

8.5 Setting Up a Chi-Square Test for Homogeneity or Independence

Name: _____ Class: _____ Date: _____ **Total: 10 marks**

KEY WORDS independence 独立性 • homogeneity 同质性 • chi-square 卡方
• degrees of freedom 自由度

Objective

Build the skills to answer exam questions on **setting up a chi-square test for homogeneity or independence**.

You must be able to:

- distinguish a test of **homogeneity** 同质性 from a test of **independence** 独立性
- calculate the degrees of freedom $(r - 1)(c - 1)$

1 Worked examples

Study these first. Each one shows the method for a question type used later.

■ Homogeneity vs independence

- **Homogeneity** — compares the distribution of one categorical variable across **several groups** (several samples).
- **Independence** — tests whether **two** categorical variables are associated within **one** population (one sample).

Both use the same statistic $\chi^2 = \sum \frac{(O - E)^2}{E}$ and the same degrees of freedom:

$$\text{df} = (r - 1)(c - 1).$$

■ Which one, and its df

A 3×4 table (3 rows, 4 columns) has $\text{df} = (3 - 1)(4 - 1) = 6$. If the rows are **separate samples** (say, three schools), it is a test of **homogeneity**; if the table comes from **one sample** cross-classified by two variables, it is a test of **independence**.

2 Practice

2.1 State the difference between a test of homogeneity and a test of independence. [2]

2.2 Write the degrees of freedom for an $r \times c$ table. [1]

2.3 For a 3×4 table, find the degrees of freedom. [2]

3 Exam-style questions

3.1 The degrees of freedom for a two-way chi-square with r rows and c columns is [1]

- **A** rc
- **B** $(r - 1)(c - 1)$
- **C** $r + c$
- **D** $rc - 1$

3.2 A test of whether two categorical variables are associated in one population is a test of [1]

- **A** homogeneity
- **B** independence
- **C** goodness of fit
- **D** means

3.3 A 2×3 table is analysed.

(a) Find the degrees of freedom.

[2]

(b) Name the test that compares distributions across several groups.

[1]

Solutions

2.1 homogeneity compares one variable's distribution across several groups; independence tests whether two variables are associated in one population.

2.2 $(r - 1)(c - 1)$.

2.3 $(3 - 1)(4 - 1) = 2 \times 3 = 6$.

3.1 B.

3.2 B.

3.3 (a) $(2 - 1)(3 - 1) = 2$. (b) a test of homogeneity.