programmed to fill shampoo bottles such that the amount of shampoo in each bottle is normally distributed with mean 0.60 liter and standard deviation 0.04 liter. Let the random variable A represent the amount of shampoo, in liters, that is inserted into a bottle by the filling machine.

- (a) A bottle is considered to be underfilled if it has less than 0.50 liter of shampoo. Determine the probability that a randomly selected bottle of shampoo will be underfilled. Show your work.

After the bottles are filled, they are placed in boxes of 10 bottles per box. After the bottles are placed in the boxes, several boxes are placed in a crate for shipping to a beauty supply warehouse. The manufacturing company's contract with the beauty supply warehouse states that one box will be randomly selected from a crate. If 2 or more bottles in the selected box are underfilled, the entire crate will be rejected and sent back to the manufacturing company.

- (b) The beauty supply warehouse manager is interested in the probability that a crate shipped to the warehouse will be rejected. Assume that the amounts of shampoo in the bottles are independent of each other.

- (i) Define the random variable of interest for the warehouse manager and state how the random variable is distributed.

- (ii) Determine the probability that a crate will be rejected by the warehouse manager. Show your work.

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Continue your response to **QUESTION 3** on this page.

To reduce the number of crates rejected by the beauty supply warehouse manager, the manufacturing company is considering adjusting the programming of the filling machine so that the amount of shampoo in each bottle is normally distributed with mean 0.56 liter and standard deviation 0.03 liter.

- (c) Would you recommend that the manufacturing company use the original programming of the filling machine or the adjusted programming of the filling machine? Provide a statistical justification for your choice.

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5. A company that manufactures smartphones developed a new battery that has a longer life span than that of a traditional battery. From the date of purchase of a smartphone, the distribution of the life span of the new battery is approximately normal with mean 30 months and standard deviation 8 months. For the price of \$50, the company offers a two-year warranty on the new battery for customers who purchase a smartphone. The warranty guarantees that the smartphone will be replaced at no cost to the customer if the battery no longer works within 24 months from the date of purchase.
- (a) In how many months from the date of purchase is it expected that 25 percent of the batteries will no longer work? Justify your answer.
- (b) Suppose one customer who purchases the warranty is selected at random. What is the probability that the customer selected will require a replacement within 24 months from the date of purchase because the battery no longer works?
- (c) The company has a gain of \$50 for each customer who purchases a warranty but does not require a replacement. The company has a loss (negative gain) of \$150 for each customer who purchases a warranty and does require a replacement. What is the expected value of the gain for the company for each warranty purchased?

3. A grocery store purchases melons from two distributors, J and K. Distributor J provides melons from organic farms. The distribution of the diameters of the melons from Distributor J is approximately normal with mean 133 millimeters (mm) and standard deviation 5 mm.

(a) For a melon selected at random from Distributor J, what is the probability that the melon will have a diameter greater than 137 mm?

Distributor K provides melons from nonorganic farms. The probability is 0.8413 that a melon selected at random from Distributor K will have a diameter greater than 137 mm. For all the melons at the grocery store, 70 percent of the melons are provided by Distributor J and 30 percent are provided by Distributor K.

(b) For a melon selected at random from the grocery store, what is the probability that the melon will have a diameter greater than 137 mm?

(c) Given that a melon selected at random from the grocery store has a diameter greater than 137 mm, what is the probability that the melon will be from Distributor J?

2. Nine sales representatives, 6 men and 3 women, at a small company wanted to attend a national convention. There were only enough travel funds to send 3 people. The manager selected 3 people to attend and stated that the people were selected at random. The 3 people selected were women. There were concerns that no men were selected to attend the convention.

(a) Calculate the probability that randomly selecting 3 people from a group of 6 men and 3 women will result in selecting 3 women.

(b) Based on your answer to part (a), is there reason to doubt the manager's claim that the 3 people were selected at random? Explain.

(c) An alternative to calculating the exact probability is to conduct a simulation to estimate the probability. A proposed simulation process is described below.

Each trial in the simulation consists of rolling three fair, six-sided dice, one die for each of the convention attendees. For each die, rolling a 1, 2, 3, or 4 represents selecting a man; rolling a 5 or 6 represents selecting a woman. After 1,000 trials, the number of times the dice indicate selecting 3 women is recorded.

Does the proposed process correctly simulate the random selection of 3 women from a group of 9 people consisting of 6 men and 3 women? Explain why or why not.

3. Schools in a certain state receive funding based on the number of students who attend the school. To determine the number of students who attend a school, one school day is selected at random and the number of students in attendance that day is counted and used for funding purposes. The daily number of absences at High School A in the state is approximately normally distributed with mean of 120 students and standard deviation of 10.5 students.

(a) If more than 140 students are absent on the day the attendance count is taken for funding purposes, the school will lose some of its state funding in the subsequent year. Approximately what is the probability that High School A will lose some state funding?

(b) The principals' association in the state suggests that instead of choosing one day at random, the state should choose 3 days at random. With the suggested plan, High School A would lose some of its state funding in the subsequent year if the mean number of students absent for the 3 days is greater than 140. Would High School A be more likely, less likely, or equally likely to lose funding using the suggested plan compared to the plan described in part (a)? Justify your choice.

(c) A typical school week consists of the days Monday, Tuesday, Wednesday, Thursday, and Friday. The principal at High School A believes that the number of absences tends to be greater on Mondays and Fridays, and there is concern that the school will lose state funding if the attendance count occurs on a Monday or Friday. If one school day is chosen at random from each of 3 typical school weeks, what is the probability that none of the 3 days chosen is a Tuesday, Wednesday, or Thursday?