

Question 6**Intent of Question**

The primary goals of this question were to assess a student's ability to (1) recognize the population to which results from a random sample may be generalized; (2) describe a disadvantage of using a sample mean rather than a sample median to indicate typical values when the sample distribution is skewed; (3) describe how the theoretical sampling distribution of the sample median could be constructed; (4) construct an approximate confidence interval for a population median using results from a bootstrap procedure; and (5) interpret a confidence interval.

Solution**Part (a):**

Because random sampling was used, the results of the sample may be generalized to the population of rental prices for one-bedroom apartments in the city that are listed on this particular website at the time the sample was taken.

Part (b):

Because the distribution of the 50 rental prices in the sample is skewed to the right, the sample median provides a better indicator of typical rental prices than the sample mean. Some very large rental prices results in a sample mean that is substantially larger than the more typical rental prices. As a result the sample mean would overestimate the typical rental price, whereas the sample median would be a more accurate representation of typical rental prices.

Part (c):

To determine the sampling distribution of median rental prices for random samples of 50 one-bedroom apartments from this population, Emma would need to obtain every possible sample of 50 one-bedroom apartments from this website and compute the median of each sample. The collection of all possible sample medians is the theoretical sampling distribution for sample median.

Part (d):

- (i) $(0.05)(15,000) = 750$ and $(0.95)(15,000) = 14,250$. The 5th percentile is a value, say $x_{0.05}$, such that at least 750 values in the table are less than or equal to $x_{0.05}$ and at least 14,250 are greater than or equal to $x_{0.05}$. Cumulate frequencies starting with the smallest sample median listed in the table and going toward the largest (going down columns) until you first reach 750 values, to obtain $x_{0.05} = \$2,500$.
- (ii) Similarly, $x_{0.95} = \$2,950$.

Part (e):

The percentage of bootstrap medians between (and including) the values found in part (d) for the 5th and 95th percentiles is

$$\frac{14,404}{15,000} \times 100\% \approx 96.03\%$$

2019 SCORING GUIDELINES**Question 6 (continued)****Part (f):**

From the results in part (d) and part (e), an approximate 96 percent confidence interval for the median rental price of all one-bedroom apartments listed on this website for this city is (\$2,500, \$2,950). We are approximately 96 percent confident that the median rental price of all one-bedroom apartments listed on this website for this city is between \$2,500 and \$2,950.

Scoring

This question is scored in four sections. Section 1 consists of parts (a) and (b), section 2 consists of part (c), section 3 consists of parts (d) and (e), and section 4 consists of part (f). Sections 1, 2, 3, and 4 are each scored as essentially correct (E), partially correct (P), or incorrect (I).

Section 1 is scored as follows:

Essentially correct (E) if the response satisfies the following three components:

1. The correct population (listings of one-bedroom apartments on the website) is identified in part (a).
2. In part (b), identifying that using the sample mean instead of the sample median overestimates the typical rental price.
3. The disadvantage of using the sample mean that is reported in part (b) is correctly linked to some feature of the distribution (e.g. skewness) that is evident in the histogram.

Partially correct (P) if the response satisfies only two of the three components.

Incorrect (I) if the response does not meet the criteria for E or P.

Note: Responses that refer to the mean being larger than the median in a skewed right distribution alone is not sufficient to satisfy component 2.

Section 2 is scored as follows:

Essentially correct (E) if the response satisfies the following two components:

1. Indicates that Emma would need to obtain every possible sample of 50 one-bedroom apartments.
2. Indicates that Emma would need to compute the median rental price for each sample.

Partially correct (P) if the response satisfies only one of the two components.

Incorrect (I) if the response does not satisfy the criteria for E or P.

Section 3 is scored as follows:

Essentially correct (E) if the response satisfies the following two components:

1. Correct values for the 5th percentile and the 95th percentile are reported in part (d).
2. The correct percentage of bootstrap samples that produced sample medians at or between the two values, if they are plausible, reported in part (d) is reported in part (e).

Partially correct (P) if the response satisfies only one of the two components.

Incorrect (I) if the response does not satisfy the criteria for E or P.

Note: Plausible values for part (d) will be considered values between 2,345 and 3,062.5.

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Section 4 is scored as follows:

Essentially correct if the response in part (f) satisfies the following three components:

1. Uses \$2500 and \$2950 or the values of the percentiles reported in part (d) as the endpoints of the confidence interval.
2. Indicates an approximate 90 or 96 percent level of confidence or a level of confidence consistent with part (e).
3. Makes a correct statement in context indicating that the confidence interval is for the median.

Partially correct if the response satisfies only two of the three components.

Incorrect if the response does not satisfy the criteria for E or P.

Note: Since rental prices from the population are discrete values, the true confidence level of the interval from part (d) is unknown. A correctly calculated part (e) is a way to estimate the confidence level; from Emma's sample the confidence level is estimated to be approximately 96 percent. The process described in part (d) for calculating the interval will result in a confidence level of at least 90 percent. For these reasons, confidence levels of either 90 or 96 percent satisfy component 2.

Each essentially correct (E) section counts as 1 point, and a partially correct (P) section counts as $\frac{1}{2}$ point.

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| 4 | Complete Response |
| 3 | Substantial Response |
| 2 | Developing Response |
| 1 | Minimal Response |

If a response is between two scores (for example, $2\frac{1}{2}$ points), use a holistic approach to decide whether to score up or down depending on the strength of the response and communication.