

2018

AP[®]

CollegeBoard

AP Statistics

Scoring Guidelines

2018 SCORING GUIDELINES**Question 1****Intent of Question**

The primary goals of this question were to assess a student's ability to (1) identify various values in regression computer output; (2) interpret the intercept of a regression line in context; (3) interpret the coefficient of determination (r^2) in context; and (4) identify an outlier from a scatterplot.

Solution**Part (a):**

The estimate of the intercept is 72.95. It is estimated that the average time to finish checkout if there are no other customers in line is 72.95 seconds.

Part (b):

The coefficient of determination is $r^2 = 73.33\%$. This value indicates that 73.33% of the variability in the times it takes customers to finish checkout, including time waiting in line, can be explained by knowing how many customers are in line in front of the selected customer.

Part (c):

The outlier is the point with $x = 3$ and y close to 0. This point is considered an outlier because the combination of x and y values differs from the pattern of the rest of the data. Specifically, the value of y (time to finish checkout) is much lower than would be expected when there are $x = 3$ customers in line in front of the selected customer, given the remaining data.

Scoring

Parts (a), (b), and (c) are scored as essentially correct (E), partially correct (P), or incorrect (I).

Part (a) is scored as follows:

Essentially correct (E) if the response satisfies the following three components:

1. Correctly identifies 72.95 as the intercept.
2. Communicates the concept of a y -intercept in a context that includes both time and zero customers.
3. Indicates that the value of the intercept is a prediction by using language such as “predicted,” “estimated,” or “average” value of y .

Partially correct (P) if the response includes only two of the three components.

Incorrect (I) if the response includes at most one of the three components.

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Question 1 (continued)

Notes:

- Regression equations (such as $\hat{y} = 72.95 + 174.40x$) cannot be used to satisfy identification of the intercept in component 1, unless the intercept is explicitly labeled as such.
- A regression equation cannot be used to satisfy component 3.
- Incorrect regression equations are treated as extraneous and do not affect the scoring of any component.
- A response that interprets 72.95 as a slope does not satisfy components 1 or 2.

Part (b) is scored as follows:

Essentially correct (E) if the response satisfies the following three components:

1. Correctly identifies 73.33% as the coefficient of determination.
2. Provides a correct (possibly generic) interpretation of r^2 .
3. Interpretation includes context.

Partially correct (P) if the response satisfies only two of the three components;

OR

if the response satisfies the three components, but reverses the roles of number of customers in line and time to finish checkout in the interpretation.

Incorrect (I) if the response satisfies at most one of the three components.

Notes:

- In component 2 the correct interpretation of the coefficient of determination can take any of several equivalent forms, such as:
 - The percent variability in y that is attributed to the linear relationship between y and x or between x and y .
 - The proportion of the total variability in the dependent variable y that is explained by the independent variable x .
 - The proportion of variation in y that is accounted for by the linear model.
 - The proportionate reduction of total variation of the y values that is associated with the use of the independent variable x .
 - The proportionate reduction in the sum of the squares of vertical deviations obtained by using the least-squares line instead of the naïve prediction of $\bar{y}$.
- In component 2 common *incorrect* interpretations of the coefficient of determination include:
 - The percent variability in the *predicted* y values that is explained by the linear relationship between y and x .
 - The percent variability in the *data* that is explained by the linear relationship between y and x .
 - The percent variability that is explained by the linear relationship between y and x .
 - The percent variability in y that is *on average* explained by the linear relationship between y and x .
- For component 3 context must include mention of time or customers.

2018 SCORING GUIDELINES**Question 1 (continued)**

Part (c) is scored as follows:

Essentially correct (E) if the response satisfies the following two components:

1. Correctly identifies the outlier.
2. Describes an unusual feature of the identified scatter plot point, relative to the remaining data points, that is sufficient to identify it as the outlier. Examples include:
 - The combination of x and y values is unusual compared to the other points.
 - The value of y is much lower than would be expected (or predicted), given the remaining data.
 - The residual for the point is unusually large relative to the other residuals.

Partially correct (P) if the response satisfies component 1 but does not satisfy component 2.

Incorrect (I) if the response does not meet the criteria for E or P.

Notes:

- In the absence of any point being circled on the graph, component 1 can still be satisfied by explicitly referring to the coordinates of the outlier. Valid coordinates for outlier identification must specify an x value of 3 and a y value that is strictly between 0 and 250.
- A response that does not make a comparison to the remaining data points, such as stating the outlier has a large residual or is nowhere near the regression line, does not satisfy component 2.
- A response that makes a comparison to the remaining data points based upon an unusual feature that is *insufficient* for outlier identification, such as stating the point is the only point with that particular y value, does not satisfy component 2.
- In the absence of explicit numerical calculation, a response that appeals to the influence that the outlier has on the regression coefficient estimates or on the sample correlation coefficient does not satisfy component 2.

2018 SCORING GUIDELINES**Question 1 (continued)****4 Complete Response**

Three parts essentially correct

3 Substantial Response

Two parts essentially correct and one part partially correct

2 Developing Response

Two parts essentially correct and no parts partially correct

OR

One part essentially correct and one or two parts partially correct

OR

Three parts partially correct

1 Minimal Response

One part essentially correct

OR

No parts essentially correct and two parts partially correct

2018 SCORING GUIDELINES

Question 2

Intent of Question

The primary goals of this question were to assess a student's ability to (1) calculate the sample size when given the endpoints of a confidence interval for a proportion; (2) explain how bias could be present in a particular survey method; and (3) estimate a proportion from sample data collected using a method designed to decrease bias.

Solution

Part (a):

Using the standard formula for a confidence interval for one proportion, the interval (0.584 to 0.816) is found as follows. $\hat{p} \pm z^* \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}}$ where $\hat{p} = \frac{0.584 + 0.816}{2} = 0.7$, the margin of error is $0.816 - 0.7 = 0.116$, and $z^* = 1.96$.

Solving $1.96 \sqrt{\frac{0.7(1 - 0.7)}{n}} = 0.116$ yields $n = \frac{(1.96)^2 (0.7)(1 - 0.7)}{(0.116)^2} \approx 59.95$. The sample size was 60.

Part (b):

Bias might have been introduced because students responded directly to the environmental science teacher. Because the students would know that an environmental science teacher cares about the environment, they might say yes when they actually don't recycle. This would result in a point estimate that is greater than the proportion of all students who would respond yes to the question.

Part (c):

(i) The expected number is $(300)\left(\frac{1}{2}\right) = 150$.

(ii) The point estimate is based on expecting 150 students to be required to say no and 150 students to truthfully answer the question. Of the 213 answers of no, we expect that $213 - 150 = 63$ were from students who truthfully answered the question. That means we expect that the remaining $150 - 63 = 87$ students truthfully answered the question and responded yes. So the point estimate for the proportion of all students at the high school who would respond yes to the question is $\frac{87}{150} = 0.58$.

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Question 2 (continued)

Scoring

Parts (a), (b), and (c) are scored as essentially correct (E), partially correct (P), or incorrect (I).

Part (a) is scored as follows:

Essentially correct (E) if the response satisfies the following five components:

1. Uses a standard error in the form $\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$ where $\hat{p}$ is between 0 and 1.
2. Shows evidence that $\hat{p} = 0.7$ was correctly used in the standard error.
3. Shows evidence that 0.116 was correctly used as the margin of error in the calculation.
4. Shows evidence that $z^* = 1.96$ was correctly used as the critical value in the calculation.
5. Includes a single, positive whole-number answer.

Partially correct (P) if the response satisfies only three or four of the five components.

Incorrect (I) if the response satisfies at most two of the five components.

Notes:

- Using an equation in the form $n = \frac{z^2 \hat{p}(1-\hat{p})}{\text{MOE}^2}$ satisfies component 1.
- A value of 0.21 in the numerator of the standard error implies that $\hat{p} = 0.7$ was correctly used in the standard error and satisfies component 2.
- An equation such as $0.816 = 0.7 + \text{MOE}$ implies that 0.116 was correctly used for the margin of error and satisfies component 3.
- Statements that suggest a whole-number answer is approximate (such as, “about 60” or “ ≈ 60 ”) satisfy component 5.
- Algebraic work between the set-up and final answer does not need to be shown to satisfy component 5.
- When calculating the values 0.7, 0.116, or 1.96, ignore minor arithmetic errors or transcription errors if they can be identified by the work shown.

