

4.10 Introduction to the Binomial Distribution

Name: _____ Class: _____ Date: _____ **Total: 11 marks**

KEY WORDS binomial 二项 • random 随机 • probability 概率 • independent 独立
• random variable 随机变量

Objective

Build the skills to answer exam questions on **the binomial distribution**.

You must be able to:

- recognise a **binomial** 二项 setting: n independent trials, each a success or failure, with fixed p
- calculate $P(X = x) = \binom{n}{x} p^x (1 - p)^{n-x}$

1 Worked examples

Study these first. Each one shows the method for a question type used later.

■ The binomial setting

A binomial random variable X counts the number of **successes** in n trials that are:

- **fixed** in number (n),
- **two-outcome** (success/failure),
- **independent**, with
- a **constant** probability of success p .

■ The binomial formula

$$P(X = x) = \binom{n}{x} p^x (1 - p)^{n-x}.$$

For $n = 3$, $p = 0.5$, $x = 2$: $\binom{3}{2} (0.5)^2 (0.5)^1 = 3 \times 0.125 = 0.375$.

2 Practice

2.1 State the four conditions of a binomial setting. [2]

2.2 Write the binomial probability formula. [1]

2.3 For $n = 4$, $p = 0.5$, find $P(X = 4)$. [2]

3 Exam-style questions

3.1 A binomial random variable counts the number of [1]

- **A** trials
- **B** successes in n trials
- **C** failures only
- **D** outcomes

3.2 In a binomial setting, the probability of success p is [1]

- **A** changing each trial
- **B** constant across trials
- **C** zero
- **D** exactly 1

3.3 A fair coin is flipped 3 times; X = number of heads.

(a) State n and p .

[1]

(b) Find $P(X = 0)$.

[2]

(c) Name the distribution.

[1]

Solutions

2.1 fixed number of trials; two outcomes; independent trials; constant p .

2.2 $P(X = x) = \binom{n}{x} p^x (1 - p)^{n-x}$.

2.3 $P(X = 4) = \binom{4}{4} (0.5)^4 = (0.5)^4 = 0.0625$.

3.1 B.

3.2 B.

3.3 (a) $n = 3$, $p = 0.5$. (b) $P(X = 0) = (0.5)^3 = 0.125$. (c) binomial.