

6.4 Setting Up a Test for a Population Proportion

Name: _____ Class: _____ Date: _____ Total: 9 marks

KEY WORDS null 零假设 • test statistic 检验统计量 • null hypothesis 原假设
• alternative hypotheses 备择假设

Objective

Build the skills to answer exam questions on **setting up a test for a population proportion**.

You must be able to:

- state the **null** 零假设 and **alternative hypotheses** 备择假设
- write the test statistic $z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$
- state the conditions for the test

1 Worked examples

Study these first. Each one shows the method for a question type used later.

■ Hypotheses

- Null** $H_0 : p = p_0$ (a specific value — “no effect”).
- Alternative** $H_a : p \neq p_0$ (or $< p_0$, or $> p_0$).

■ Test statistic

$$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$$

The standard error uses the **null value** p_0 . Conditions: **random, independent** (10% rule), and **large counts** ($np_0 \geq 10$, $n(1-p_0) \geq 10$).

2 Practice

2.1 State the null hypothesis for a test about a proportion. [1]

2.2 Write the test-statistic formula for a proportion. [1]

2.3 State one condition needed to run the test. [1]

3 Exam-style questions

3.1 The null hypothesis H_0 usually states that [1]

- A** there is an effect
- B** there is no effect (a specific value)
- C** the sample is biased
- D** nothing at all

3.2 The test statistic for a proportion computes the standard error using [1]

- A** $\hat{p}$
- B** p_0 (the null value)
- C** zero
- D** the sample size only

3.3 A test checks whether $p \neq 0.5$, using $\hat{p} = 0.6$ and $n = 100$.

(a) Write H_0 . [1]

(b) Write H_a . [1]

(c) Compute the test statistic z . [2]

Solutions

2.1 $H_0 : p = p_0$ (a specific value).

$$\mathbf{2.2} \ z = \frac{\hat{p} - p_0}{\sqrt{p_0(1 - p_0)/n}}.$$

2.3 any one of: random sample, independence (10% rule), large counts.

3.1 B.

3.2 B.

$$\mathbf{3.3} \text{ (a) } H_0 : p = 0.5. \text{ (b) } H_a : p \neq 0.5. \text{ (c) } z = \frac{0.6 - 0.5}{\sqrt{0.5(0.5)/100}} = \frac{0.1}{0.05} = 2.0.$$