2018 SCORING GUIDELINES**Question 2 (continued)**

Part (b) is scored as follows:

Essentially correct (E) if the response satisfies the following three components:

1. Explains why the responses to the survey might differ from the truth about student recycling in this context (for example, the survey was not anonymous, the question was asked by an authority figure).
2. Explains how the responses to the survey might differ from the truth about student recycling (for example, “students might say yes when they actually don’t recycle,” “students lie and say yes,” “students don’t recycle but lie to the teacher”).
3. Describes the effect of the bias on the point estimate (or the proportion, percentage, number of yes responses in the sample) and doesn’t contradict the bias described.

Partially correct (P) if the response satisfies only two of the three components.

Incorrect (I) if the response satisfies at most one of the three components.

Notes:

- To satisfy component 1 the response must provide a reason that is based on a bias created by the teacher asking students in person. For example, a response that addresses the wording of the question, voluntary response, or sampling variability does not satisfy component 1.
- To satisfy component 2 the response needs to explicitly contrast what the students say with what they do.
- Evidence used to address component 3 cannot also be used to address component 2. For example, a response that says “Students might lie, producing an estimate that is too high” addresses the effect of the bias on the point estimate but should not be combined with the statement about students lying to infer that students do not actually recycle. However, a response that says “Students may lie and say yes, producing an estimate that is too high” satisfies both components 2 and 3.
- If the response is clearly about the population proportion and not about the point estimate, component 3 cannot be satisfied.
- Statements such as “the interval will be too high” do not satisfy component 3 because they don’t specifically address the point estimate.

2018 SCORING GUIDELINES**Question 2 (continued)**

Part (c) is scored as follows:

Essentially correct (E) if the response gives an answer of 150 in (c-i) and gives an answer of 0.58 (or equivalent) in (c-ii).

Partially correct (P) if the response gives an answer of 150 in (c-i) and gives an answer of 0.42 (or equivalent) in (c-ii);

OR

if the response does not give an answer of 150 in (c-i) but gives an answer of 0.58 (or equivalent) with supporting work in (c-ii).

Incorrect (I) if the response does not meet the criteria for E or P.

Notes:

- In part (c-i) the answer must be a single number. Responses such as “at least 150” or “147–153” are incorrect. However, responses such as “about 150” or “ ≈ 150 ” are acceptable.
- In part (c-ii) the proportion can be described verbally (e.g., “87 out of 150”).
- In part (c-ii) if the response clearly indicates that 0.58 (or 0.42) is the population proportion, lower the overall score in part (c) by one level (that is, from E to P, or from P to I). Using probability notation such as $P(\text{yes})$ does not clearly indicate a population proportion.
- In part (c-ii) if the response includes a point estimate of 0.58 or 0.42 but uses a confidence interval as the final answer, lower the overall score in part (c) by one level (that is, from E to P, or from P to I).
- If the answer is incorrect in part (c-i) and the answer in part (c-ii) uses numerator = 87 and denominator = 300 – answer to (c-i), the response should be scored P.

2018 SCORING GUIDELINES**Question 2 (continued)****4 Complete Response**

Three parts essentially correct

3 Substantial Response

Two parts essentially correct and one part partially correct

2 Developing Response

Two parts essentially correct and no parts partially correct

OR

One part essentially correct and one or two parts partially correct

OR

Three parts partially correct

1 Minimal Response

One part essentially correct

OR

No parts essentially correct and one or two parts partially correct

2018 SCORING GUIDELINES

Question 3

Intent of Question

The primary goals of this question were to assess a student's ability to (1) compute a probability based on a weighted mixture of two populations; (2) compute a conditional probability; and (3) recognize a binomial random variable and compute the probability associated with it.

Solution

Part (a):

Let L denote left-handed, M denote multiple birth, and S denote single birth.
The probability that a randomly selected child born in the region is left-handed is:

$$P(L) = P(M)P(L | M) + P(S)P(L | S) = (0.035)(0.22) + (0.965)(0.11) = 0.0077 + 0.10615 = 0.11385.$$

Part (b):

From part (a), $P(L) = 0.11385$. Therefore,

$$P(M | L) = \frac{P(L \text{ and } M)}{P(L)} = \frac{(0.035)(0.22)}{0.11385} = \frac{0.0077}{0.11385} \approx 0.0676.$$

Part (c):

Let X represent the number of children who are left-handed in a random sample of 20 children from the region. X has a binomial distribution with $n = 20$ and $p = 0.11385$ (found in part (a)). Using the binomial distribution,

$$\begin{aligned} P(X \geq 3) &= 1 - P(X \leq 2) \\ &= 1 - \binom{20}{0}(0.11385)^0(0.88615)^{20} - \binom{20}{1}(0.11385)^1(0.88615)^{19} - \binom{20}{2}(0.11385)^2(0.88615)^{18} \\ &\approx 1 - 0.598 \approx 0.402. \end{aligned}$$

2018 SCORING GUIDELINES**Question 3 (continued)****Scoring**

Parts (a), (b), and (c) each scored as essentially correct (E), partially correct (P), or incorrect (I).

Part (a) is scored as follows:

Essentially correct (E) if the probability is computed correctly, *AND* work is shown that includes correct numerical values using a formula, end results from a tree diagram, or some other appropriate strategy.

Partially correct (P) if the response provides a reasonable strategy for finding the probability, such as a formula or tree diagram, but uses one or more inappropriate values;

OR

if the response gives the correct probability but not enough work is shown to determine how that probability was found.

Incorrect (I) if the response does not meet the criteria for E or P.

Note: A reasonable strategy needs to include summing the results of two multiplications.

Part (b) is scored as follows:

Essentially correct (E) if the probability is computed correctly, with work shown that includes appropriate numerical values for both the numerator and denominator.

Partially correct (P) if the response includes a numerator and denominator in calculating the conditional probability, with one appropriate term (numerator or denominator) and the other inappropriate.

Incorrect (I) if the response does not meet the criteria for E or P.

Note: Appropriate values include incorrectly calculated values from part (a).

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Question 3 (continued)

Part (c) is scored as follows:

Essentially correct (E) if the response satisfies the following five components:

1. Uses a calculation based on the binomial distribution to find the probability of the number of children in the sample who are left-handed.
2. Specifies appropriate values for n and p .
3. Uses correct endpoint value for the probability.
4. Uses correct direction to calculate the probability of at least three left-handed children.
5. Correctly calculates a binomial probability consistent with the previous work.

Partially correct (P) if the response satisfies component 1 and only two or three of the other four components;
OR

if components 2, 3, 4, and 5 are met, and the response does not explicitly indicate the binomial distribution is used by name or formula.

Incorrect (I) if the response does not meet the criteria for E or P.

Notes:

- “Appropriate” values include incorrectly calculated values from part (a) or a recalculated probability from part (b).
- An unlabeled numerical value in a calculator statement cannot be used to satisfy a component.
- A response which calculates $P(X \leq 3)$ satisfies component 3 but does not satisfy component 4.
- A normal approximation to the binomial is not appropriate because $np = 20 \times 0.11385 = 2.277 < 5$.
A response using the normal approximation can score at most P. To earn a score of P, the response must include all of the following:
 - a correct mean and standard deviation based on the binomial parameters
 - clear indication of boundary and direction with a z-score or diagram
 - the probability computed correctly

Notes for all parts:

- If the resulting probability or part of the calculation of the probability uses a value that is not between 0 and 1, inclusive, the score for that part is lowered by one level (that is, from E to P, or from P to I).
- An arithmetic or transcription error in a response can be ignored if correct work is shown. For example, $0.0077 + 0.10615 = 0.1385$.

2018 SCORING GUIDELINES**Question 3 (continued)****4 Complete Response**

Three parts essentially correct

3 Substantial Response

Two parts essentially correct and one part partially correct

2 Developing Response

Two parts essentially correct and no parts partially correct

OR

One part essentially correct and one or two parts partially correct

OR

Three parts partially correct

1 Minimal Response

One part essentially correct

OR

No parts essentially correct and two parts partially correct

2018 SCORING GUIDELINES**Question 4****Intent of Question**

The primary goals of this question were to assess a student's ability to (1) determine whether a cause-and-effect conclusion can be made based on how a study was conducted and (2) set up, perform, and interpret the results of a hypothesis test, in the context of the problem.

Solution**Part (a):**

Yes, it would be reasonable to conclude that the new procedure causes a reduction in recovery time, for patients similar to those in the study. The patients in the study were randomly assigned to the two procedures, which reduces the chance that confounding variables will affect the results. Therefore the statistically significant reduction in mean recovery time can be attributed to the new procedure being superior to the standard procedure.

