AP STATISTICS • EXERCISE SHEET

5.5 Sampling Distributions for Sample Proportions

Name: _____ Class: _____ Date: _____ **Total: 9 marks**

KEY WORDS sampling distribution 抽样分布

Objective

Build the skills to answer exam questions on **the sampling distribution of a sample proportion**.

You must be able to:

- use the mean $\mu_{\hat{p}} = p$
- use the standard deviation $\sigma_{\hat{p}} = \sqrt{\dfrac{p(1-p)}{n}}$
- state the condition for it to be approximately **normal**

1 Worked examples

Study these first. Each one shows the method for a question type used later.

■ **Centre and spread**

For the sample proportion $\hat{p}$:

$$\mu_{\hat{p}} = p, \qquad \sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}.$$

■ **Normal condition**

The distribution of $\hat{p}$ is approximately **normal** when $np \geq 10$ and $n(1-p) \geq 10$.

■ **Example**

$p = 0.5$, $n = 100$: $\sigma_{\hat{p}} = \sqrt{\dfrac{0.25}{100}} = 0.05$.

2 Practice

2.1 Write the mean of the sampling distribution of $\hat{p}$. [1]

2.2 For $p = 0.4$ and $n = 100$, find $\sigma_{\hat{p}}$. [2]

2.3 State the condition for the distribution of $\hat{p}$ to be approximately normal. [1]

3 Exam-style questions

3.1 The mean of the sampling distribution of $\hat{p}$ is [1]

- **A** $\hat{p}$
- **B** p
- **C** n
- **D** 0

3.2 The standard deviation of $\hat{p}$ is [1]

- **A** $\dfrac{p(1-p)}{n}$
- **B** $\sqrt{\dfrac{p(1-p)}{n}}$
- **C** $\sqrt{np}$
- **D** $\dfrac{p}{n}$

3.3 A population has $p = 0.2$; samples of size $n = 64$ are taken.

(a) Find $\mu_{\hat{p}}$. [1]

(b) Find $\sigma_{\hat{p}}$. [2]

Solutions

2.1 $\mu_{\hat{p}} = p$.

2.2 $\sigma_{\hat{p}} = \sqrt{\dfrac{0.4 \times 0.6}{100}} = \sqrt{0.0024} = 0.049$.

2.3 $np \geq 10$ and $n(1-p) \geq 10$.

3.1 **B**.

3.2 **B**.

3.3 (a) 0.2. (b) $\sigma_{\hat{p}} = \sqrt{\dfrac{0.2 \times 0.8}{64}} = \sqrt{0.0025} = 0.05$.