AP® STATISTICS
2016 SCORING GUIDELINES

Question 4

Intent of Question

The primary goals of this question were to assess a student's ability to (1) calculate a probability using basic probability rules or the geometric distribution; (2) recognize that a probability calculation for independent events does not depend on the previous outcomes of those events; and (3) assess whether a claim about the probability of a single event is reasonable based on a calculated probability of a series of those events.

Solution

Part (a):

If the failure rate for the super igniters is 15 percent, then the probability that each igniter fails is 0.15, and the probability that it does not fail is 0.85. Therefore the probability that the first 30 igniters tested do not fail is $(0.85)^{30} \approx 0.0076$. The solution can also be written as $(1 - 0.15)^{30} \approx 0.0076$.

Part (b):

Given that there are no failures in the first 30 trials, the probability that the first failure occurs on the 31st trial is 0.15, and the probability that it does not occur on the 31st but occurs on the 32nd trial is $(0.85)(0.15) = 0.1275$. Therefore the probability that the first failure occurs on the 31st or 32nd super igniter tested is $0.15 + 0.1275 = 0.2775$.

Note that this is equivalent to asking for the probability that the first failure occurs on the first or second trial, which is $0.15 + (0.85)(0.15) = 0.2775$.

Part (c):

The result of the probability calculation in part (a) provides a reason to believe that the failure rate of the super igniters is less than 15 percent. The calculated probability of 0.0076 shows that there is less than a 1 percent chance that 30 or more igniters in a row would not fail if the failure rate was 15 percent. This probability is smaller than conventional significance levels such as $\alpha = 0.05$ or $\alpha = 0.01$, and thus is small enough to make it reasonable to believe that the failure rate of the super igniters is less than 15 percent.

Scoring

Parts (a), (b), and (c) are scored as essentially correct (E), partially correct (P), or incorrect (I).

Part (a) is scored as follows:

Essentially correct (E) if the response gives the correct probability *AND* correct justification.

Partially correct (P) if the response correctly notes that the answer is the probability that there will be 30 successes in 30 attempts, but does not carry out a correct probability calculation;
OR

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Question 4 (continued)

if the response defines the random variable X as the trial with the first failure, identifies X as having a geometric distribution with $p = 0.15$, and writes the desired probability as $P(X > 30)$, but does not carry out a correct probability calculation;
OR
if the response defines the random variable X as the number of failures in the first 30 attempts, identifies X as a binomial random variable with $p = 0.15$ and $n = 30$, and writes the desired probability as $P(X = 0)$, but does not carry out a correct probability calculation;
OR
if the response gives the correct probability but, in specifying a geometric or binomial distribution, has an incorrect or incomplete definition of parameters or value(s) of the random variable.

Incorrect (I) if the response does not meet the criteria for E or P.

Note: Justification can be given using the multiplication rule; *OR* by defining X to be the trial with the first failure, recognizing that X has a geometric distribution, and using that information to find $P(X > 30)$; *OR* by defining X to be the number of failures in the first 30 attempts, and then finding $P(X = 0)$ using either probability rules or the binomial distribution with $n = 30$ and $p = 0.15$.

Part (b) is scored as follows:

Essentially correct (E) if the response gives the correct probability AND correct justification.

Partially correct (P) if the response makes a reasonable attempt to calculate a geometric, binomial, or conditional probability, but does not successfully carry out the calculation;
OR
if the response gives the correct probability but, in specifying a geometric or binomial distribution, has an incorrect or incomplete definition of parameters or value(s) of the random variable.

Incorrect (I) if the response finds an incorrect probability resulting from an unreasonable attempt to calculate a geometric, binomial, or conditional probability or otherwise does not meet the criteria for E or P.

Note: Similar to part (a) justification can be given using probability rules; *OR* by stating that X is geometric where X is the trial with the first failure, then finding $P(X = 1 \text{ or } X = 2)$; *OR* by stating that X is the number of failures in two trials and finding $1 - P(X = 0)$ or $P(X = 1 \text{ or } X = 2)$ using the binomial distribution.

Part (c) is scored as follows:

Essentially correct (E) if the response states that it is reasonable to believe that the failure rate is less than 15 percent *AND* bases this decision on the fact that the probability of 30 consecutive successful launches with a failure rate of 15 percent (that is, answer from part (a)) is small *AND* does so in the context of the situation.

Partially correct (P) if the response otherwise satisfies the criteria for an (E) but does so without any context;
OR

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Question 4 (continued)

if the response states a significance level and makes a decision in a context that is appropriate to the given probability in part (a) and the stated significance level but does not explicitly compare the probability and the significance level (no linkage).

Incorrect (I) if the response does not explicitly make a decision about whether it is reasonable to conclude that the failure rate is less than 15 percent (For example: "As seen in Part (a), if the failure rate is 15 percent then the probability of 30 successful launches in a row is very small.");
OR
if the response otherwise does not meet the criteria for E or P.

Notes:

- Justification based on the probability can come by stating a significance level and noting that the probability is smaller than the significance level *OR* by simply stating that the probability of 0.0076 is small *OR* by referring to the expected number of failures (4.5) as being very unlikely because zero failures is more than two standard deviations below 4.5.
- If the response bases the decision on the expected number of failures (4.5) for $n = 30$ and $p = 0.15$ without referencing why zero failures would be considered to be too far below 4.5 to give reason to doubt the stated 15 percent failure rate, the response is scored P.
- If the calculation in part (a) is incorrect, the answer in part (c) needs to be consistent with the answer in part (a), unless the value is recalculated in part (c).

4 **Complete Response**

Three parts essentially correct

3 **Substantial Response**

Two parts essentially correct and one part partially correct

2 **Developing Response**

Two parts essentially correct and no parts partially correct
OR
One part essentially correct and one or two parts partially correct
OR
Three parts partially correct

1 **Minimal Response**

One part essentially correct
OR
No parts essentially correct and two parts partially correct