Part (b):

Step 1: State a correct pair of hypotheses.

Let μ_S represent the mean recovery time among all patients similar to those in the study if they were to receive the standard treatment.

Let μ_N represent the mean recovery time among all patients similar to those in the study if they were to receive the new treatment.

The hypotheses to be tested are $H_0 : \mu_S = \mu_N$ versus $H_a : \mu_S > \mu_N$.

Step 2: Identify a correct test procedure (by name or by formula) and check appropriate conditions.

The appropriate procedure is a two-sample t -test for a difference between means.

Because this is an experiment, the first condition is that subjects were randomly assigned to one treatment group or the other. In this case the condition is satisfied because we were told that the subjects were randomly assigned to either the standard or new procedure.

The second condition is that the recovery times of the two populations are normally distributed or the sample sizes are sufficiently large to presume that the distribution of the difference in the sample means is approximately normal. In this case the condition is met because the sample sizes of 110 and 100 are both sufficiently large.

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Question 4 (continued)

Step 3: Correct mechanics, including the value of the test statistic, degrees of freedom, and p -value (or rejection region).

$$\text{The test statistic is } t = \frac{\bar{x}_S - \bar{x}_N}{\sqrt{\frac{s_S^2}{n_S} + \frac{s_N^2}{n_N}}} = \frac{217 - 186}{\sqrt{\frac{34^2}{110} + \frac{29^2}{100}}} \approx 7.13.$$

The p -value is the area greater than 7.13 for a t -distribution with $df = 207.18$, which is essentially 0 (8.36×10^{-12}).

Step 4: State a correct conclusion in the context of the problem, using the result of the statistical test.

Because the p -value is very small, we have sufficient evidence to conclude that for patients similar to the ones in the study, those receiving the new procedure would have less recovery time, on average, than those receiving the standard procedure.

Scoring

This question is scored in three sections. Section 1 consists of part (a); section 2 consists of step 1, step 2, and the test statistic in step 3 in part (b); and section 3 consists of the p -value in step 3 and step 4 in part (b). Sections 1, 2, and 3 are each scored essentially correct (E), partially correct (P), or incorrect (I).

Section 1 is scored as follows:

Essentially correct (E) if the response satisfies the following three components:

1. Correctly states that it is reasonable to make a causal conclusion.
2. Justifies the causal conclusion based on random assignment of patients to procedures (or procedures to patients);
OR
justifies the causal conclusion by stating that a randomized experiment was conducted.
3. Includes the context of the situation.

Partially correct (P) if the response satisfies component 1 AND provides WEAK justification of the causal conclusion by stating that there was random assignment or a randomized experiment was conducted, but with no context;

OR

by stating that an experiment was conducted or there was assignment (without the word “randomized”) AND the response includes context of the situation;

OR

by stating that the study design reduces the chance of confounding variables or balances the effects of uncontrolled variables across both groups in context without explicitly referring to the random assignment.

Incorrect (I) if the response does not meet the criteria for E or P.

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Question 4 (continued)

Notes:

- If the response states that it is *not* reasonable to make a causal conclusion because the result could have been due to random chance *AND* explains that there is evidence for a causal conclusion based on random assignment of patients to procedures or by stating that a randomized experiment was conducted, then the response is scored E.
- If the response discusses aspects of an experiment other than random assignment (such as, control, replication, or large samples), then those aspects are considered extraneous and the response can be scored E unless those aspects are incorrect for this study (such as, blocking is a requirement, or the study used blocking, or the study used a placebo) in which case the score should be lowered one level (that is, from E to P, or from P to I).
- If the response correctly states in context that it is reasonable to make a causal conclusion but includes incorrect or contradictory justification (such as, random selection of patients), then the response is scored I.

Section 2 is scored as follows:

Essentially correct (E) if the response satisfies the following four components:

1. Parameters are defined correctly.
2. Hypotheses imply equality in the null and correct direction in the alternative.
3. Correct test is identified by name or formula.
4. Correct test statistic for a difference in means is calculated.

Partially correct (P) if the response satisfies only two or three of the four components.

Incorrect (I) if the response satisfies at most one of the four components.

Notes:

- If standard symbols are used for the parameters with appropriate group labels (such as, μ_S, μ_N), component 1 is satisfied.
- If the correct test is identified, but the response states an incorrect formula or uses incorrect notation in the formula, component 3 is not satisfied.
- A pooled two-sample t -test is acceptable for component 3, but the student must also state and comment on the plausibility of the equal population variances assumption.
- If the response identifies a z -test for equal means as the correct test identification, component 3 is not satisfied but component 4 could be satisfied.

Confidence Interval approach:

- If a single two-sample t -interval for the difference in means is used, components 3 and 4 can be satisfied. Component 3 is satisfied if the t -interval is correctly identified by name or formula. Component 4 is satisfied if the correct interval is calculated. If an alpha level is stated, then an appropriate adjustment to the confidence level must be made because the appropriate test is one-sided.
- If two one-sample t -intervals are used, while not a recommended approach, component 3 is not satisfied but component 4 could be satisfied. Component 4 is satisfied if both intervals are calculated correctly.

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Question 4 (continued)

Section 3 is scored as follows:

Essentially correct (E) if the response satisfies the following three components:

1. Makes reference to an approximately correct p -value that is consistent with the test statistic and alternative hypothesis for a difference in means.
2. Correctly justifies the conclusion based on the size of the p -value or the test statistic.
3. Correctly states the conclusion in context.

Partially correct (P) if the response satisfies only two of the three components.

Incorrect (I) if the response does not meet the criteria for E or P or includes a justification not based on the inferential results.

Notes:

Component 1:

- Is satisfied if the response makes reference to a large test statistic without referring to a p -value.

Component 2:

- No alpha level is needed to provide justification of the conclusion based on the size of the p -value.
- Is satisfied if the response states the p -value without reference to size, but it is contiguous to the conclusion and clearly indicates a continuous train of thought.
- A correct interpretation of the p -value with a complete explanation that obtaining a test statistic at least this extreme is unlikely due to chance alone is considered justification based on the size of the p -value.
- If an incorrect interpretation of the p -value is given, the score is lowered one level (that is, from E to P, or from P to I).
- A decision about the null hypothesis (reject H_0 or fail to reject H_0) is not required, but if an incorrect decision is stated based on the given p -value then component 2 is not satisfied.
- If a rejection region approach is used, a reasonable critical value replaces the p -value.

Component 3:

- A correct conclusion must be related to the alternative hypothesis in order to satisfy component 3.
- The following responses do not satisfy component 3:
 - States or implies that the null hypothesis is *accepted*
 - States or implies that the alternative hypothesis has been *proven*
 - States the conclusion in past tense (unless the response did not satisfy a component of section 2 for the use of past tense)

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Question 4 (continued)

Confidence Interval approach:

- If a single two-sample t -interval for the difference in means is used:
 - Component 1 is satisfied if the response indicates that zero is either included or not included in the calculated interval.
 - Component 2 is satisfied if the response indicates that the bounds are either both above or both below zero (consistent with alternative hypothesis) and uses that as justification for the conclusion.
 - Component 3 is satisfied if the conclusion is stated in context.
- If two one-sample t -intervals are used (which is not recommended) the response is scored at most P if all three components are satisfied, otherwise scored I:
 - Component 1 is satisfied if the response states that the intervals do not overlap.
 - Component 2 is satisfied if the conclusion indicates that the confidence interval for the new procedure lies below the confidence interval for the standard procedure.
 - Component 3 is satisfied if the conclusion is stated in context.

Note: If the three sections of the response are scored as E, to earn a score of 4 as a complete response, both conditions in step 2 must be correctly stated and justified. Additional condition(s) inappropriate for a two-sample t -test must not be stated. Otherwise, the response earns a score of 3 a substantial response.

2018 SCORING GUIDELINES

Question 4 (continued)

4 Complete Response

Three sections essentially correct with conditions for inference

3 Substantial Response

Three sections essentially correct without conditions for inference

OR

Two sections essentially correct and one section partially correct

2 Developing Response

Two sections essentially correct and no sections partially correct

OR

One section essentially correct and one or two sections partially correct

OR

Three sections partially correct

1 Minimal Response

One section essentially correct

OR

No sections essentially correct and one or two sections partially correct

2018 SCORING GUIDELINES

Question 5

Intent of Question

The primary goals of this question were to assess a student's ability to (1) determine which of two histograms represents data with a larger median; (2) calculate the mean of a combined data set when the separate means and sample sizes are known; and (3) calculate the probability that an individual randomly chosen from a finite population will have a value within one standard deviation of the mean, when provided with values for the mean, standard deviation, and all members of the population.

