

4.9 Combining Random Variables

Name: _____ Class: _____ Date: _____ **Total: 9 marks**

KEY WORDS random variable 随机变量 • random 随机 • independent 独立

Objective

Build the skills to answer exam questions on **combining random variables**.

You must be able to:

- find the mean of $aX + bY$ as $a\mu_X + b\mu_Y$
- find the variance of $aX + bY$ for **independent** X, Y as $a^2\sigma_X^2 + b^2\sigma_Y^2$
- describe a **linear transformation** $Y = a + bX$: $\mu_Y = a + b\mu_X$, $\sigma_Y = |b|\sigma_X$

1 Worked examples

Study these first. Each one shows the method for a question type used later.

■ Means add

$$\mu_{aX+bY} = a\mu_X + b\mu_Y \quad (\text{always}).$$

■ Variances add (if independent)

$$\sigma_{aX+bY}^2 = a^2\sigma_X^2 + b^2\sigma_Y^2 \quad (\text{independent}).$$

Note that **variances**, not standard deviations, add.

■ Linear transformation

For $Y = a + bX$: $\mu_Y = a + b\mu_X$ and $\sigma_Y = |b|\sigma_X$ (adding a constant does not change the spread).

2 Practice

2.1 State the mean of $X + Y$ in terms of μ_X and μ_Y . [1]

2.2 If $\mu_X = 5$ and $\mu_Y = 3$, find the mean of $X + Y$. [1]

2.3 For $Y = 2X$ with $\sigma_X = 4$, find σ_Y . [2]

3 Exam-style questions

3.1 The mean of $X + Y$ is [1]

- **A** $\mu_X \cdot \mu_Y$
- **B** $\mu_X + \mu_Y$
- **C** $\mu_X - \mu_Y$
- **D** μ_X / μ_Y

3.2 For **independent** X and Y , the variance of $X + Y$ is [1]

- **A** $\sigma_X + \sigma_Y$
- **B** $\sigma_X^2 + \sigma_Y^2$
- **C** $\sigma_X \cdot \sigma_Y$
- **D** $\sigma_X^2 - \sigma_Y^2$

3.3 X has $\mu_X = 10$ and $\sigma_X = 2$. Let $Y = 3X$.

(a) Find μ_Y . [1]

(b) Find σ_Y . [2]

Solutions

2.1 $\mu_X + \mu_Y$.

2.2 $5 + 3 = 8$.

2.3 $\sigma_Y = |2|\sigma_X = 2 \times 4 = 8$.

3.1 B.

3.2 B.

3.3 (a) $\mu_Y = 3 \times 10 = 30$. (b) $\sigma_Y = |3| \times 2 = 6$.