

4.6 Independent Events and Unions of Events

Name: _____ Class: _____ Date: _____ Total: 9 marks

KEY WORDS independent 独立 • addition rule 加法法则 • probability 概率

Objective

Build the skills to answer exam questions on **independent events and unions of events**.

You must be able to:

- describe **independence** 独立: $P(A | B) = P(A)$, so $P(A \cap B) = P(A) \cdot P(B)$
- apply the **addition rule** 加法法则, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

1 Worked examples

Study these first. Each one shows the method for a question type used later.

■ Independence

A and B are **independent** if knowing that one occurred does not change the probability of the other: $P(A | B) = P(A)$. Then

$$P(A \cap B) = P(A) \cdot P(B).$$

■ The addition rule

For the **union** (A or B or both):

$$P(A \cup B) = P(A) + P(B) - P(A \cap B).$$

2 Practice

2.1 State the condition for A and B to be independent. [1]

2.2 For independent events with $P(A) = 0.5$ and $P(B) = 0.4$, find $P(A \cap B)$. [2]

2.3 Write the addition rule for $P(A \cup B)$. [1]

3 Exam-style questions

3.1 For independent events, $P(A \cap B) =$ [1]

- A $P(A) + P(B)$
- B $P(A) \cdot P(B)$
- C 0
- D $P(A | B)$

3.2 The addition rule is $P(A \cup B) =$ [1]

- A $P(A) + P(B)$
- B $P(A) + P(B) - P(A \cap B)$
- C $P(A) \cdot P(B)$
- D $P(A) - P(B)$

3.3 $P(A) = 0.6$, $P(B) = 0.3$, and $P(A \cap B) = 0.2$.

(a) Find $P(A \cup B)$. [2]

(b) State whether A and B are independent (check $P(A \cap B) = P(A) \cdot P(B)$). [1]

Solutions

2.1 knowing that one has occurred does not change the probability of the other.

2.2 $P(A \cap B) = 0.5 \times 0.4 = 0.2$.

2.3 $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

3.1 B.

3.2 B.

3.3 (a) $0.6 + 0.3 - 0.2 = 0.7$. (b) no — $P(A) \cdot P(B) = 0.18 \neq 0.2$.