Solution**Part (a):**

The median teaching year for High School A is any value with 100 data values at or below it and 100 data values at or above it. The median teaching year for High School B is the 111th value in the ordered list of values. For High School A the median is in the interval that starts at 7 and ends just before 10, because there are only 94 data values below 7 and 106 data values of at least 7. Therefore the median cannot be less than 7. For High School B the median is in the interval that starts at 4 and ends just before 7 because there are more than half (113) of the data values less than 7. Therefore the median must be less than 7. So High School A must be the one with a median of 7, and High School B must be the one with a median of 6.

Another way to determine which school has the median of 7 is to notice that the distribution for High School B is highly skewed to the right, whereas the distribution for High School A is bimodal with a few possible outliers on the right. A distribution that is highly right-skewed is likely to have a substantially larger mean than median. The mean of both distributions is given as 8.2 years, so it makes sense that the highly right-skewed distribution (High School B) is the one with the bigger gap between the mean and median and, therefore, the one with the lower median of 6.

Part (b):

The mean for the original 200 teachers was given as 8.2 years, and the mean for the additional 18 teachers is 2.5 years. Therefore the mean for the combined data set is:

$$\frac{(200)(8.2) + (18)(2.5)}{200 + 18} = \frac{1,640 + 45}{218} \approx 7.73 \text{ years.}$$

Part (c):

The interval mean plus or minus 1 standard deviation on either side of the mean is 8.2 ± 7.2 , or from 1.0 year to 15.4 years. Because teaching year is recorded as an integer, the interval includes teaching years 1 to 15. The number of teachers in that interval can be found by adding the heights of the five bars in the histogram for the intervals from 1 to 16, which includes $79 + 34 + 28 + 29 + 19 = 189$. Therefore the probability is $\frac{189}{221} \approx 0.8552$.

2018 SCORING GUIDELINES**Question 5 (continued)****Scoring**

Parts (a), (b), and (c) are scored as essentially correct (E), partially correct (P), or incorrect (I).

Part (a) is scored as follows:

Essentially correct (E) if the response satisfies the following three components:

1. States that the median is 6 for High School B and the median is 7 for High School A.
2. Provides a reasonable explanation of how the decision was made.
3. Provides the definition of the median or explicitly applies the definition of a median as a criterion in reaching their decision.

OR

Essentially correct (E) if the response satisfies the following three components:

1. States that the median is 6 for High School B and the median is 7 for High School A.
2. States that High School B shows a skewed distribution (or High School A shows a less skewed distribution).
3. Provides a reasonable explanation of how the more skewed distribution (High School B) would be the one with a larger separation between the mean and median.

Partially correct (P) if the response satisfies the first component and only one of the other two components required for E.

Incorrect (I) if the response does not meet the criteria for E or P.

Note: An incorrect statistical statement in the response will result in E being lowered to P, but not P being lowered to I. For example,

- If either distribution is described as left skewed, normal, or approximately normal;
- If the discussion would indicate a median different than 7 for High School A or a median different than 6 for High School B.

Part (b) is scored as follows:

Essentially correct (E) if the response satisfies the following two components:

1. The correct answer that the mean is 7.73.
2. Enough work to show that the answer was obtained as a weighted average of the two individual means.

Partially correct (P) if the response satisfies only one of the two components.

Incorrect (I) if the response does not satisfy the requirements for E or P.

2018 SCORING GUIDELINES**Question 5 (continued)**

Part (c) is scored as follows:

Essentially correct (E) if the response satisfies the following three components:

1. Calculates that the appropriate interval is 1 to 15.4 or 1 to 15 teaching years.
2. Correctly sums the counts of data values in the numerator based on the intervals provided.
3. Computes the probability using 221 as the denominator.

Partially correct (P) if the response satisfies only two of the three components;

OR

if the response reports the correct probability (0.8552) without supporting work.

Incorrect (I) if the response satisfies at most one of the three components.

Notes:

- If the response attempts to use the Empirical Rule or normal distribution to provide the desired probability, the response is scored I.
- If an incorrect count is shown in component 2, for instance by including the interval from 16 to 19, then component 3 is satisfied if that incorrect count is divided by 221 to find the reported probability.
- It is acceptable if the count is slightly off because of difficulty reading the exact heights of the bars in the histogram.
- If only one of component 2 or component 3 is missing, but the correct probability (0.8552) is reported, the response can be scored E.
- If the response recognizes that all values in the histogram bins up to 16 fall within one standard deviation of the mean and reports the interval as 1 to 16, component 1 is satisfied.

2018 SCORING GUIDELINES**Question 5 (continued)****4 Complete Response**

Three parts essentially correct

3 Substantial Response

Two parts essentially correct and one part partially correct

OR

Part (a) essentially correct and two parts partially correct

2 Developing Response

Two parts essentially correct and no parts partially correct

OR

Part (b) or part (c) essentially correct and one or two parts partially correct

OR

Three parts partially correct

1 Minimal Response

One part essentially correct

OR

No parts essentially correct and one or two parts partially correct

2018 SCORING GUIDELINES

Question 6

Intent of Question

The primary goals of this question were to assess a student's ability to (1) describe what constitutes a Type II error for a specific hypothesis test; (2) specify a rejection region in terms of values of the sample mean; (3) compute the power of a test for a specific value in the alternative hypothesis; (4) recognize the definition of power; and (5) understand the impact of increasing the sample size on the power of a test.

Solution**Part (a):**

A Type II error occurs when the alternative hypothesis is true, but the null hypothesis is not rejected. In this situation a Type II error would happen if the mean systolic blood pressure of the population of employees is greater than 122 mmHg, but the null hypothesis that it is 122 mmHg is not rejected. In other words a Type II error would happen if the mean blood pressure for the population of employees is higher than the national average, but the test does not conclude that it is higher.

Part (b):

The test is one-sided and the standard deviation is known, so the null hypothesis will be rejected if the test statistic $z = \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}} > 1.645$. With $\mu = 122$, $\sigma = 15$, and $n = 100$, we get $\frac{\bar{x} - 122}{1.5} > 1.645$. Therefore, $\bar{x} > 124.4675$.

Part (c):

If the actual population mean is 125, with $\sigma = 15$ and $n = 100$, then the sampling distribution of $\bar{x}$ is approximately normal with mean of 125 and standard deviation $\frac{15}{\sqrt{100}} = 1.5$. Therefore,

$$P(\bar{x} > 124.4675) = P\left(\frac{\bar{x} - \mu_{\bar{x}}}{\sigma_{\bar{x}}} > \frac{124.4675 - 125}{1.5}\right) = P(z > -0.355) = 0.64.$$

Part (d):

The probability found in part (c) is called the power of the test.

Part (e):

If the sample size is increased from 100 to something larger, the probability of rejecting the null hypothesis when the population mean is 125 will be higher than it is for a sample of size 100. Intuitively, more data provide a higher probability of a correct conclusion. The technical explanation is that the rejection region will still be $z > 1.645$, but the sampling distributions of the sample mean will have a smaller standard deviation; therefore, the minimum value of $\bar{x}$ for which we would reject the null hypothesis would be lower and, in return, the probability the null hypothesis is rejected will increase.

2018 SCORING GUIDELINES

Question 6 (continued)

Scoring

This question is scored in three sections. Section 1 consists of part (a) and part (b), section 2 consists of part (c), and section 3 consists of part (d) and part (e). Sections 1, 2, and 3 are scored as essentially correct (E), partially correct (P), or incorrect (I).

Section 1 is scored as follows:

Essentially correct (E) if the response satisfies the following four components:

1. Part (a) includes in the description of a Type II error the fact that the alternative hypothesis is true, either generically or in context of the situation.
2. Part (a) includes in the description of a Type II error the fact that the null hypothesis is not rejected, either generically or in context of the situation.
3. Part (b) includes a correct z-score for the upper 5 percent tail and indicates the correct direction for the rejection region.
4. Part (b) includes $\mu_{\bar{x}} = 122$, $\sigma_{\bar{x}} = 1.5$, and the resulting $\bar{x}$ value.

Partially correct (P) if the response satisfies only two or three of the four components.

Incorrect (I) if the response does not meet the criteria for E or P.

