

7.1 Introducing Statistics: Should I Worry About Error?

Name: _____ Class: _____ Date: _____ Total: 8 marks

KEY WORDS distribution 分布 • degrees of freedom 自由度

Objective

Build the skills to answer exam questions on **inference for a mean and the t-distribution**.

You must be able to:

- explain why inference for a mean uses the **t-distribution** t 分布 when σ is unknown
- state that the sample standard deviation s estimates σ
- describe how the t-distribution approaches the normal as n grows

1 Worked examples

Study these first. Each one shows the method for a question type used later.



■ Why t, not z

When the population standard deviation σ is **unknown** (almost always), we estimate it with the **sample** standard deviation s . This extra uncertainty means we use the **t-distribution** instead of the normal.

■ The t-distribution

It is symmetric but has **heavier tails** than the normal, with **degrees of freedom** $= n - 1$. As n grows, it approaches the normal distribution.

2 Practice

2.1 State why we use the t-distribution instead of z for means. [1]

2.2 State what replaces σ when it is unknown. [1]

2.3 State what happens to the t-distribution as n increases. [1]

3 Exam-style questions

3.1 When the population σ is unknown, inference for a mean uses the [1]

- A z-distribution
- B t-distribution
- C binomial distribution
- D uniform distribution

3.2 The t-distribution is [1]

- A narrower than the normal
- B wider than the normal (heavier tails)
- C always identical to the normal
- D skewed

3.3 A sample of size 20 estimates a mean, with σ unknown.

(a) Name the distribution used. [1]

(b) State what estimates σ . [1]

(c) State the degrees of freedom. [1]

Solutions

2.1 because σ is unknown and estimated by s , adding uncertainty.

2.2 the sample standard deviation s .

2.3 it approaches the normal distribution.

3.1 B.

3.2 B.

3.3 (a) the t-distribution. (b) the sample standard deviation s . (c) $n - 1 = 19$.