

5.5 Sampling Distributions for Sample Proportions

Name: _____ Class: _____ Date: _____ Total: 9 marks

KEY WORDS sampling distribution 抽样分布

Objective

Build the skills to answer exam questions on **the sampling distribution of a sample proportion**.

You must be able to:

- use the mean $\mu_{\hat{p}} = p$
- use the standard deviation $\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$
- state the condition for it to be approximately **normal**

1 Worked examples

Study these first. Each one shows the method for a question type used later.

■ Centre and spread

For the sample proportion $\hat{p}$:

$$\mu_{\hat{p}} = p, \quad \sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}.$$

■ Normal condition

The distribution of $\hat{p}$ is approximately **normal** when $np \geq 10$ and $n(1-p) \geq 10$.

■ Example

$p = 0.5, n = 100$: $\sigma_{\hat{p}} = \sqrt{\frac{0.25}{100}} = 0.05$.

2 Practice

2.1 Write the mean of the sampling distribution of $\hat{p}$. [1]

2.2 For $p = 0.4$ and $n = 100$, find $\sigma_{\hat{p}}$. [2]

2.3 State the condition for the distribution of $\hat{p}$ to be approximately normal. [1]

3 Exam-style questions

3.1 The mean of the sampling distribution of $\hat{p}$ is [1]

- | | |
|---|-----------|
| A | $\hat{p}$ |
| B | p |
| C | n |
| D | 0 |

3.2 The standard deviation of $\hat{p}$ is [1]

- | | |
|---|---------------------------|
| A | $\frac{p(1-p)}{n}$ |
| B | $\sqrt{\frac{p(1-p)}{n}}$ |
| C | $\sqrt{np}$ |
| D | $\frac{p}{n}$ |

3.3 A population has $p = 0.2$; samples of size $n = 64$ are taken.

(a) Find $\mu_{\hat{p}}$. [1]

(b) Find $\sigma_{\hat{p}}$. [2]