Notes:

- If the response in part (a) does not include context, the number of components satisfied is reduced by one (that is, from four to three, or from three to two, and so on). Context includes a reference to units, to blood pressure, to employees, etc.
- If a response in part (a) is clearly referring to an individual's blood pressure as opposed to the mean blood pressure of all employees, neither components 1 nor 2 are satisfied.

Section 2 is scored as follows:

Essentially correct (E) if the response satisfies the following three components:

1. Recognizes that the null hypothesis will be rejected when $\bar{x} \geq 124.4675$, as found in part (b).
2. Provides the correct sampling distribution for the sample mean when the true mean is 125, including correct values for the mean and standard deviation, either explicitly or by plugging them into the test statistic formula.
3. Provides evidence of using the normal curve and finds the correct probability value.

Partially correct (P) if the response satisfies only two of the three components.

Incorrect (I) if the response does not meet the criteria for E or P.

Notes:

- Components 1 and 3 can still be satisfied if errors made in finding the rejection region in part (b) are carried into part (c).
- A calculator statement that does not include labels for input values does not satisfy component 2 but may still satisfy components 1 and 3.

2018 SCORING GUIDELINES

Question 6 (continued)

Section 3 is scored as follows:

Essentially correct (E) if the response satisfies the following four components:

1. Part (d) specifies power as the name of the probability.
2. Part (e) correctly states that the probability would be greater.
3. Part (e) correctly implies that the standard deviation of the sampling distribution decreases, either explicitly or by substituting values into a formula.
4. Part (e) indicates the minimum value of $\bar{x}$ for which the null hypothesis is rejected decreases, either explicitly or by substituting values into a formula.

Note: Component 4 can still be satisfied if a response indicates that a maximum value of $\bar{x}$ for which the null hypothesis is rejected increases if this direction is consistent with answers in parts (b) and (c).

Partially correct (P) if the response satisfies only two or three of the four components.

Incorrect (I) if the response does not meet the criteria for E or P.

2018 SCORING GUIDELINES

Question 6 (continued)

4 Complete Response

Three sections essentially correct

3 Substantial Response

Two sections essentially correct and one section partially correct

2 Developing Response

Two sections essentially correct and no sections partially correct

OR

One section essentially correct and one or two sections partially correct

OR

Three sections partially correct

1 Minimal Response

One section essentially correct

OR

No sections essentially correct and two sections partially correct

OR

Sections 1 and 2 incorrect, and section 3 partially correct with exactly three of the four components satisfied

OR

Section 1 partially correct with exactly three of the four components satisfied, and sections 2 and 3 incorrect

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SCORING GUIDELINES**Question****Intent of Question**

The primary goals of this question were to assess a student's ability to (1) describe the distribution of a quantitative variable based on a histogram and (2) determine the effect of changing one data value on mean and the median.

Solution**Part (a):**

The distribution of Robin's tip amounts is skewed to the right. There is a gap between the largest amount (in the \$20 to \$22.50 interval) and the second largest tip amount (in the \$12.50 to \$15 interval), and the largest tip amount appears to be an outlier. The median tip amount is between \$2.50 and \$5.00. Robin's tip amounts vary from a minimum of between \$0 and \$2.50 to a maximum of between \$20.00 and \$22.50. About 78 percent of the tip amounts are between \$0 and \$5.

Part (b):

◻ mean: If the \$8 tip had been \$18, the mean would increase by \$10 divided by 60, or $\$ \frac{1}{6}$, or about 17 cents.

◻ median: If the \$8 tip had been \$18, the median would not change because the current median is between \$2.50 and \$5.00, and both \$8 and \$18 are greater than that.

Scoring

Parts (a) and (b) are scored as essentially correct (E), partially correct (P), or incorrect (I).

Part (a) is scored as follows:

Essentially correct (E) if the response includes reasonable comments on the following five components:

1. Shape (skewed right)
2. Outlier (at least one) *OR* gap (one tip amount greater than \$20, next highest at most \$15)
3. Center between \$2.50 and \$5.00 (median) or between \$2.62 and \$5.13 (mean)
4. Variability, by noting that the tip amounts vary from about \$0 to at most \$22.50, or that a majority of tip amounts are between \$0 and a value greater than or equal to \$5, or by providing a correct numerical approximation of a measure of variability
5. Context (tip amounts)

Partially correct (P) if the response includes only three or four of the five components.

Incorrect (I) if the response includes at most two of the five components.

SCORING GUIDELINES**Question 1 (continued)**

Part (b) is scored as follows:

Essentially correct (E) if the response includes the following four components:

1. Comments that the mean will increase
2. Correctly justifies why the mean will increase
3. Comments that the median will not change
4. Correctly justifies why the median will not change

Partially correct (P) if the response includes only two or three of the four components.

Incorrect (I) if the response includes at most one of the four components.

4 Complete Response

Both parts essentially correct

3 Substantial Response

One part essentially correct and one part partially correct

2 Developing Response

One part essentially correct and one part incorrect

OR

Both parts partially correct

1 Minimal Response

One part partially correct and one part incorrect

SCORING GUIDELINES

Question

Intent of Question

The primary goals of this question were to assess a student's ability to (1) identify, set up, perform, and interpret the results of an appropriate hypothesis test to address a particular question and (2) assess the effectiveness of treatments in a controlled experiment.

Solution

Part (a):

Step 1: States a correct pair of hypotheses.

H_0 : The proportion of children who would choose each snack is the same regardless of which type of ad is viewed.

H_a : The proportion of children who would choose each snack differs based on which type of ad is viewed.

Step 2: Identifies a correct test procedure (by name or formula) and checks appropriate conditions.

The appropriate procedure is a chi-square test of homogeneity.

The conditions for this test are satisfied because (1) the question states that the children were randomly assigned to groups, and (2) expected counts for the six cells of the table are all at least 5, as seen in the following table that lists expected counts beside observed counts.

Group	Choco-Zuties	Apple-Zuties	Total
A	21 (18.67)	4 (6.33)	25
B	13 (18.67)	12 (6.33)	25
C	22 (18.67)	3 (6.33)	25
Total	56	19	75

Step 3: Calculates the appropriate test statistic and p -value.

The test statistic is calculated as $\chi^2 = \sum \frac{(O - E)^2}{E}$, which is

$$\begin{aligned}\chi^2 &\approx \\ &0.292 + 0.860 + \\ &1.720 + 5.070 + \\ &0.595 + 1.754 \approx 10.291.\end{aligned}$$

The p -value is $P(\chi^2_{df2} \geq 10.291) \approx 0.006$.

Step 4: States a correct conclusion in the context of the study, using the result of the statistical test

Because the p -value is very small (for instance, much smaller than $\alpha = 0.05$) we reject the null hypothesis at the .05 level (and at the .01 level). The data provide convincing statistical evidence that the proportions who would choose each snack differ based on which ad is viewed.

SCORING GUIDELINES

Question 2 (continued)

Part (b):

When neither ad was viewed, $\frac{22}{25}$ or 88 percent of the children chose Choco-Zuties whereas only _____ percent chose Apple-Zuties.

When the Choco-Zuties ad was viewed, 84 percent of the children chose Choco-Zuties, which was very similar to the percentage that chose them without viewing any ad. So watching the Choco-Zuties ad did not affect the snack choice very much.

When the Apple-Zuties ad was viewed, only $\frac{13}{25}$ or 52 percent of the children chose Choco-Zuties, and _____ percent chose Apple-Zuties. Watching the Apple-Zuties ad seemed to increase the proportion of children choosing Apple-Zuties.

Scoring

This question is scored in four sections. Section 1 consists of steps 1 and 2 in part (a); section 2 consists of step 3 in part (a); section 3 consists of step 4 in part (a); and section 4 consists of part (b). Sections 1, 2, 3 and 4 are scored as essentially correct (E), partially correct (P), or incorrect (I).

Section 1 is scored as follows:

Essentially correct (E) if the response includes the following three components:

1. Both hypotheses are stated correctly with at least one in context.
2. Identifies the correct test procedure (by name or by formula).
3. The technical conditions are checked (all expected counts are greater than or equal to 5).

Partially correct (P) if the response includes only two of the three components.

Incorrect (I) if the response correctly includes at most one of the three components.

Notes for component 1:

- It is acceptable if hypotheses are stated in terms of association/independence instead of proportions.
- Context must be included in at least one of the hypotheses, but is not required to be in both.
- If the hypotheses contain language that suggests that the response refers to the sample data, the component is not satisfied.

