

6.8 Confidence Intervals for the Difference of Two Proportions

Name: _____ Class: _____ Date: _____ Total: 8 marks

KEY WORDS confidence interval 置信区间

Objective

Build the skills to answer exam questions on a **confidence interval for the difference of two proportions**.

You must be able to:

- use the point estimate $\hat{p}_1 - \hat{p}_2$
- build the interval $(\hat{p}_1 - \hat{p}_2) \pm z^* \sqrt{\frac{\hat{p}_1(1 - \hat{p}_1)}{n_1} + \frac{\hat{p}_2(1 - \hat{p}_2)}{n_2}}$
- use the interval to judge whether the two proportions differ

1 Worked examples

Study these first. Each one shows the method for a question type used later.

■ The two-proportion interval

$$(\hat{p}_1 - \hat{p}_2) \pm z^* \sqrt{\frac{\hat{p}_1(1 - \hat{p}_1)}{n_1} + \frac{\hat{p}_2(1 - \hat{p}_2)}{n_2}}.$$

■ Judging a difference

If the interval **contains** 0, the two proportions may not differ (no significant difference). If it is **entirely above** (or **below**) 0, there is a significant difference.

2 Practice

2.1 Write the point estimate for the difference of two proportions. [1]

2.2 State how to judge whether the two proportions differ using the interval. [1]

2.3 For $\hat{p}_1 = 0.6$ and $\hat{p}_2 = 0.5$, find the point estimate of the difference. [1]

3 Exam-style questions

3.1 A confidence interval for $p_1 - p_2$ is centred on [1]

- A $\hat{p}_1 \cdot \hat{p}_2$
- B $\hat{p}_1 - \hat{p}_2$
- C 0
- D $\hat{p}_1 + \hat{p}_2$

3.2 If a 95% CI for $p_1 - p_2$ contains 0, the two proportions [1]

- A definitely differ
- B may not differ (no significant difference)
- C are exactly equal
- D are both zero

3.3 A 95% CI for $p_1 - p_2$ is (0.02, 0.10).

(a) State which proportion is larger. [1]

(b) State whether 0 is in the interval. [1]

(c) State whether there is a significant difference. [1]

Solutions

2.1 $\hat{p}_1 - \hat{p}_2$.

2.2 if the interval contains 0, there is no significant difference; if not, there is.

2.3 $0.6 - 0.5 = 0.1$.

3.1 B.

3.2 B.

3.3 (a) p_1 (the interval is positive). (b) no. (c) yes.