

AP® STATISTICS
2016 SCORING GUIDELINES

Question 5

Intent of Question

The primary goals of this question were to assess a student's ability to (1) construct and interpret a confidence interval for a population proportion; (2) explain why one of the conditions for inference is necessary; and (3) explain why a suggested procedure for constructing a confidence interval is incorrect.

Solution

Part (a):

The appropriate procedure is a one-sample z-interval for a population proportion. The problem stated the conditions for inference have been met, so they do not need to be checked. A 95 percent

confidence interval for the population proportion is given as $\hat{p} \pm z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$, which is

$$0.37 \pm 1.96 \sqrt{\frac{(0.37)(0.63)}{1,048}} \approx 0.37 \pm 0.03 = (0.34, 0.40).$$

We are 95 percent confident that the population proportion of all adults in the U.S. who would have chosen the economy statement is between 0.34 and 0.40.

Part (b):

The condition is necessary because the formula for the confidence interval relies on the fact that the binomial distribution can be approximated by a normal distribution which then results in the sampling distribution of $\hat{p}$ being approximately normal. The approximation does not work well unless both $n\hat{p}$ and $n(1-\hat{p})$ are at least 10.

Part (c):

The suggested procedure is not appropriate because one of the requirements for using a two-sample z-interval for a difference between proportions is that the two proportions are based on two independent samples. In the situation described the two proportions come from a single sample and thus are not independent.

Scoring

This question is scored in three sections. Section 1 consists of the mechanics of the confidence interval in part (a), section 2 consists of interpreting the confidence interval in part (a), and section 3 consists of parts (b) and (c).

Section 1 is scored as follows:

Essentially correct (E) if the response includes the following two components:

1. States the correct procedure by name or formula
2. Calculates the confidence interval

Partially correct (P) if the response includes only one of the two components.

Incorrect (I) if the response does not meet the criteria for E or P.

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Question 5 (continued)

Notes:

- A formula with correct values is sufficient for component 2.
- Component 2 can never be satisfied if the response contains an incorrect formula.

Section 2 is scored as follows:

Essentially correct (E) if the response includes the following four components for the interval interpretation:

1. Estimates a proportion
2. Infers about the population
3. States 95 percent confidence
4. Includes context

Partially correct (P) if the response satisfies components 1 and 2 *AND* satisfies only one of components 3 or 4;

OR

if the response gives a correct interpretation of the confidence *level* in context without interpreting the specific interval.

Incorrect (I) if the response does not meet the criteria for E or P.

Notes:

- Any indication of an inference to the adults sampled rather than the population does not satisfy component 2 and is scored as I.
- Stating values that are unrealistic as proportions (including blanks) in the interpretation lowers the score by one level (from E to P or from P to I).
- When both the interpretation of the interval and the level are given, only the interpretation of the interval is scored.

Section 3 is scored as follows:

Essentially correct (E) if the response includes the following two components:

1. Part (b) states that the condition implies the sampling distribution of $\hat{p}$ is approximately normal *OR* the normal approximation to the binomial distribution is appropriate.
2. Part (c) indicates the procedure is not appropriate because the two proportions come from a single sample (dependent) rather than two (independent) samples.

Partially correct (P) if the response includes only one of the two components.

Incorrect (I) if the response does not meet the criteria for E or P.

Notes:

- Referring to the sampling distribution needing to be normal without explicitly stating $\hat{p}$ satisfies component 1.
- Referring to normal approximation must specify "to binomial" to satisfy component 1.
- Discussing the two-sample z-interval versus the two-proportion z-interval is extraneous information.

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- In part (c) component 2 can never be satisfied if a procedure that is not possible for one sample of categorical data is suggested.

4 Complete Response

Three parts essentially correct

3 Substantial Response

Two parts essentially correct and one part partially correct

2 Developing Response

Two parts essentially correct and no parts partially correct

OR

One part essentially correct and one or two parts partially correct

OR

Three parts partially correct

1 Minimal Response

One part essentially correct

OR

No parts essentially correct and two parts partially correct