Note for component 2: It is acceptable if the test is identified as a test of independence instead of a test of homogeneity.

Notes for component 3:

- The random assignment condition was stated so need not be explicitly mentioned.
- Stating the expected count condition is met is not sufficient for component 3. The condition must be checked by reporting the expected counts and either:
 - noting that all are greater than or equal to 5;

SCORING GUIDELINES

Question 2 (continued)

- noting that all are greater than or equal to 1 and at most 20 percent are less than 5.
- Noting that the smallest expected count is 6.33 and that it is greater than or equal to 5 is sufficient to satisfy this component.
- Component 3 is not satisfied if the expected counts are reported as integers.
- This component is not satisfied if the response includes any of the following inappropriate conditions:
 - The response implies that a random sample was taken, e.g., “SRS – check.”
 - The response refers to independence of groups or independence of ads as a required condition.
 - The response indicates that a sample size greater than 30 ensures normality or the response implies normality as a condition.
- A response stating that children are independent can be ignored in the scoring of this component.

Section 2 is scored as follows:

Essentially correct (E) if the response includes the following three components:

1. Correct chi-square test-statistic
2. Correct degrees of freedom
3. Correct p -value

Partially correct (P) if the response includes only two of the three components.

Incorrect (I) if the response correctly includes at most one of the three components.

Notes:

- If the response makes an error in one calculation, future calculations are considered correct if they follow correctly from the initial miscalculation.
- A chi-square critical value approach is acceptable: The critical value for $\alpha = 0.05$ is $\chi^2 = 5.99$.

Section 3 is scored as follows:

Essentially correct (E) if the response provides a correct conclusion about the alternative hypothesis context, *AND* provides justification based on linkage between the p -value and conclusion.

Partially correct (P) if the response provides a correct conclusion in context, but without justification based on linkage to the p -value;

OR

if the response provides a correct conclusion, with linkage to the p -value, but not in context;

OR

if the response provides a correct decision stated in terms of the null hypothesis in context, with linkage to the p -value, but no conclusion is made about the alternative hypothesis.

Incorrect (I) if the response does not meet the criteria for E or P.

SCORING GUIDELINES

Question 2 (continued)

Notes:

- If the conclusion is consistent with the p -value from section 2, and also in context with justification based on linkage to the p -value, section 3 is scored as E.
- If no alpha level is given, the solution must be explicit about the linkage by giving a correct interpretation of the p -value or by explaining how the conclusion follows from the p -value such as saying: “Because the p -value is small, we reject the null hypothesis.” or “Because the p -value is large, we do not reject the null hypothesis.”
- If a conclusion contains language that suggests that the response refers to the sample data, conclusion component is not correct, unless the same error occurred in the statement of hypotheses in section 1.

Section 4 is scored as follows:

Essentially correct (E) if the response, in context, concludes that ad type A had little effect and ad type B had an effect, both supported by the observed proportions (or counts) from the table.

Partially correct (P) if the response correctly concludes that ad type A had little effect, and ad type B had an effect but does not provide correct numerical justification;

OR

if the response compares all of the proportions (or counts) as required, but without correctly concluding ad effectiveness;

OR

if the response correctly describes the effect of one of the ad types A or B (in context, with correct numerical justification) but not the other;

OR

if the response compares all of the proportions (or counts) as required, but not in context.

Incorrect (I) if the response does not meet the criteria for E or P.

Each essentially correct (E) section counts as 1 point, and a partially correct (P) section counts as $\frac{1}{2}$ point.

- | | |
|----------|-----------------------------|
| 4 | Complete Response |
| 3 | Substantial Response |
| 2 | Developing Response |
| 1 | Minimal Response |

If a response is between two scores (for example, $2\frac{1}{2}$ points), use a holistic approach to decide whether score up or down, depending on the strength of the response and communication.

SCORING GUIDELINES

Question

Intent of Question

The primary goals of this question were to assess a student's ability to (1) identify explanatory and response variables from the description of a research study; (2) indicate and justify whether a study is observational or experimental; and (3) explain what confounding means in the context of a particular study with a specific confounding variable.

Solution

Part (a):

The explanatory variable is the person's degree of cigarette smoking. The response variable is whether the person develops Alzheimer's disease during the course of the study.

Part (b):

This is an observational study because the people in the study were not assigned to a certain degree of cigarette smoking. Rather, the degree of cigarette smoking for each person was passively observed and recorded, not manipulated by the researchers.

Part (c):

Confounding variable is one that is related to the explanatory variable and possibly influences response variable. In this case it seems plausible that people who exercise more regularly might be more health conscious, therefore, less likely to smoke cigarettes than people who do not exercise regularly. Similarly, it's possible that people who exercise more regularly are less likely to develop Alzheimer's disease than people who do not exercise regularly. If both of these relationships turn out to be true, then smoking cigarettes would be associated with developing Alzheimer's disease due to the association of both variables with exercise, even if there were no cause-and-effect relationship between smoking cigarettes and developing Alzheimer's disease.

Scoring

Parts (a), (b), and (c) are scored as essentially correct (E), partially correct (P), or incorrect (I).

Part (a) is scored as follows:

Essentially correct (E) if both variables are described correctly. A correct description includes some degree of status of the variables, such as smoking versus not smoking and developing Alzheimer's versus not developing Alzheimer's.

Partially correct (P) if one variable is described correctly and one is not described correctly;

OR

neither variable is described correctly but smoking is mentioned as the explanatory variable, and Alzheimer's is mentioned as the response variable;

OR

the explanatory and response variables are correctly described but are interchanged.

Incorrect if the response does not meet the criteria for E or P.

SCORING GUIDELINES

Question 3 (continued)

Note: Describing the variables only as smoking and Alzheimer's without a reference to levels/status not sufficient for E. But making the connection between smoking and explanatory and between Alzheimer's and response is sufficient for P.

Part (b) is scored as follows:

Essentially correct (E) if the response selects the correct type of study (observational) and provides the correct explanation that smoking status was not assigned *OR* that smoking status was only observed.

Partially correct (P) if the response selects the correct type of study (observational) and provides correct explanation but does not refer to smoking status as the variable that would have been assigned had it been an experiment; for instance, by simply stating that treatment was not assigned;

OR

if the response selects the correct type of study (observational) and provides an explanation that says that smoking is observed (or that smoking status and Alzheimer's are observed) without a modifier for observed (such as, only, just, merely, simply) *AND* without indicating that treatments were not assigned.

Incorrect (I) if the response does not meet the criteria for E or P.

Notes:

- A response that states the explanatory variable was not assigned without naming smoking status is sufficient for E if the explanatory variable is correctly defined in part (a).
- A response that states that smoking status is *ONLY* observed (or that smoking status and Alzheimer's are *ONLY* observed) is sufficient for E.
- A response that says that Alzheimer's is observed without mentioning smoking status is scored I.
- A response that provides an incorrect statistical explanation (such as, "the study is observational because an experiment must have a control group") lowers the score in part (b) by one level (from E to P or from P to I).
- If Alzheimer's is given as the explanatory variable and smoking is given as the response variable in part (a), then part (b) should be scored accordingly with the two variables interchanged.
- Any phrase that refers to the "effect of smoking on Alzheimer's" or "the association between smoking and Alzheimer's" (rather than smoking status) should be ignored.

Part (c) is scored as follows:

Essentially correct (E) if the response includes the following two components:

1. Provides a reasonable explanation that exercise status is related to smoking status.
2. States that exercise status might influence whether the person develops Alzheimer's disease.

Partially correct (P) if the response describes only one of the two components;

OR

if the response only describes how smoking and exercise jointly influence whether the person develops Alzheimer's;

SCORING GUIDELINES**Question 3 (continued)**

if the response mentions both of the associations in each component, but also includes an incorrect statistical statement. For instance, part (a) is correct, but in part (c), the explanatory and response variables are interchanged.

Incorrect (I) if the response does not meet the criteria for E or P.

4 Complete Response

Three parts essentially correct

3 Substantial Response

Two parts essentially correct and one part partially correct

2 Developing Response

Two parts essentially correct and no parts partially correct

OR

One part essentially correct and one or two parts partially correct

OR

Three parts partially correct

1 Minimal Response

One part essentially correct

OR

No parts essentially correct and two parts partially correct

SCORING GUIDELINES

Question 4

Intent of Question

The primary goals of this question were to assess a student's ability to (1) calculate a probability using basic probability rules or the geometric distribution; (2) recognize that a probability calculation for independent events does not depend on the previous outcomes of those events; and (3) assess whether a claim about the probability of a single event is reasonable based on a calculated probability of a series of those events.

