

7.5 Carrying Out a Test for a Population Mean

Name: _____ Class: _____ Date: _____

Total: 9 marks

KEY WORDS distribution 分布

Objective

Build the skills to answer exam questions on **carrying out a test for a population mean**.

You must be able to:

- compute the t test statistic and find the **p-value** from the t-distribution
- compare the p-value to α and reach a decision
- state a conclusion in context

1 Worked examples

Study these first. Each one shows the method for a question type used later.

■ The procedure

1. State $H_0 : \mu = \mu_0$ and H_a ; check conditions.
2. Compute $t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$ with $\text{df} = n - 1$.
3. Find the **p-value** from the **t-distribution**.
4. Compare to α : reject H_0 if p-value $< \alpha$, else fail to reject.
5. State the conclusion **in context**.

■ A worked test

Test $H_0 : \mu = 50$ against $H_a : \mu \neq 50$ with $\bar{x} = 52$, $s = 4$, $n = 16$. Then

$$t = \frac{52 - 50}{4/\sqrt{16}} = \frac{2}{1} = 2.0, \quad \text{df} = 15.$$

The two-sided p-value $\approx 0.06 > 0.05$, so we fail to reject H_0 .

2 Practice

2.1 State the steps to carry out a t-test for a mean. [2]

2.2 State the distribution that gives the p-value. [1]

2.3 A t-test gives a p-value of 0.04 with $\alpha = 0.05$. State the decision. [1]

3 Exam-style questions

3.1 The p-value for a mean test comes from the [1]

- A normal distribution
- B t-distribution
- C binomial distribution
- D χ^2 distribution

3.2 A t-test rejects H_0 when the p-value is [1]

- A above α
- B below α
- C exactly zero
- D exactly one

3.3 A t-test of $\mu > 100$ gives a p-value of 0.02 at $\alpha = 0.05$.

- (a) State the decision. [1]
(b) State whether the result is significant. [1]
(c) State the conclusion in context. [1]

Solutions

2.1 state hypotheses and check conditions; compute t ; find the p-value; compare to α and conclude.

2.2 the t-distribution.

2.3 reject H_0 ($0.04 < 0.05$).

3.1 B.

3.2 B.

3.3 (a) reject H_0 . (b) yes. (c) there is convincing evidence that $\mu > 100$.