AP STATISTICS • EXERCISE SHEET

4.9 Combining Random Variables

Name: _____ Class: _____ Date: _____ **Total: 9 marks**

KEY WORDS random variable 随机变量 • random 随机 • independent 独立

Objective

Build the skills to answer exam questions on **combining random variables**.

You must be able to:

- find the mean of $aX + bY$ as $a\mu_X + b\mu_Y$
- find the variance of $aX + bY$ for **independent** X, Y as $a^2\sigma_X^2 + b^2\sigma_Y^2$
- describe a **linear transformation** $Y = a + bX$: $\mu_Y = a + b\mu_X$, $\sigma_Y = |b|\sigma_X$

1 Worked examples

Study these first. Each one shows the method for a question type used later.

■ **Means add**

$$\mu_{aX+bY} = a\mu_X + b\mu_Y \quad \text{(always)}.$$

■ **Variances add (if independent)**

$$\sigma^2_{aX+bY} = a^2\sigma_X^2 + b^2\sigma_Y^2 \quad \text{(independent)}.$$

Note that **variances**, not standard deviations, add.

■ **Linear transformation**

For $Y = a + bX$: $\mu_Y = a + b\mu_X$ and $\sigma_Y = |b|\sigma_X$ (adding a constant does not change the spread).

2 Practice

2.1 State the mean of $X + Y$ in terms of μ_X and μ_Y. [1]

2.2 If $\mu_X = 5$ and $\mu_Y = 3$, find the mean of $X + Y$. [1]

2.3 For $Y = 2X$ with $\sigma_X = 4$, find σ_Y. [2]

3 Exam-style questions

3.1 The mean of $X + Y$ is [1]

A B
C D

A $\mu_X \cdot \mu_Y$
B $\mu_X + \mu_Y$
C $\mu_X - \mu_Y$
D μ_X / μ_Y

3.2 For **independent** X and Y, the variance of $X + Y$ is [1]

A B
C D

A $\sigma_X + \sigma_Y$
B $\sigma_X^2 + \sigma_Y^2$
C $\sigma_X \cdot \sigma_Y$
D $\sigma_X^2 - \sigma_Y^2$

3.3 X has $\mu_X = 10$ and $\sigma_X = 2$. Let $Y = 3X$.

(a) Find μ_Y. [1]

(b) Find σ_Y. [2]