

7.8 Setting Up a Test for the Difference of Two Population Means

Name: _____ Class: _____ Date: _____

Total: 10 marks

Objective

Build the skills to answer exam questions on **setting up a test for a difference of two means**.

You must be able to:

- state the hypotheses $H_0 : \mu_1 = \mu_2$ and H_a
- write the test statistic $t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$

1 Worked examples

Study these first. Each one shows the method for a question type used later.

■ Hypotheses

- Null** $H_0 : \mu_1 = \mu_2$ (no difference).
- Alternative** $H_a : \mu_1 \neq \mu_2$ (or one-sided).

■ Test statistic

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

The standard error again adds the two **variances** (s^2/n).

2 Practice

2.1 State the null hypothesis for a two-sample mean test.

[1]

2.2 Write the test-statistic formula for a difference of means.

[1]

2.3 For $\bar{x}_1 = 50$, $s_1 = 6$, $n_1 = 36$, $\bar{x}_2 = 45$, $s_2 = 8$, $n_2 = 64$, find the standard error $\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$.

[2]

3 Exam-style questions

3.1 The null hypothesis for a two-sample mean test is

[1]

- A** $\mu_1 \neq \mu_2$
- B** $\mu_1 = \mu_2$
- C** $\mu_1 = 0$
- D** $\bar{x}_1 = \bar{x}_2$

3.2 The test statistic for a difference of means uses the standard error

[1]

- A** $\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$
- B** $s_1 + s_2$
- C** $s_1 \cdot s_2$
- D** $\sqrt{np}$

3.3 $\bar{x}_1 = 70$, $s_1 = 10$, $n_1 = 25$, $\bar{x}_2 = 65$, $s_2 = 10$, $n_2 = 25$.

(a) Find the standard error.

[2]

(b) Compute the test statistic t .

[2]

Solutions

2.1 $H_0 : \mu_1 = \mu_2.$

2.2 $t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{s_1^2/n_1 + s_2^2/n_2}}.$

2.3 $\sqrt{\frac{36}{36} + \frac{64}{64}} = \sqrt{1+1} = \sqrt{2} \approx 1.41.$

3.1 B.

3.2 A.

3.3 (a) $\sqrt{\frac{100}{25} + \frac{100}{25}} = \sqrt{8} \approx 2.83.$ (b) $t = \frac{70 - 65}{2.83} \approx 1.77.$