Solution

Part (a):

If the failure rate for the super igniters is 15 percent, then the probability that each igniter fails is 0.15, and the probability that it does not fail is 0.85. Therefore the probability that the first 30 igniters tested do not fail is $(0.85)^{30} \approx 0.0076$. The solution can also be written as $(1 - 0.15)^{30} \approx 0.0076$.

Part (b):

Given that there are no failures in the first 30 trials, the probability that the first failure occurs on the trial is 0.15, and the probability that it does not occur on the 31st but occurs on the 32nd trial $(0.85)(0.15) = 0.1275$. Therefore the probability that the first failure occurs on the 31st or 32nd super igniter tested is $0.15 + 0.1275 = 0.2775$.

Note that this is equivalent to asking for the probability that the first failure occurs on the first or second trial, which is $0.15 + (0.85)(0.15) = 0.2775$.

Part (c):

The result of the probability calculation in part (a) provides a reason to believe that the failure rate of the super igniters is less than 15 percent. The calculated probability of 0.0076 shows that there is less than a 1 percent chance that 30 or more igniters in a row would not fail if the failure rate was 15 percent. This probability is smaller than conventional significance levels such as $\alpha = 0.05$ or $\alpha = 0.01$, and thus is small enough to make it reasonable to believe that the failure rate of the super igniters is less than 15 percent.

Scoring

Parts (a), (b), and (c) are scored as essentially correct (E), partially correct (P), or incorrect (I).

Part (a) is scored as follows:

Essentially correct (E) if the response gives the correct probability *AND* correct justification.

Partially correct (P) if the response correctly notes that the answer is the probability that there will be 30 successes in 30 attempts, but does not carry out a correct probability calculation;

OR

SCORING GUIDELINES

Question 4 (continued)

if the response defines the random variable X as the trial with the first failure, identifies X as having a geometric distribution with $p = 0.15$, and writes the desired probability as $P(X > 30)$, but does not carry out a correct probability calculation;

OR

if the response defines the random variable X as the number of failures in the first 30 attempts, identifies X as a binomial random variable with $p = 0.15$ and $n = 30$, and writes the desired probability as $P(X = 0)$, but does not carry out a correct probability calculation;

OR

if the response gives the correct probability but, in specifying a geometric or binomial distribution, has an incorrect or incomplete definition of parameters or value(s) of the random variable.

Incorrect (I) if the response does not meet the criteria for E or P.

Note: Justification can be given using the multiplication rule; OR by defining X to be the trial with the first failure, recognizing that X has a geometric distribution, and using that information to find $P(X > 30)$; OR by defining X to be the number of failures in the first 30 attempts, and then finding $P(X = 0)$ using either probability rules or the binomial distribution with $n = 30$ and $p = 0.15$.

Part (b) is scored as follows:

Essentially correct (E) if the response gives the correct probability AND correct justification.

Partially correct (P) if the response makes a reasonable attempt to calculate a geometric, binomial, or conditional probability, but does not successfully carry out the calculation;

OR

if the response gives the correct probability but, in specifying a geometric or binomial distribution, has an incorrect or incomplete definition of parameters or value(s) of the random variable.

Incorrect (I) if the response finds an incorrect probability resulting from an unreasonable attempt to calculate a geometric, binomial, or conditional probability or otherwise does not meet the criteria for E or P.

Note: Similar to part (a) justification can be given using probability rules; OR by stating that X is geometric where X is the trial with the first failure, then finding $P(X = 1 \text{ or } X = 2)$; OR by stating that s is the number of failures in two trials and finding $1 - P(X = 0)$ or $P(X = 1 \text{ or } X = 2)$ using the binomial distribution.

Part (c) is scored as follows:

Essentially correct (E) if the response states that it is reasonable to believe that the failure rate is less than 15 percent AND bases this decision on the fact that the probability of 30 consecutive successful launches with a failure rate of 15 percent (that is, answer from part (a)) is small AND does so in the context of the situation.

Partially correct (P) if the response otherwise satisfies the criteria for an (E) but does so without any context;

SCORING GUIDELINES

Question 4 (continued)

if the response states a significance level and makes a decision in a context that is appropriate to the given probability in part (a) and the stated significance level but does not explicitly compare the probability and the significance level (no linkage).

Incorrect (I) if the response does not explicitly make a decision about whether it is reasonable to conclude that the failure rate is less than 15 percent (For example: “As seen in Part (a), if the failure rate is 15 percent then the probability of 30 successful launches in a row is very small.”);

OR

if the response otherwise does not meet the criteria for E or P.

Notes:

- Justification based on the probability can come by stating a significance level and noting that the probability is smaller than the significance level *OR* by simply stating that the probability of 0.0076 is small *OR* by referring to the expected number of failures (4.5) as being very unlikely because zero failures is more than two standard deviations below 4.5.
- If the response bases the decision on the expected number of failures (4.5) for $n = 30$ and $p = 0.15$ without referencing why zero failures would be considered to be too far below 4.5 to give reason to doubt the stated 15 percent failure rate, the response is scored P.
- If the calculation in part (a) is incorrect, the answer in part (c) needs to be consistent with the answer in part (a), unless the value is recalculated in part (c).

4 Complete Response

Three parts essentially correct

3 Substantial Response

Two parts essentially correct and one part partially correct

2 Developing Response

Two parts essentially correct and no parts partially correct

OR

One part essentially correct and one or two parts partially correct

OR

Three parts partially correct

1 Minimal Response

One part essentially correct

OR

No parts essentially correct and two parts partially correct

SCORING GUIDELINES

Question 5

Intent of Question

The primary goals of this question were to assess a student's ability to (1) construct and interpret a confidence interval for a population proportion; (2) explain why one of the conditions for inference necessary; and (3) explain why a suggested procedure for constructing a confidence interval is incorrect.

Solution

Part (a):

The appropriate procedure is a one-sample z-interval for a population proportion. The problem stated the conditions for inference have been met, so they do not need to be checked. A 95 percent

confidence interval for the population proportion is given as $\hat{p} \pm z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$, which is

$$0.37 \pm 1.96 \sqrt{\frac{(0.37)(0.63)}{1,048}} \approx 0.37 \pm 0.03 = (0.34, 0.40).$$

We are 95 percent confident that the population proportion of all adults in the U.S. who would have chosen the economy statement is between 0.34 and 0.40.

Part (b):

The condition is necessary because the formula for the confidence interval relies on the fact that the sampling distribution can be approximated by a normal distribution which then results in the sampling distribution of $\hat{p}$ being approximately normal. The approximation does not work well unless both $n\hat{p}$ and $n(1-\hat{p})$ are at least 10.

Part (c):

The suggested procedure is not appropriate because one of the requirements for using a two-sample interval for a difference between proportions is that the two proportions are based on two independent samples. In the situation described the two proportions come from a single sample and thus are not independent.

Scoring

This question is scored in three sections. Section 1 consists of the mechanics of the confidence interval in part (a), section 2 consists of interpreting the confidence interval in part (a), and section 3 consists of parts (b) and (c).

Section 1 is scored as follows:

Essentially correct (E) if the response includes the following two components:

1. States the correct procedure by name or formula
2. Calculates the confidence interval

Partially correct (P) if the response includes only one of the two components.

Incorrect (I) if the response does not meet the criteria for E or P.

SCORING GUIDELINES

Question 5 (continued)

Notes:

- A formula with correct values is sufficient for component 2.
- Component 2 can never be satisfied if the response contains an incorrect formula.

Section 2 is scored as follows:

Essentially correct (E) if the response includes the following four components for the interval interpretation:

1. Estimates a proportion
2. Infers about the population
3. States 95 percent confidence
4. Includes context

Partially correct (P) if the response satisfies components 1 and 2 *AND* satisfies only one of components 3 or 4;

OR

if the response gives a correct interpretation of the confidence *level* in context without interpreting the specific interval.

Incorrect (I) if the response does not meet the criteria for E or P.

Notes:

- Any indication of an inference to the adults sampled rather than the population does not satisfy component 2 and is scored as I.
- Stating values that are unrealistic as proportions (including blanks) in the interpretation lower the score by one level (from E to P or from P to I).
- When both the interpretation of the interval and the level are given, only the interpretation of the interval is scored.

Section 3 is scored as follows:

Essentially correct (E) if the response includes the following two components:

1. Part (b) states that the condition implies the sampling distribution of $\hat{p}$ is approximately normal *OR* the normal approximation to the binomial distribution is appropriate.
2. Part (c) indicates the procedure is not appropriate because the two proportions come from single sample (dependent) rather than two (independent) samples.

Partially correct (P) if the response includes only one of the two components.

Incorrect (I) if the response does not meet the criteria for E or P.

Notes:

- Referring to the sampling distribution needing to be normal without explicitly stating $\hat{p}$ satisfies component 1.
- Referring to normal approximation must specify “to binomial” to satisfy component 1.
- Discussing the two sample interval versus the two proportion interval is extraneous information.

SCORING GUIDELINES**Question 5 (continued)**

- In part (c) component 2 can never be satisfied if a procedure that is not possible for one sample of categorical data is suggested.

4 Complete Response

Three parts essentially correct

3 Substantial Response

Two parts essentially correct and one part partially correct

2 Developing Response

Two parts essentially correct and no parts partially correct

OR

One part essentially correct and one or two parts partially correct

OR

Three parts partially correct

1 Minimal Response

One part essentially correct

OR

No parts essentially correct and two parts partially correct

SCORING GUIDELINES**Question****Intent of Question**

The primary goals of this question were to assess a student's ability to (1) use a scatterplot to comment on report about the relationship between two variables and interpret the slope for the least-squares regression line summarizing this relationship; (2) describe the relationship between two variables scatterplot when a categorical variable is introduced and compare a characteristic of the distribution variable for different categories of individuals in a scatterplot; and (3) describe how the associations between two variables for each category of individuals in a scatterplot differ from the overall association in the same scatterplot.

Solution**Part (a):**

The scatterplot supports the newspaper report about number of semesters needed to complete an academic program and starting salary because it shows a positive association between these two variables.

Part

The slope is 1.1594. For each additional semester needed to complete an academic program, predicted starting salary increases by €1,159.40.

Part (c):

For the business majors alone, there is a strong, negative, linear association between number of semesters and starting salary. Business majors who need a greater number of semesters to complete an academic program tend to have lower starting salaries.

Part (d):

Business majors have the lowest median starting salary at around €38,000, followed by physics majors at around €48,000, and then chemistry majors with the highest median starting salary at around €55,000.

Part (e):

The newspaper report should be modified to account for major. Overall, majors that take longer to complete tend to have higher starting salaries, with chemistry the highest, physics the next highest, and business the lowest. However, within a major, students who take a greater number of semesters tend to have lower starting salaries.

Scoring

This question is scored in three sections. Section 1 consists of parts (a) and (b), section 2 consists of parts (c) and (d), and section 3 consists of part (e). Sections 1, 2, and 3 are scored as essentially correct (E), partially correct (P), or incorrect (I).

SCORING GUIDELINES

Question 6 (continued)

Section 1 is scored as follows:

Essentially correct (E) if the response includes the following five components:

1. In part (a) the response addresses the positive association.
2. In part (a) the response uses the positive association to justify that the scatterplot support newspaper report.
3. In part (b) the response correctly identifies the numerical value of the slope from the computer output.
4. In part (b) the response interprets the slope as the change in starting salary for each additional semester, in context.
5. In part (b) the interpretation of slope includes nondeterministic language (e.g., “predicted starting salary” or equivalent) when interpreting the slope.

Partially correct (P) if the response includes three or four of the five components.

Incorrect (I) if the response does not meet the criteria for E or P.

Notes:

- In part (a) the response can use phrases such as “positive association (correlation, relationship),” “increasing relationship,” or describe a positive association (e.g., “the starting salaries are higher when there is a greater number of semesters”) to satisfy component 1. However, describing the relationship between only two points does not satisfy this component.
- In part (a) comments about linearity and strength should be ignored, even if the response implies these are required (e.g., “Yes, because the relationship is strong, positive, and linear.”).
- In part (a) responses that answer “no,” “maybe,” “kind of,” “somewhat,” or equivalent do not satisfy component 2.
- In part (a) a response that says “no, because the three clusters of points each have a negative association” does not satisfy component 1 or component 2.
- In part (a) no context is required, but variable names are required in part (b) to satisfy component 4.
- In part (b) a response that states the equation $\hat{y} = 34.018 + 1.1594x$ does not satisfy component 3 unless the slope is specifically identified or used in the interpretation.
- In part (b) a response that incorrectly identifies the numerical value of the slope can still satisfy components 4 and 5 using the incorrect value.
- In part (b) the 1-unit increase in number of semesters must be stated or implied to satisfy component 4 (e.g., for each semester, for every semester). A response that states or implies an unspecified number of semesters does not satisfy this component (for example, as semesters increase).
- In part (b) examples of nondeterministic language include “predicted starting salary,” “expected starting salary,” “estimated starting salary,” “typical starting salary,” “average starting salary,” “starting salary, on average,” “our model predicts,” and so on. However, “about,” “approximately,” and “according to the model” do not satisfy component 5.
- In part (b) no units are required for the change in predicted starting salary (which means OK to say 1.1594 euros, 1159.40, 1.1594, 1,159.40 dollars, and so on).
- In all parts it is acceptable if a response refers to salary rather than starting salary.

SCORING GUIDELINES

Question 6 (continued)

Section 2 is scored as follows:

Essentially correct (E) if the response includes the following five components:

1. In part (c) the response states that the association is negative.
2. In part (c) the response states that the association is strong *OR* linear *OR* both.
3. In part (c) the response refers to both variables (semesters, salary) in context.
4. In part (d) the response correctly compares the three majors.
5. In part (d) the response provides reasonable values for the median salaries or refers to “median starting salaries” when describing a characteristic of the graph.

Partially correct (P) if the response includes three or four of the five components.

Incorrect (I) if the response does not meet the criteria for E or P.

Notes:

- In part (c) the response can use phrases such as “negative association (correlation, relationship),” “decreasing relationship,” “inversely related,” or describe a negative association (for example, “the starting salaries are smaller when there is a greater number of semesters”) satisfy component 1. However, describing the relationship between only two points does not satisfy this component.
- In part (c) responses that describe the relationship incorrectly as weak or nonlinear do satisfy component 2, even if the other characteristic is described correctly.
- Only drawing a line on the scatterplot does not satisfy component 2.
- In part (d) a response that says “chemistry has the highest median starting salary and business has the lowest median starting salary” (or the equivalent) implies that physics is in the middle, and satisfies component 4.
- In part (d) if no values are provided for the medians, the response must use the phrase “median starting salary” at least once to satisfy component 5.
- In all parts it is acceptable if a response refers to salary rather than starting salary.

Section 3 is scored as follows:

Essentially correct (E) if the response states that there is a negative association for each of the majors *AND* the response notes the overall positive association.

Partially correct (P) if the response states that there is a negative association for each of the majors *BUT* does not note the overall positive association;

OR

if the response states that there is a negative association for one or two specific majors (for example, for business majors) *AND* the response notes the overall positive association.

Incorrect (I) if the response does not meet the criteria for E or P.

Notes:

- Additional ways to note the overall positive association include stating that the original report was correct, stating that majors that take longer to complete tend to have higher starting salaries, or the equivalent.

SCORING GUIDELINES**Question 6 (continued)**

- A response that does not explicitly name the majors or refer to the three majors but makes a general statement such as “However, for an academic major, there is a negative association” satisfies the requirement to state the negative association for each of the majors.
- A response that states the original report was wrong, or incorrect, or should be retracted, cannot satisfy the requirement to note the overall positive association. However, a response that says the original report might be misleading (or the equivalent) but still notes the overall positive association satisfies the requirement to note the overall positive association.
- In all parts it is acceptable if a response refers to salary rather than starting salary.

4 Complete Response

All three sections essentially correct

3 Substantial Response

Two sections essentially correct and one section partially correct

OR

Sections 1 and 2 partially correct and section 3 essentially correct

OR

Sections 1 and 3 essentially correct and section 2 incorrect

OR

Sections 2 and 3 essentially correct and section 1 incorrect

2 Developing Response

Sections 1 and 2 essentially correct and section 3 incorrect

OR

Section 1 essentially correct and sections 2 and 3 partially correct

OR

Sections 1 and 3 partially correct and section 2 essentially correct

OR

One section essentially correct and one section partially correct

OR

All three sections partially correct

1 Minimal Response

One section essentially correct and two sections incorrect

OR

Two sections partially correct and one section incorrect

OR

One section partially correct and two sections incorrect